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Complex Systems Modeling

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A Method for Obtaining User Preferences in Multicriteria Problems with Hierarchy

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Agenda

- Problem
- Assumptions
- Approach
- Nucleolar solution concept
- Algorithm
- Cumulating derived information
- Conclusions

Problem

- Elicitation of preferences
- Numerous criteria
- Structure (tree) of criteria
- Large number of alternatives

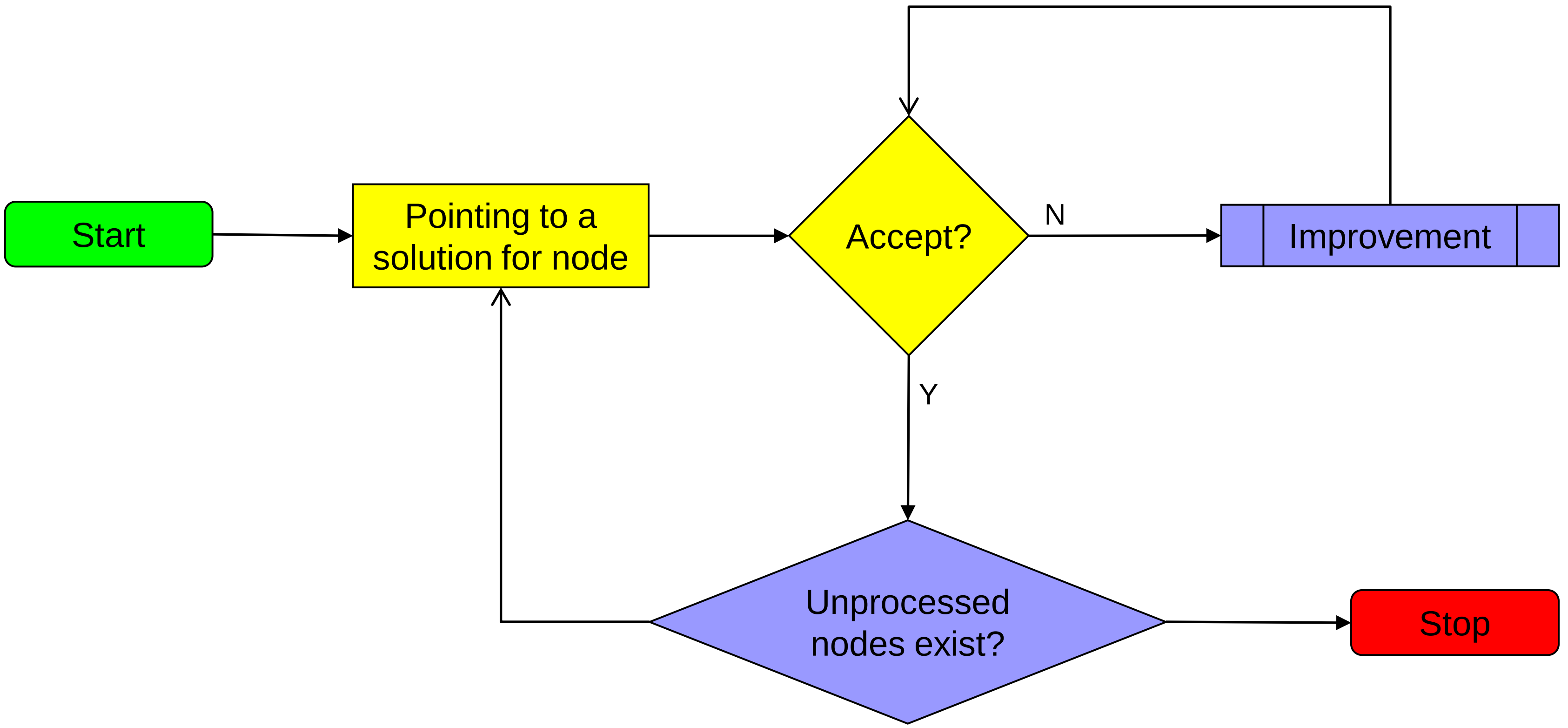
Assumptions (1 of 2)

- Cooperation of user and the method underlying software generating user interface
- Single-criteria optimization on the lowest-level of criteria (indicators' level)
- Narrowing user focus to only a few outcome vectors presented in the node
- Nucleolar optimization in every node (criterion)
- Hints for improvement is assessed from user

Assumptions (2 of 2)

- Transformation of user and his/her choices of most prefered solution from the set of achieved undomminated solutions (based on the hints) to importance coefficients
- Analysis modes
  - Browse
    - Bottom-up (from the indicators' level to the zero-level criterion)
    - Selective (in areas of user interest)
  - Nodes
    - Indicators
    - **Non-agregated** criteria
- Failure of the linear weighting aggregations (AHP etc.)

