

On a Descriptive Model of a Measurable Value Function under Uncertainty for Complex Decision Analysis

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Abstract

In this paper a descriptive extension of the expected utility model to account for various paradoxes is proposed using the concept of strength of preference. We deal with the case where probability of occurrence for each event is unknown. When we describe the degree of ignorance and uncertainty by the basic probability of Dempster-Shafer theory, the problem is how to represent the value of a set element to construct a measurable value function under uncertainty based on this concept. For identifying a value function under uncertainty, we need to find the preference relations among set elements, which is not an easy task. If the number of elements contained in the set element contains considerable number of elements, it is not practical to find the value function as a function of the set elements. To cope with this difficulty we use some appropriate axiom of dominance.

Keywords: Decision analysis; Modeling; Behavioral model; Utility theory; Expected utility paradox; Decision making under uncertainty

1. Introduction

The expected utility model has been widely used as a normative model of decision analysis under risk for modeling individual decision making. But, various paradoxes [1, 3] have been reported for the expected utility model, and it is argued that the expected utility model is not an adequate behavioral (descriptive) model. In this paper a behavioral model is shown to resolve expected utility paradox. Some realistic potential applications to public sectors are mentioned.

Incorporating the Dempster-Shafer probability theory in our descriptive model of a value function under uncertainty, we could model the lack of belief that could not be modelled by Bayes' probability theory. As the result our descriptive model could resolve the Ellsburg paradox.

Recent increase of carbon dioxide concentration around the globe is getting serious and it is said that the resulting greenhouse effect and global warming may cause serious damages in our life. Therefore, we need to restrict the emission of carbon dioxide and other greenhouse gas somehow. By using a value function under uncertainty we could deal with a set element of like $\{d_1, d_2\}$, where

d_1 : get damage caused by unusual weather due to global warming.

d_2 : get damage caused by unusual weather which is not related with global warming.

Actually, when we get damage caused by unusual weather, we do not know whether it is due to global warming or not. Basic probability could be assigned to such a set element $\{d_1, d_2\}$. Then, we could construct a measurable value function under uncertainty for evaluating the alternative policies to decrease the emission of carbon dioxide for avoiding global warming. The measurable value function under uncertainty could also evaluate the preference of various type of decision makers; ordinary, pessimistic or optimistic ones.

2. Expected utility model

Utility function could provide a mean of modeling value judgement of a decision maker quantitatively. Scientific approach for value judgement has been discussed rigorously in the area of economics. People would get feeling of

psychological satisfaction by consuming economic goods, by receiving service and so forth. This degree of satisfaction obtained is called “utility.” This concept plays fundamental role in the theory of consumers’ behavior.

Let x_1 and x_2 be the amount of goods A and B consumed, respectively, $u(x_1, x_2)$ be the corresponding value of the consumer’s utility function, p_1 and p_2 be the price of unit amount of goods A and B, respectively, and b be the amount of budget. Then, the consumer may want to

$$\text{maximize } u(x_1, x_2) \quad (1)$$

subject to

$$p_1x_1 + p_2x_2 \leq b \quad (2)$$

That is, the consumer behavior has been explained in such a way that he/she would act to maximize his own utility, Eq. (1), subject to the budget constraint, Eq. (2).

In order to derive the equilibrium condition of a consumer’s behavior, the ordinal utility function would be enough to evaluate, but for obtaining a preferred solution of a multiple criteria decision making problem, we need a cardinal utility function. Furthermore, since in a decision making problem under risk the consequence or outcome would be obtained under some probability distribution, we need to evaluate the so-called expected utility. For this purpose we need to provide a cardinal utility function. Von Neumann and Morgenstern [8] have first developed axioms such that the expected utility hypothesis for the decision making problem under risk would be meaningful.

Let X be a set of consequences, $u : X \rightarrow \text{Re}$ be a cardinal utility function, then the expected utility with respect to the probability on X is written as

$$E(u, p) = \sum_{x \in X} p(x)u(x) \quad (3)$$

which is called the expected utility. Let $P = \{p_1, p_2, \dots\}$ be the set of probability on X , then the following theorem provides the existence and uniqueness of a cardinal utility function [8].

