Catastrophe Bond Pricing based on Behaviors’ Model

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What is Catastrophe bond

- Catastrophe bond (CAT Bond) is a corporate bond that requires investors to forgive some or all principal or interest in the event that catastrophe losses surpass the trigger specified in the bond.

- Issuer
  - SPV(Special Purpose Vehicle established by Insurer, Reinsurer)

- Bond holder
  - Fund, insurers, individual

- Aim
  - risk transfer:
What is CAT bond
Catastrophe Bond Pricing Problems

• Catastrophes are rare events
  – Impossible to predict potential losses
  – Catastrophic event have spatial and temporal characteristics
  – Historical losses cannot be used for prediction
  – Losses depend on land use decisions, economic growth

• Volume problem
  • How many bond need to be issued
Model Structure

Huge scale of issue volume and coupon rate combination → Constraints → Accepted combination of issue volume and coupon rate
Constraints

1. Issuer Survival factor: Insurer bankruptcy probability
   
   $P(R(\tau) < 0) < \alpha$

   Where
   
   ◦ $R(\tau)$ is the risk reserve when catastrophe occur

2. Existing factor: Bond exists constraints (individual serious loss probability)
   
   $P(IL(\tau) > \bar{L}) < \beta$

   Where
   
   ◦ $IL(\tau)$ is the losses that individual suffer when catastrophe occur
Some related equations

- Calculation of SPV’s payment to insurer once catastrophe occur at time \( t \)
  \[
  B(t) = n \cdot B_0 \cdot [a \cdot (1 + r_f)^{t-T} + b \cdot r_c \cdot \sum_{i=0}^{T-t} (1 + r_f)^{-i}] 
  \]

- Where
  - \( n \) is the issue volume of Catastrophe bond
  - \( r_c \) is the coupon rate of Catastrophe bond
  - \( r_f \) is risk free rate, and
  - \( B_0 \) is the face value of Catastrophe bond
  - \( B(t) \) is the payment the insurer get from the Cat bond once catastrophe occur at time \( t \)
  - \( a \) is the part that once the catastrophe occur, the portion of principal the bond holder will loss
  - \( b \) is the part that once the catastrophe occur, the portion of coupon the bond holder will loss
Some related equations

- The function to calculate Individual losses

\[ IL(t) = \begin{cases} 
L(t) \cdot (1 - \varphi) & \text{when } R(t) \geq 0 \\
L(t) \cdot (1 - \varphi) + R(t) & \text{when } R(t) < 0 
\end{cases} \]

- Where
  - \( \varphi \) is the insurance coverage rate
  - \( L(\tau) \) is the loss occur at time \( \tau \)
  - \( R(t) \) is the risk reserve at time \( t \)
Some related equations

- Calculation of risk reserve

\[
R(\tau) = \text{Capital} \cdot (1+r_f)^\tau + \pi \cdot \sum_{i=1}^{\tau} (1+r_f)^i - n \cdot r_c \cdot B_0 \cdot \sum_{i=1}^{\tau} (1+r_f)^{i-1} + B(\tau) - L(\tau) \cdot \varphi
\]

- Where
  - \( \tau \) is the time the first time catastrophe occur
  - \( r_f \) is risk free rate
  - Capital is the original capital in the first beginning.
  - \( \pi \) is the catastrophe insurance premium income
  - \( n \) is the issue volume of Catastrophe bond
  - \( r_c \) is the coupon rate of Catastrophe bond
  - \( B_0 \) is the face value of Catastrophe bond
  - \( B(\tau) \) is the payment the insurer get from the Cat bond once catastrophe occur at time
  - \( \varphi \) is the insurance coverage rate
  - \( L(\tau) \) is the loss occur at time
Simulation Result

Loss: Weibull Distribution
Application on Typhoon Risk in China

Input:
- Issue volume: 100 billion to 10 trillion RMB
- Coupon rate: 2% to 30% (annually)
- Loss: Typhoon loss in China since 1980 to 2005 adjusted by GDP

Result
- 197 accepted combination of issue volume and coupon rate
- Obviously negative relationship between issue volume and coupon rate
Application on Typhoon Risk in China

All the risk reserve is positive
Moral hazard exist
Adjusted model

- Moral hazard constraint

\[ P(B(\tau) > L(\tau)\cdot\varphi) < \gamma \]

- Where
  - \( B(\tau) \) is the payment the insurer gets from the CAT bond when catastrophe occurs
  - \( L(\tau) \) is the catastrophe loss when catastrophe occurs
Problems and future work

- Model optimization
- Application on more risks
- Market reaction research