Approach (1)
Node Process



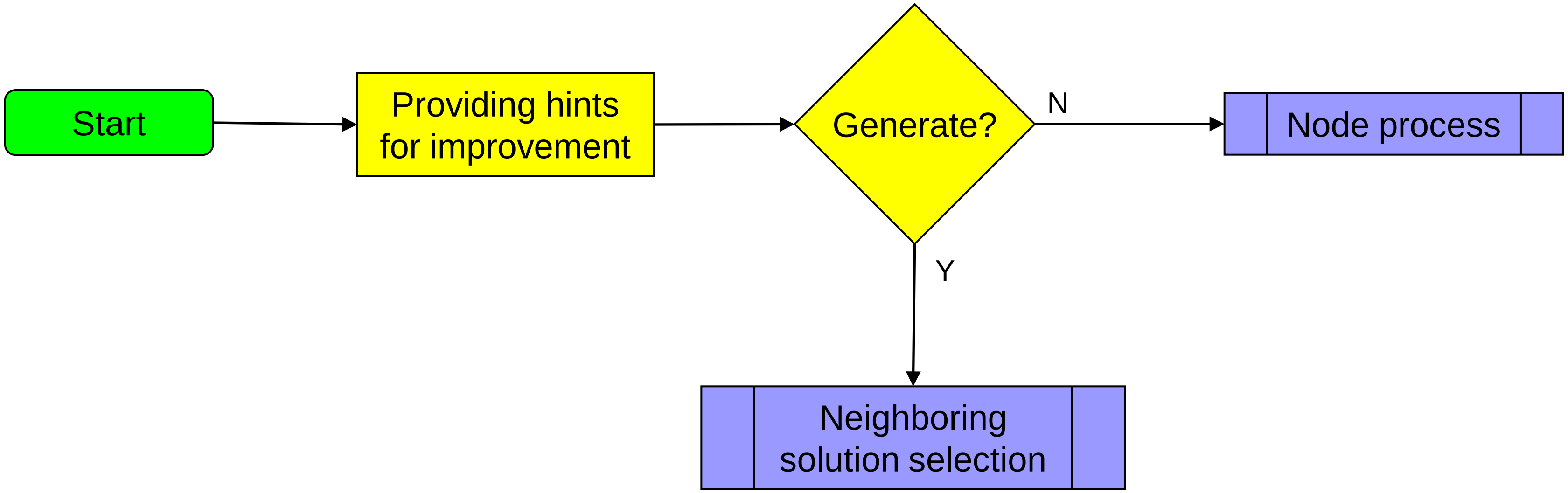
GUI (1)

Pointing to a solution for node (1)