Theorem: Existence and uniqueness of a cardinal utility function

Let P be the set of all probability on X , (P, \succeq) be preference structure on P , then for any $p, q \in P$, the necessary and sufficient condition of the existence of cardinal utility function $u : X \rightarrow \text{Re}$ such that

$$p \succeq q \Leftrightarrow E(u, p) \geq E(u, q), \quad \forall p, q \in P \quad (4)$$

is given as follows:

NM1: (P, \succeq) is weak order.

NM2: $p \succ q \Rightarrow \alpha p + (1-\alpha)r \succ \alpha q + (1-\alpha)r, \quad \forall r \in P, \quad \alpha \in (0,1)$

NM3: $p \succ q \succ r \Rightarrow \alpha p + (1-\alpha)r \succ q \succ \beta p + (1-\beta)r, \quad \text{for some } \alpha, \beta \in (0,1)$

Furthermore, such u is unique within the positive linear transformation (there exist h and $k > 0$ such that $u' = h + ku$) and is called von Neumann-Morgenstern utility function.

The expected utility model based on the expected utility hypothesis is a useful model as a normative model, that is, to find a decision to be made. However, since various paradoxes [1, 3] have been reported for the expected utility model, it is argued that the expected utility model is not an adequate behavioral (descriptive) model. Actually, there exist many phenomena that violate expected utility hypothesis such as Allais paradox [1] and Ellsberg paradox [3]. In the next section we propose a generalized model of measurable value function under risk and measurable value function under uncertainty to overcome the difficulty of expected utility paradoxes.

3. Behavioral models to resolve expected utility paradoxes

3.1. Measurable value function under risk

The expected utility model has been widely used as a normative model of decision analysis under risk. But, various paradoxes have been reported for the expected utility model, and it is argued that the expected utility model is not an adequate descriptive model.

In this section a descriptive extension of the expected utility model to account for various paradoxes is shown using the concept of strength of preference.

Let X be a set of all consequences, $x \in X$, and A be a set of all risky alternatives; a risky alternative $\ell \in A$ is written as

$$\ell = (x_1, x_2, \dots, x_n; p_1, p_2, \dots, p_n) \quad (5)$$

that yields consequence $x_i \in X$ with probability $p_i, i = 1, 2, \dots, n$, where $\sum p_i = 1$.

Let A^* be a nonempty subset of $A \times A$, and \succeq and \succeq^* be binary relations on A and A^* , respectively. Relation \succeq could also be a binary relation on X . We interpret $\ell_1 \succeq \ell_2$ ($\ell_1, \ell_2 \in A$) to mean that ℓ_1 is preferred or indifferent to ℓ_2 , and $\ell_1 \ell_2 \succeq^* \ell_3 \ell_4$ ($\ell_1, \ell_2, \ell_3, \ell_4 \in A$) to mean that the strength of preference for ℓ_1 over ℓ_2 is greater than or equal to the strength of preference for ℓ_3 over ℓ_4 .

We postulate that (A, A^*, \succeq^*) takes a positive difference structure that is based on the axioms described by Krantz et al. [5]. The axioms imply that there exists a real-valued function F on A such that for all $\ell_1, \ell_2, \ell_3, \ell_4 \in A$, if $\ell_1 \succeq \ell_2$ and $\ell_3 \succeq \ell_4$, then

$$\ell_1 \ell_2 \succeq^* \ell_3 \ell_4 \Leftrightarrow F(\ell_1) - F(\ell_2) \geq F(\ell_3) - F(\ell_4) \quad (6)$$

Since F is unique up to a positive linear transformation, it is a cardinal function. It is natural to hold for $\ell_1, \ell_2, \ell_3 \in A$ that

$$\ell_1 \ell_3 \succeq^* \ell_2 \ell_3 \Leftrightarrow \ell_1 \succeq \ell_2. \quad (7)$$

Then from Eq. (6) we obtain

$$\ell_1 \succeq \ell_2 \Leftrightarrow F(\ell_1) \geq F(\ell_2). \quad (8)$$

Thus, F is a value function on A and, in view of Eq. (6), it is a measurable value function.

