An Extension of Data Envelopment Analysis using an Interactive Tri-Criteria Linear Programming Package (TRIMAP)

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DEA-CCR MODEL

$$Max \quad h_0 = \sum_{r=1}^{s} u_r y_{rj_0}$$

sujeito a

$$\sum_{i=1}^{m} v_i x_{ij_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \; j = 1, .., n$$

$$u_r, v_i \geq \; 0, \; \forall r, i$$

Relative Efficiency evaluation concerning Decision Making Units (DMU's) – maximizing a ratio of a weighted sum of outputs to a weighted sum of inputs (here weights are "multipliers").

Constant returns of scale

Some Drawbacks:

-a lot of ties among efficient DMU's,

- multipliers nil valued.

MOLP-DEA (based on Li and Reeves)

$$Min\ d_0$$

$$Min\ Max\ d_j$$

$$Min \sum_{j=1}^{n} d_j$$

subject to:

$$\sum_{i=1}^{m} v_i x_{ij_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0,\ j=1,\dots,n$$

$$u_r,\ v_i \geq 0, \quad \forall r,i,j$$

Motivation: dealing with the following concerns in classical DEA – multipliers with a nil value and discrimination improvement in the presence of few DMUs.

Objective functions:

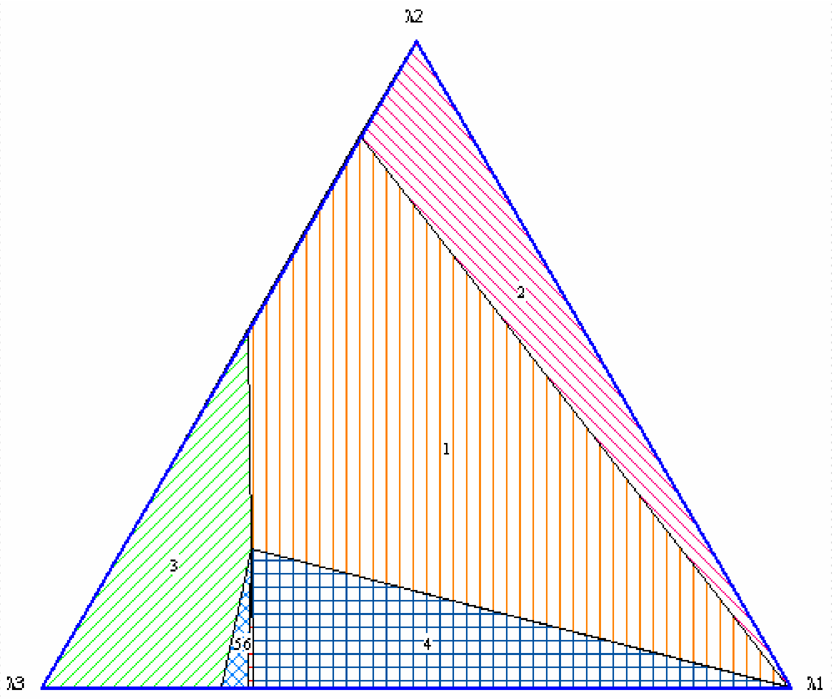
1- DMU efficiency= $1\text{-}d_0$

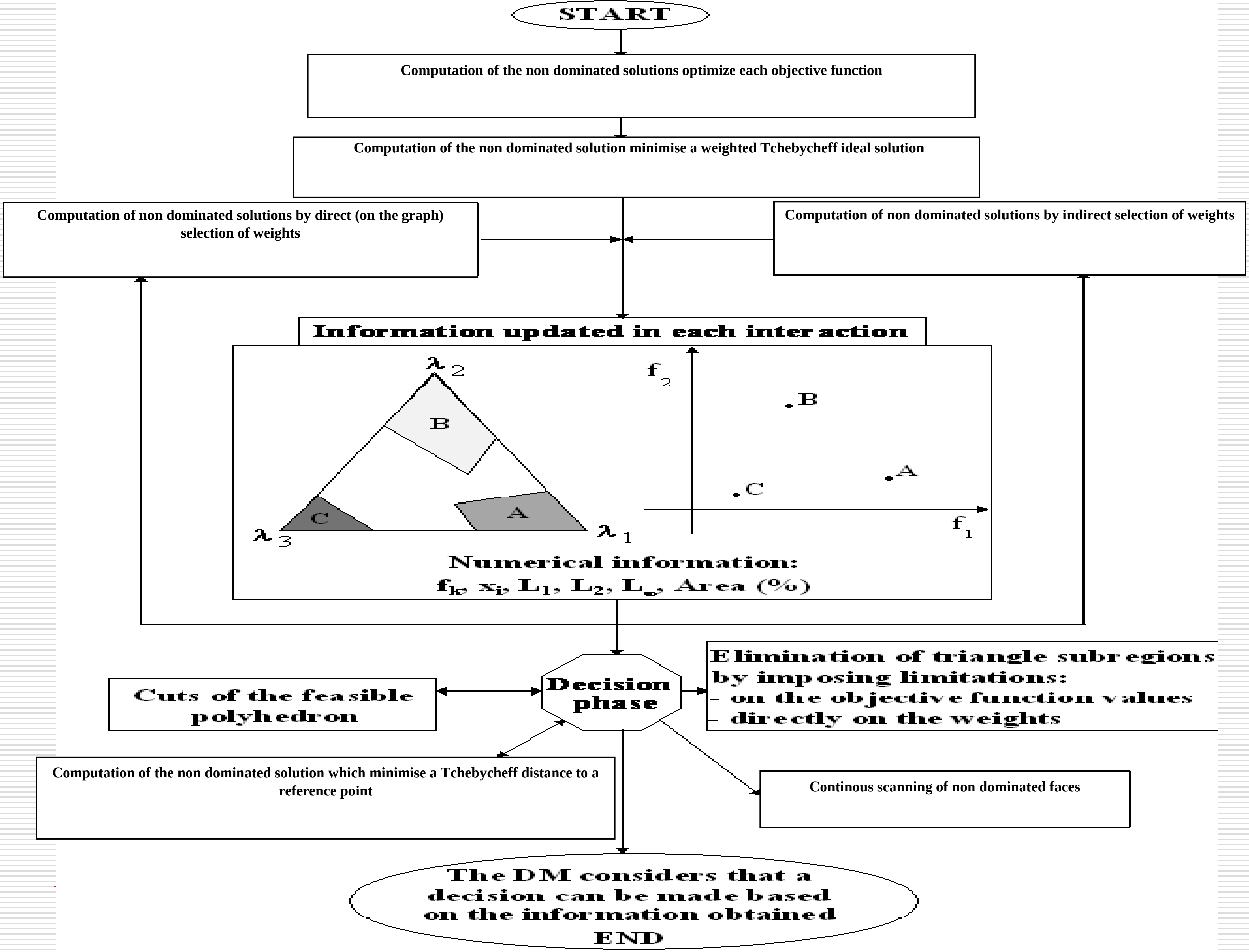
2- equity function

3- a global benevolence function

TRIMAP

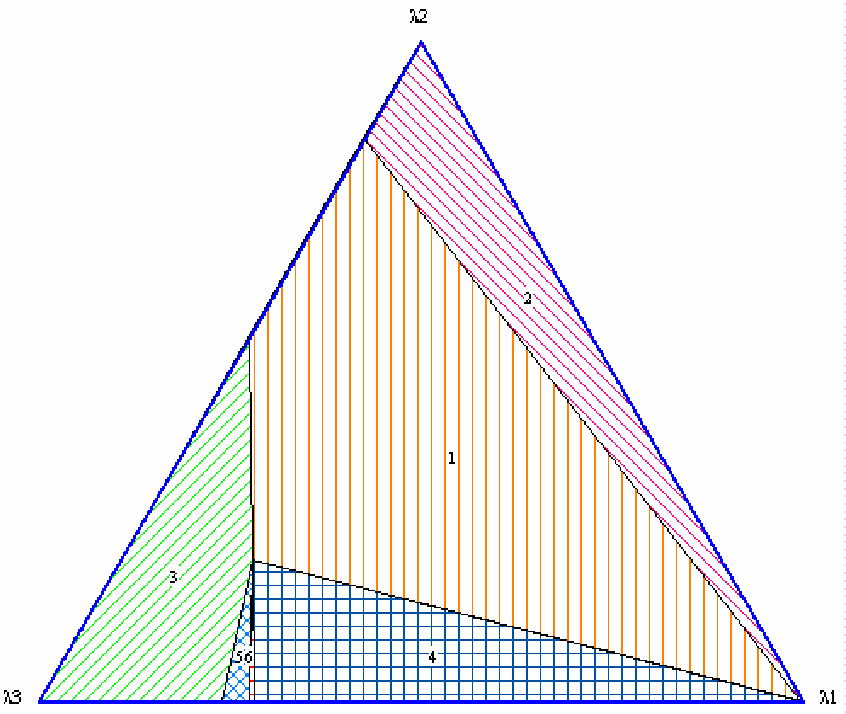
- A Tri-criteria linear programming package.
- It is an interactive decision environment...