Accept Improve

	Solution	
	Value	**...**
Indicator 1		
Indicator 2		
...	...	...
Indicator N		

Approach (2)
Improvement



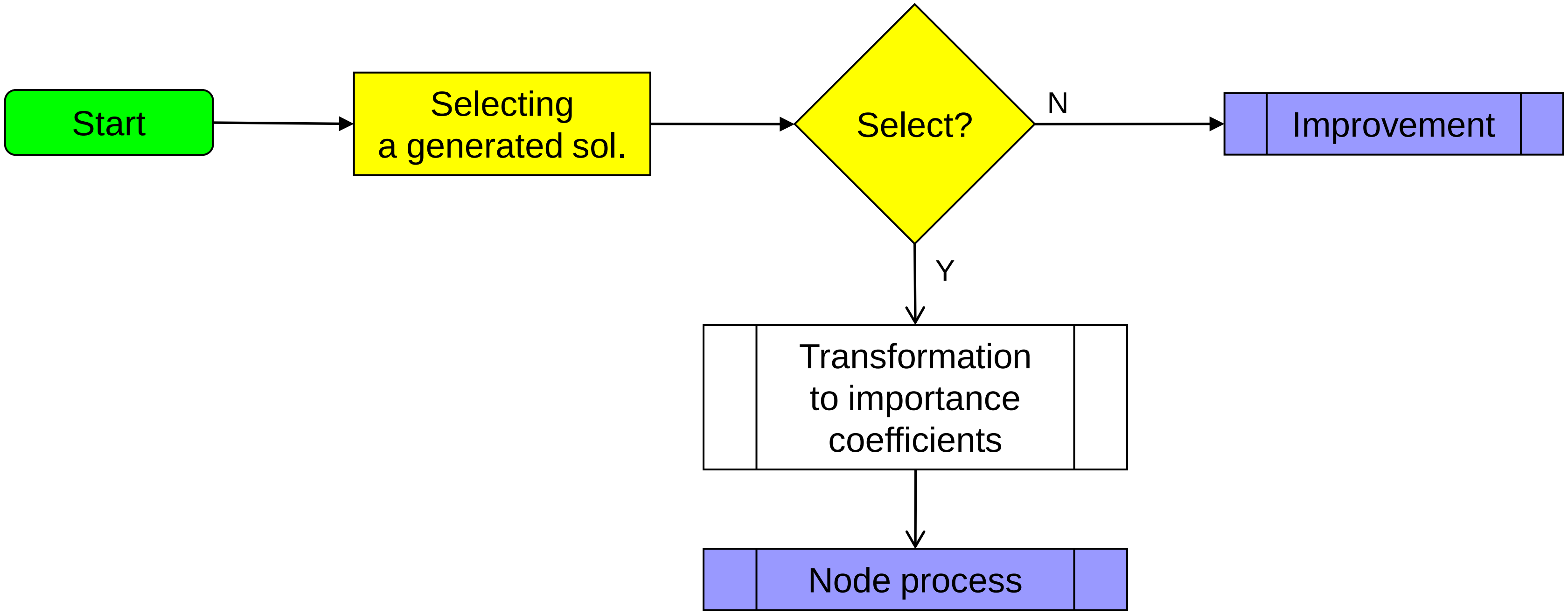
GUI (2)

Providing hints for improvement

Cancel improvement | Choose

I	~I	~D	D		Solution	
					Value	...
○	○	○	○	Indicator 1		
●	○	○	○	Indicator 2		
…	…	…	…	...	…	…
○	○	○	○	Indicator N		

Approach (3)
Neighboring solution selection



GUI (3)
Selecting a generated solution

Cancel Select

							...			
	Solution		Generated 1		Generated 2		...		Generated N	
	Value	...	Value	...	Value	...	...	...	Value	...
Indicator 1							...	...		
Indicator 2							...	...		
...	...	...	...	...	...	...	...	...	...	...
Indicator N							...	...		