We assume that the decision maker will try to maximize the value (or utility) of a risky alternative $\ell \in A$, which is given by the general form as follows:

$$\max_{\ell \in A} F(\ell) = \max_{\ell \in A} \sum_i f(x_i, p_i) \quad (9)$$

where $f(x, p)$ denotes the value (strength of preference) for a consequence x which comes out with probability p . This function is called the *measurable value function under risk*. The main objectives here are to give an appropriate decomposition and interpretation of $f(x, p)$ and to explore its descriptive implications to account for the various paradoxes.

The model Eq. (9) is reduced to the expected utility form by setting

$$f(x, p) = pu(x) \quad (10)$$

when $u(x)$ is regarded as a von Neumann-Morgenstern utility function. The prospect theory of Kahneman and Tversky [4] is obtained by setting

$$f(x, p) = \pi(p)v(x) \quad (11)$$

where $\pi(p)$ denotes a weighting function for probability and $v(x)$ a value

function for consequence. In this model the value of each consequence is multiplied by a decision weight for probability (not by probability itself).

Extending this Kahneman-Tversky model we obtain a decomposition form

$$f(x, p) = w(p | x) v(x) \quad (12)$$

where

$$w(p | x) \equiv \frac{f(x, p)}{f(x, 1)} \quad (13a)$$

$$v(x) \equiv v(x | 1) \quad (13b)$$

$$v(x | p) \equiv \frac{f(x, p)}{f(x^*, p)} \quad (13c)$$

and x^* denotes the best consequence. The expected utility model, Eq. (10), and Kahneman-Tversky model, Eq. (11), are included in our model, Eq. (12), as a special case. Eq. (13b) implies that $v(x)$ denotes a measurable value function under certainty. Therefore, our model, Eq. (12), also includes Dyer and Sarin's model [2] as a special case.

The model Eq. (12) could also be written as

$$f(x, p) = w(p) v(x | p) \quad (14)$$

where

$$w(p) \equiv w(p | x^*). \quad (15)$$

We assume that

$$f(x, 0) = 0, \quad \forall x \in X \quad (16a)$$

$$f(x^R, p) = 0, \quad \forall p \in [0, 1] \quad (16b)$$

where $x^R \in X$ denotes the reference point (e.g. status quo). The better region on X compared with x^R is called the gain domain and the worse region the loss domain. We also assume that $f(x, p) \geq 0$ on the gain domain and $f(x, p) < 0$ on the loss domain.

It will be shown that the conditional weighting function $w(p | x)$ describes the strength of preference for probability under the given conditional level of consequence, and $v(x | p)$ describes the strength of preference for consequence under the given conditional level of probability.

For interpreting the descriptive model $f(x, p)$ we need to interpret F such that Eq. (6) holds. For all $x_1, x_2, x_3, x_4 \in X$, $\alpha \in [0, 1]$ and $y \in X$ such that

$x_1 \succeq x_2 \succeq x_3 \succeq x_4$, we consider four alternatives:

$$\begin{aligned} \ell_1 &= (x_1, y; \alpha, 1 - \alpha), & \ell_2 &= (x_2, y; \alpha, 1 - \alpha), \\ \ell_3 &= (x_3, y; \alpha, 1 - \alpha), & \ell_4 &= (x_4, y; \alpha, 1 - \alpha). \end{aligned} \quad (17)$$

In this case we obtain

$$\ell_1 \ell_2 \succeq^* \ell_3 \ell_4 \Leftrightarrow f(x_1, \alpha) - f(x_2, \alpha) \geq f(x_3, \alpha) - f(x_4, \alpha) \quad (18a)$$

$$\Leftrightarrow v(x_1 | \alpha) - v(x_2 | \alpha) \geq v(x_3 | \alpha) - v(x_4 | \alpha). \quad (18b)$$

Therefore, the value function $v(x | p)$ defined by Eq. (13c) represents the strength of preference for the risky four alternatives in Eq. (17).