START
Computation of the non dominated solutions optimize each objective function
Computation of the non dominated solution minimise a weighted Tchebycheff ideal solution
Computation of non dominated solutions by direct (on the graph) selection of weights
Computation of non dominated solutions by indirect selection of weights
Information updated in each interaction
λ2
B
C
A
λ3
λ1
f2
f1
Numerical information:
fk, xi, L1, L2, L∞, Area (%)
Cuts of the feasible polyhedron
Decision phase
Elimination of triangle subregions by imposing limitations:
– on the objective function values
– directly on the weights
Computation of the non dominated solution which minimise a Tchebycheff distance to a reference point
Continous scanning of non dominated faces
The DM considers that a decision can be made based on the information obtained
END

Alternative optimal multipliers

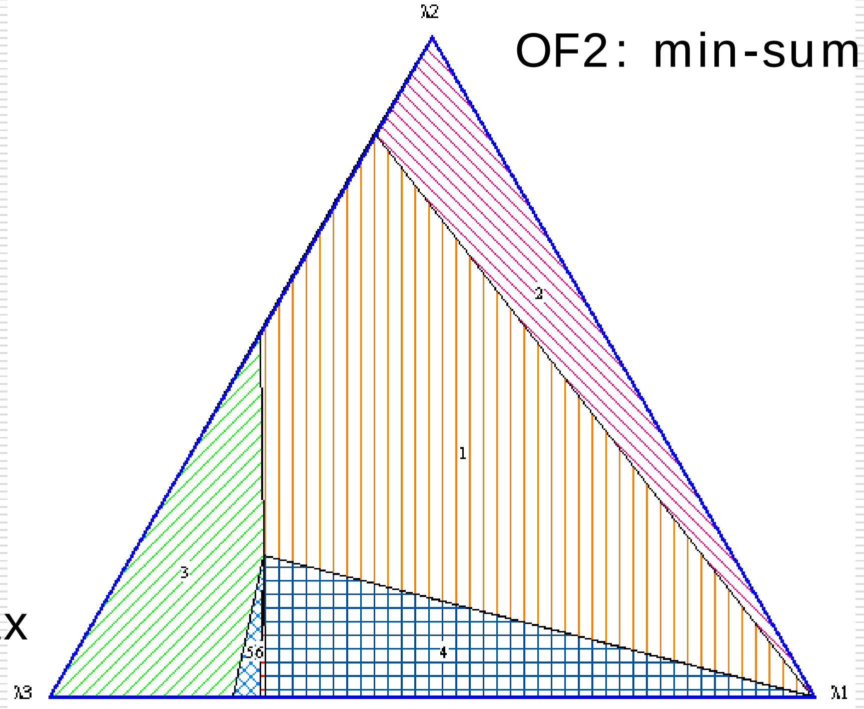
- Alternative optimal multipliers are identified by the sub-regions (three in this case) converging on objective function 1 vertex



OF1

A DMU is min-sum efficient...

- This DMU is min-sum efficient: Sub-region 2 corresponds simultaneously to objective functions 1 and 2 optimum.