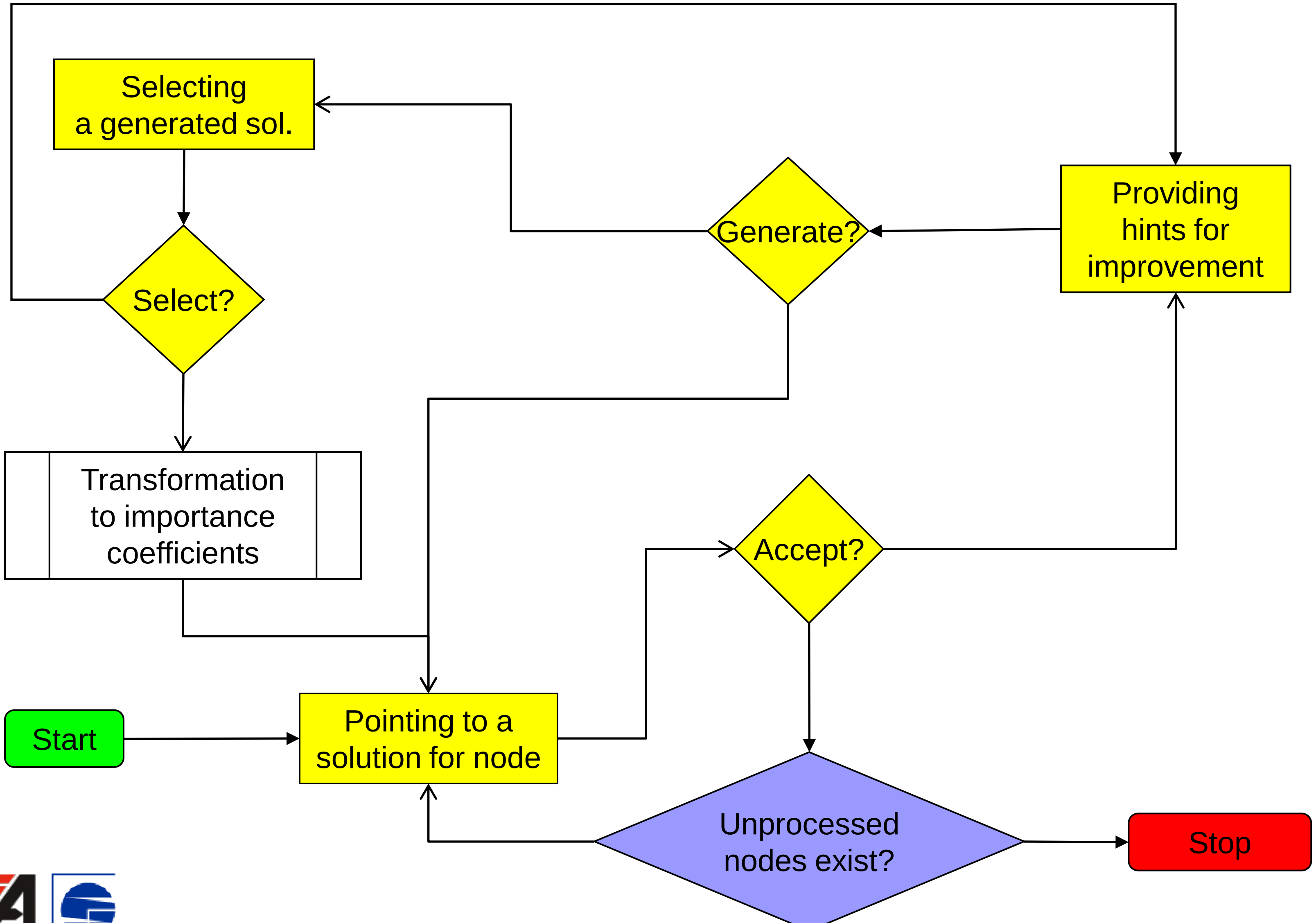
GUI (4)

Pointing to a solution for node (2)

Accept Improve

	Solution 1		**Solution 2**	
	Value	**...**	**Value**	**...**
Indicator 1				
Indicator 2				
...	...	...	...	...
Indicator N				

Approach (overview) (4)



Nucleolar solution concept

- Let us consider
  - set of solutions **D** (for example set of alternatives),
  - indicators $f_i : \mathbf{D} \rightarrow R$, $i \epsilon \mathbf{I}=\{1,2,...,I\}$ (maximization of indicator values is assumed),
  - ordering function $F: \mathbf{D} \rightarrow R^I$, in such way that

    $$F(z) = < f_{j_1}(z), f_{j_2}(z), ..., f_{j_i}(z), ..., f_{j_I}(z) > \quad j_i \in \boldsymbol{I} \quad \bigcup_{i=1}^{I} j_i = \boldsymbol{I}$$

    and for each $i<I$ condition

    $$f_{j_{i-1}}(z) \leq f_{j_i}(z)$$

    is satisfied

- Find such $z^* \epsilon D$ that for every $z \epsilon D$ condition

  $$F(z^*) \geq^L F(z)$$

  is satisfied

Cumulating derived information

Solution to the problem, i.e. subset **C** of optimal alternatives and their corresponding vector of indicators' values $<v_{j_1} * y_{j_1}, v_{j_2} * y_{j_2}, ..., v_{j_i} * y_{j_i}, ..., v_{j_I} * y_{j_I}>$ is derived by the following algorithm:

- For each $a \in A$ establish such a sequence $j_1^a, j_2^a, ..., j_i^a, ..., j_I^a$ that satisfies $\forall_{m \neq n} j_m^a \neq j_n^a \wedge \forall_{i \in II} j_i^a \in II$ and for $< d_{a1}, d_{a2}, ..., d_{ai}, ..., d_{aI} >$ satisfies $\forall_{i=1,2,...,I-1} d_{ai} \leq d_{a,i+1}$, where $d_{ai} = v_{j_i^a} * c_{aj_i^a}$
- for i=1, 2, ...,i,...,I
    - $\alpha_i := \max_{a \in A \setminus \bigcup_{n=1}^{i-1} B_n} d_{ai}$
    - $B_i = \{a \in A \setminus \bigcup_{n=1}^{i-1} B_n : d_{ai} < \alpha_i\}$
- $C = A \setminus \bigcup_{n=1}^{I} B_n$
- For any $a \in C$ and $i \in II$ do: $y_{j_i^a} := \alpha_i / v_{j_i^a}$

Cumulating derived information Assumptions and theorem

Assumptions:

1. A'={$a_1, a_2, \ldots, a_m, \ldots, a_M$} is a set of alternatives,
2. <$v_{j_1} * y_{j_1}, v_{j_2} * y_{j_2}, \ldots, v_{j_n} * y_{j_n}, \ldots v_{j_I} * y_{j_I}$> is an optimal solution on the set of alternatives A',
3. <$v_{k_1} * y_{k_1}, v_{k_2} * y_{k_2}, \ldots, v_{k_n} * y_{k_n}, \ldots v_{k_I} * y_{k_I}$> is an optimal solution on the set of alternatives A'', where $A'' \subset A'$.

Theorem:

vector <$v_{l_1} * y_{l_1}, v_{l_2} * y_{l_2}, \ldots, v_{l_n} * y_{l_n}, \ldots v_{l_I} * y_{l_I}$>

by the assumption that $\forall_{i=1,2,\ldots,I} \; y_{l_i} = y_{k_i} \wedge v_{l_i} = v_{j_i} * y_{j_i} / y_{k_i}$

is an optimal solution on A'.

Cumulating derived information Proof

Solution $<\mathbf{v}_{\mathbf{j}_1} * \mathbf{y}_{\mathbf{j}_1}, \mathbf{v}_{\mathbf{j}_2} * \mathbf{y}_{\mathbf{j}_2}, \ldots, \mathbf{v}_{\mathbf{j}_\mathbf{n}} * \mathbf{y}_{\mathbf{j}_\mathbf{n}}, \ldots \mathbf{v}_{\mathbf{j}_\mathbf{I}} * \mathbf{y}_{\mathbf{j}_\mathbf{I}}>$ has a corresponding set $\boldsymbol{C'} = \{\boldsymbol{a} \in \boldsymbol{A'}: \underset{i=1,2,\ldots,I}{\forall} y_{l_i} = c_{al_i}\}$.

Let solution $<\mathbf{v}_{\mathbf{l}_1} * \mathbf{y}_{\mathbf{l}_1}, \mathbf{v}_{\mathbf{l}_2} * \mathbf{y}_{\mathbf{l}_2}, \ldots, \mathbf{v}_{\mathbf{l}_\mathbf{n}} * \mathbf{y}_{\mathbf{l}_\mathbf{n}}, \ldots \mathbf{v}_{\mathbf{l}_\mathbf{I}} * \mathbf{y}_{\mathbf{l}_\mathbf{I}}>$ have a corresponding set $\boldsymbol{C'''} = \{\boldsymbol{a} \in \boldsymbol{A'}: \underset{i=1,2,\ldots,I}{\forall} y_{l_i} = c_{al_i}\}$.

To prove the theorem, it is enough to show that $\boldsymbol{C'''}$ is a not empty subset of optimal alternatives of, e.g., solution $<\mathbf{v}_{\mathbf{j}_1} * \mathbf{y}_{\mathbf{j}_1}, \mathbf{v}_{\mathbf{j}_2} * \mathbf{y}_{\mathbf{j}_2}, \ldots, \mathbf{v}_{\mathbf{j}_\mathbf{n}} * \mathbf{y}_{\mathbf{j}_\mathbf{n}}, \ldots \mathbf{v}_{\mathbf{j}_\mathbf{I}} * \mathbf{y}_{\mathbf{j}_\mathbf{I}}>$, (about which nothing particular is assumed) i.e.:

$$\boldsymbol{C'''} \neq \Phi \text{ and } \boldsymbol{C'''} \subset \boldsymbol{C'},$$

Cumulating derived information

The fact that $< \mathbf{v}_{k_1} * \mathbf{y}_{k_1}, \mathbf{v}_{k_2} * \mathbf{y}_{k_2}, \ldots, \mathbf{v}_{k_n} * \mathbf{y}_{k_n}, \ldots \mathbf{v}_{k_I} * \mathbf{y}_{k_I} >$ is an optimal solution on set of alternatives A'' induces that $C'' = \{a \in A'' : \forall_{i=1,2,\ldots,I} \; y_{k_i} = c_{ak_i}\} \neq \Phi$.