On the other hand, for all $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in [0, 1]$, $x \in X$ and $x^R \in X$, we consider four alternatives:

$$\begin{aligned} \ell_1' &= (x, x^R; \alpha_1, 1 - \alpha_1), & \ell_2' &= (x, x^R; \alpha_2, 1 - \alpha_2), \\ \ell_3' &= (x, x^R; \alpha_3, 1 - \alpha_3), & \ell_4' &= (x, x^R; \alpha_4, 1 - \alpha_4), \end{aligned} \quad (19)$$

then we obtain

$$\ell_1' \ell_2' \succeq^* \ell_3' \ell_4' \Leftrightarrow f(x, \alpha_1) - f(x, \alpha_2) \geq f(x, \alpha_3) - f(x, \alpha_4) \quad (20a)$$

$$\Leftrightarrow w(\alpha_1 | x) - w(\alpha_2 | x) \geq w(\alpha_3, x) - w(\alpha_4, x). \quad (20b)$$

Therefore, the weighting function defined by Eq. (13a) represents the strength of preference for the four risky alternatives in Eq. (19).

The above discussions assert that the descriptive model $f(x, p)$ represents the measurable value function under risk to evaluate the consequence $x \in X$ that comes out with probability p .

The Kahneman-Tversky model of Eq. (11) could explain a so-called certainty effect to resolve the Allais paradox [1]. Our descriptive model $f(x, p)$ could also resolve the Allais paradox.

It is well known that the expected utility model is not an appropriate model for modeling extreme events with low probability and high consequence. In [7] it is shown that our descriptive model could resolve such paradox in the application to public sector.

3.2. Measurable value function under uncertainty

In this section we deal with the case where probability of occurrence for each event is unknown. When we describe the degree of ignorance and uncertainty by

the basic probability of Dempster-Shafer theory [6], the problem is how to represent the value of a set element to construct a measurable value function under uncertainty based on this concept.

In Dempster-Shafer theory of probability let $\mu(A_i)$ be basic probability which could be assigned by any subset A_i of Θ , where Θ denotes a set containing every possible element. The basic probability $\mu(A_i)$ can be regarded as a semimobile probability mass. Let $\Lambda = 2^\Theta$ be a set containing every subset of Θ . Then, the basic probability $\mu(A_i)$ is defined on Λ and takes a value contained in $[0,1]$. When $\mu(A_i) > 0$, A_i is called the focal element or the set element and the following conditions hold:

$$\mu(\phi) = 0, \quad \sum_{A_i \in \Lambda} \mu(A_i) = 1 \quad (21)$$

where ϕ denotes an empty set.

Let the value function under uncertainty based on this basic probability be

$$f^*(B, \mu) = w'(\mu) v^*(B | \mu) \quad (22)$$

where B denotes a set element, μ denotes the basic probability, w' denotes the weighting function for the basic probability, and v^* denotes the value function with respect to a set element. The set element B is a subset of $\Lambda = 2^\Theta$. Eq. (22) is an extended version of the value function, Eq. (14), where an element is extended to a set element and the Bayes' probability is extended to the Dempster-Shafer basic probability.

For identifying v^* , we need to find the preference relations among set elements, which is not an easy task. If the number of elements contained in the set Θ is getting larger, and the set element B contains considerable number of element it is not practical to find v^* as a function of B . To cope with this difficulty we could use some appropriate axiom of dominance as follows:

Axiom of Dominance. In the set element B let the worst consequence be m_B and the best consequence be M_B . For any $B_1, B_2 \subset \Lambda = 2^\Theta$

$$m_{B_1} \preceq m_{B_2}, M_{B_1} \preceq M_{B_2} \Rightarrow B_1 \preceq B_2 \quad (23)$$

and

$$m_{B_1} \sim m_{B_2}, M_{B_1} \sim M_{B_2} \Rightarrow B_1 \sim B_2. \quad (24)$$

Our descriptive model $f^*(B, \mu)$ could resolve Ellsberg paradox [3] by

restricting a set element B to

$$\Omega = \{(m, M) \in \Theta \times \Theta : m \preceq M\} \quad (25)$$

where m and M denote the worst and the best consequence in the set element B , respectively. In this case Eq. (22) is reduced to

$$f^*(\Omega, \mu) = w'(\mu) v^*(\Omega | \mu). \quad (26)$$

Suppose we look at an index of optimism $\alpha(m, M)$, such that the following two alternatives are indifferent.