Numerical example

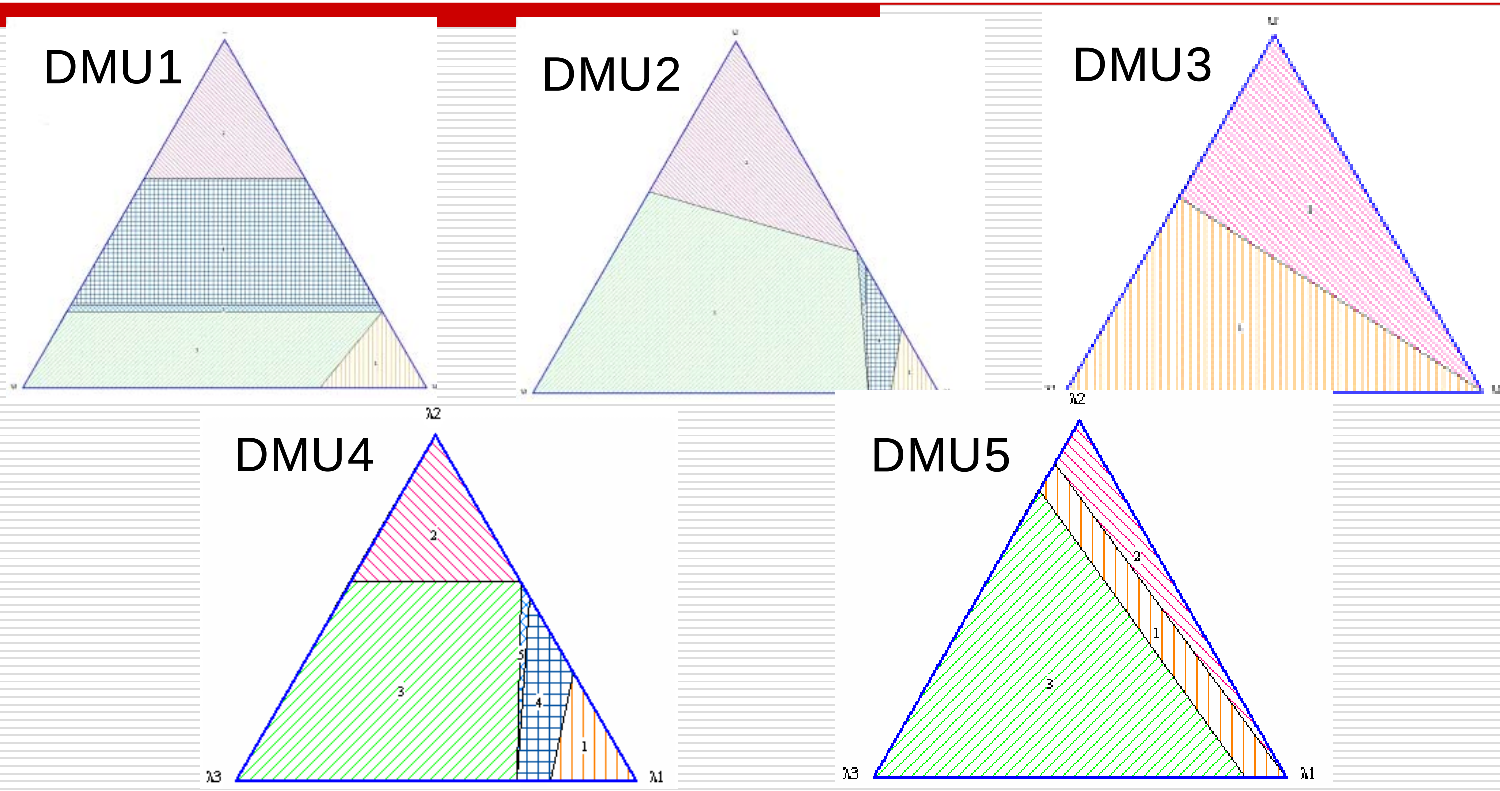
DMU	*Input* 1	*Input* 2	*Output*
1	0,5	5	8
2	2	1	4
3	3	5	20
4	4	2	8
5	1	1	4

Note: A DMU is efficient when

1-do is equal to 1

(in this case all of them are efficient)

Numerical example



DMU 3 is minimax and minisum efficient.
DMU 5 is minisum efficient.
The others are just efficient…

CASE STUDY

Brazilian privatized highways.
Efficiency evaluation (from the point of view of the users) taking into account the relations among **investments** and **traffic** as outputs and **number of crashes** and **income of the concessionary company** as inputs

DMU (Brazilian privatised highways)	Efficiency (%)	*Inputs*		*Outputs*	
		Crash/Km	Income/km	Investment/Km	Traffic/Km
CONCEPA	69,54	0,1897	0,1522	0,1400	0,0803
CONCER	100,00	0,1300	0,1312	0,2436	0,0472
CRT	71,10	0,0756	0,0984	0,0851	0,0309
Nova Dutra	71,06	0,3210	0,2708	0,3573	0,0492
Ponte	100,00	1,0000	1,0000	1,0000	1,0000

CASE STUDY

PONTE (RIO-NITEROI BRIDGE)

NOVA DUTRA (RIO–S.PAULO HIGHWAY)

CASE STUDY

CRT (RIO-TERESÓPOLIS HIGWAY)

CONCER (RIO-JUIZ DE FORA HIGHWAY)

CASE STUDY

CONCEPA (OSÓRIO-PORTO ALEGRE HIGWAY)