Because $\forall_{i=1,2,\ldots,I} \; y_{l_i} = y_{k_i}$ then $C''' = \{a \in A' : \forall_{i=1,2,\ldots,I} \; y_{k_i} = c_{ak_i}\}$.

The fact that $A' \supset A''$ induces $C''' \supset C''$ and the latter (after considering $C'' \neq \Phi$) leads to $C''' \neq \Phi$.

Cumulating derived information

The fact that $<\mathbf{v}_{j_1} * \mathbf{y}_{j_1}, \mathbf{v}_{j_2} * \mathbf{y}_{j_2}, ..., \mathbf{v}_{j_n} * \mathbf{y}_{j_n}, ... \mathbf{v}_{j_I} * \mathbf{y}_{j_I}>$ is a solution on

A' induces that $\underset{i=1,2,...,I}{\forall} B_i = \{a \in A' \setminus \bigcup_{n=1}^{i-1} B_n : d_{ai} < \alpha_i = v_{j_i} * y_{aj_i}\}$.

From the assumptions of the theorem $\underset{i=1,2,...,I}{\forall} y_{l_i} = y_{k_i} \wedge v_{l_i} = v_{j_i} * y_{j_i} / y_{k_i}$

and this induces $\underset{i=1,2,...,I}{\forall} v_{l_i} * y_{l_i} = v_{j_i} * y_{j_i} = \alpha_i$,

what (after considering definition of B_i and C') leads to a conclusion that

$\underset{i=1,2,...,I}{\forall} \underset{a \in C'''}{\forall} a \notin B_i$.

Therefore, $\underset{a \in C'''}{\forall} a \in A' \setminus \bigcup_{i=1,2,...,I} B_i = C'$ which implicates $C''' \subset C'$

and ends the proof.

Conclusions – Advantages

- Presentation discussed a method with nucleolar solution as optimization tool
- Generally this method allows to apply various approaches on a single-node (criterion) level
  - 'objective' (utopia, variability statistics, fair point, etc.)
  - clearly subjective (aspiration)

  for the node after applying proper method for optimization and proper calculation of importance coefficients
- Gradually familiarizes user with criteria and their inter-dependencies while broadening their range

Conclusions – Advantages

- Bases on selection from a small set of partial vectors of criteria values
- Enforces correction of user preferences due to necessity of consideration of an increasing number of criteria
- Enables adjustment of preferences by a reference outcomes comming from previous setting
- Allows to substitute improvement hints provided by the stakeholder and selection of one of solutions generated based on these hints by indicator’s importance coefficient

Thank you for
your attention!

The Algorithm

- Mark all criteria as not processed
- Select such criteria, which child are indicators.
- For every selected criterion:
  - Select appropriate indicator (child)
  - Calculate maximum of the indicator value, which is possible to be reached on the set of alternatives.
  - Create indicator subset of alternatives, i.e. such alternatives, for which the indicator value is equal to above maximum
  - Set correcting coefficient of this criterion equal to 1.
  - Mark the criterion as processed
- Limit set of alternatives by removing these alternatives which do not belong to any indicator subsets

The Algorithm (2)

- While exist ready criteria (marked as not processed and having all „child" criteria processed)
  - Process ready criterion (next page)
  - Mark the criterion as processed
  - Set subset of indicators which affect the criterion (indirect children)
  - For all indicators belonging to above subset set its value and correcting coefficient by taking appropriate values from child criterion.
  - Do SOLVE procedure to calculate first solution

The Algorithm (3)

- Until user does not point final solution (from presented)
  - Present (to user) all generated solutions for the criterion
  - Prompt user to select the solution
  - Prompt user to point the improvement hints
  - Do SOLVE procedure to generate solutions pointed by user hints
  - For every generated solution calculate indicators' importance coefficients
  - Present generated solutions to user
  - Consider solutions selected by user as new solutions for currently processed criterion
- End criterion processing