Tricriteria model for DMU1

$$Min\ d_1$$

$$Min\ \left(d_1+d_2+d_3+d_4+d_5\right)/5$$

$$Min\ \left(Max\ d_i\right)$$

Sujeito a:

$$0{,}1897v_1+0{,}1522v_2=1$$

$$-0{,}1897v_1-0{,}1522v_2+0{,}1400u_1+0{,}0803u_2+d_1=0$$

$$-0{,}1300v_1-0{,}1312v_2+0{,}2436u_1+0{,}0472u_2+d_2=0$$

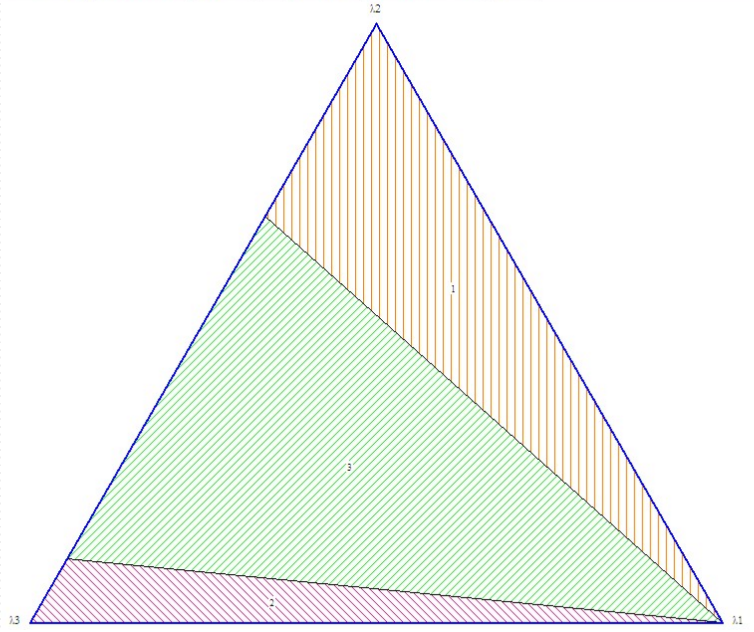
$$-0{,}0756v_1-0{,}0984v_2+0{,}0851u_1+0{,}0309u_2+d_3=0$$

$$-0{,}3210v_1-0{,}2708v_2+0{,}3573u_1+0{,}0492u_2+d_4=0$$

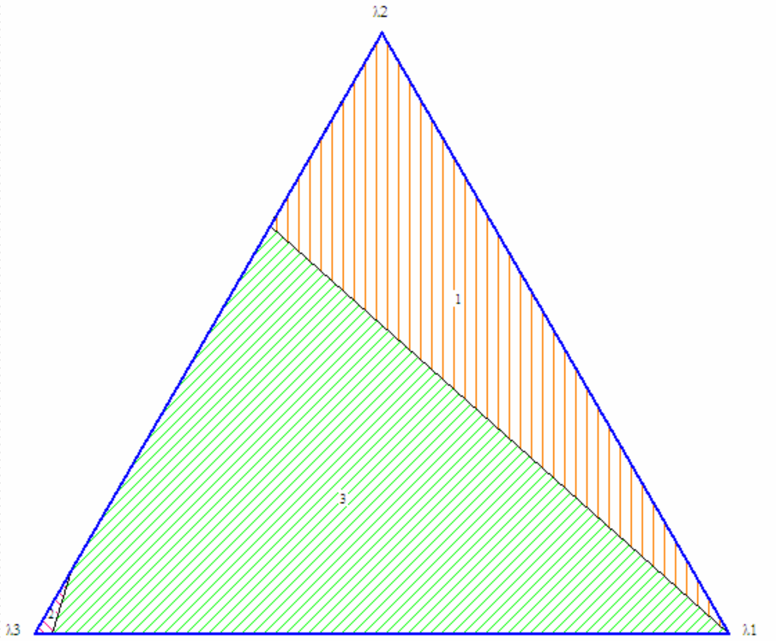
$$-v_1-v_2+u_1+u_2+d_5=0$$

$$d_1,d_2,d_{3,}d_4,d_5,v_1,v_2,u_1,u_2 \geq 0$$

Weight space decomposition for efficient DMUs



CONCER (DMU 2)



PONTE (DMU 5)

CONCER EFFICIENCY IS 1 IN ANY CASE…

New Model

Objective functions

$$Max \sum_{r=1}^{s} u_{ro} y_{rj_o}$$

$$Max\ I$$

$$Max\ O$$

$$subject\ to$$

$$\sum_{i=1}^{m} v_i x_{ij_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ j=1,...,n$$

$$I \le v_{io},\quad i=1,...,m$$

$$O \le u_{ro},\ \ r=1,...,s$$

$$u_r, v_i \ge 0,\quad \forall r,i,j$$

Motivation: avoiding nill multipliers

Objective functions:

1- Classical DEA

2- Maximizes the minor multiplier regarding the inputs

3- Maximizes minor multiplier regarding he outputs