Alternative 1. One can receive m for the worst case and M for the best case.

There exists no other information.

Alternative 2. One receives M with probability $\alpha(m, M)$ and receives m with probability $1 - \alpha(m, M)$, where $0 < \alpha(m, M) < 1$.

If one is quite optimistic, $\alpha(m, M)$ becomes nearly equal to 1, and if one is quite pessimistic $\alpha(m, M)$ becomes nearly equal to zero. If we incorporate this optimism index $\alpha(m, M)$ in Eq. (26), the value function is obtained as

$$\begin{aligned} v^*(\Omega | \mu) &= v^*((m, M) | \mu) \\ &= \alpha(m, M) v'(M | \mu) + (1 - \alpha(m, M)) v'(m | \mu) \end{aligned} \quad (27)$$

where v' denotes a value function for a single element.

Incorporating the Dempster-Shafer probability theory in the descriptive model $f^*(\Omega, \mu)$ of a value function under uncertainty, we could model the lack of belief which could not be modelled by Bayes' probability theory. As the result our descriptive model $f^*(\Omega, \mu)$ could resolve the Ellsburg paradox [3] as follows:

3.3. Resolving Ellsburg paradox

Suppose an urn contains 30 balls of red, black and white. We know that 10 of 30 balls are red, but for the other 20 balls we know only that each of these balls is either black or white. Suppose we pick a ball from this urn, and consider four events as follows:

- a. We will get 100 dollars if we pick a red ball.
- b. We will get 100 dollars if we pick a black ball.
- c. We will get 100 dollars if we pick a red or white ball.
- d. We will get 100 dollars if we pick a black or white ball.

Many people show the preference [3],

$$a \succ b, \quad d \succ c. \quad (28)$$

The probability of picking up a red ball is $1/3$. Let p_b and p_w be the probability of picking up a black ball and a white ball, respectively. Then

$$p_b + p_w = \frac{2}{3}. \quad (29)$$

The expected utility theory says that

$$a \succ b \Rightarrow \frac{1}{3}u(1M) > p_b u(1M) \Rightarrow p_b < \frac{1}{3} \quad (30)$$

$$d \succ c \Rightarrow \frac{2}{3}u(1M) > \frac{1}{3}u(1M) + p_w u(1M) \quad (31a)$$

$$\Rightarrow p_w < \frac{1}{3} \Rightarrow p_b > \frac{1}{3} \quad (31b)$$

where u denotes a von Neumann-Morgenstern utility function and $1M=100$ dollars. Eqs. (30) and (31b) are obviously contradictory. This phenomenon is called the Ellsberg paradox [3]. Therefore, the expected utility theory cannot represent the preference when the probability of each event is not known but only basic probability for a set of events is known. This phenomenon shows that one prefers the events with known probability and is called the sure-thing principle [5].

How could we explain the preference of this Ellsberg paradox by using the descriptive model $f^*(\Omega, \mu)$ of a value function under uncertainty? Let $\{R\}$ be the event of picking a red ball and $\{B, W\}$ be the set element of picking a black or white ball. Then the basic probability is written as

$$\mu(\{R\}) = \frac{1}{3}, \quad \mu(\{B, W\}) = \frac{2}{3} \quad (32)$$

In this case a set Θ containing every possible event is written as

$$\Theta = \{0, 1M\} \quad (33)$$

Table 1 Basic probability for each event

Alternative	Event		
	$\{0\}$	$\{1M\}$	$\{0, 1M\}$
a	$2/3$	$1/3$	0
b	$1/3$	0	$2/3$
c	0	$1/3$	$2/3$
d	$1/3$	$2/3$	0

Table 1 shows the basic probability of getting each event for each alternative. The value for each alternative is given by

$$V(a) = w'\left(\frac{2}{3}\right)v'\left(\{0\} \mid \frac{2}{3}\right) + w'\left(\frac{1}{3}\right)v'\left(\{1M\} \mid \frac{1}{3}\right) \quad (34a)$$

$$V(b) = w'\left(\frac{1}{3}\right)v'\left(\{0\} \mid \frac{1}{3}\right) + w'\left(\frac{2}{3}\right)v'\left(\{0,1M\} \mid \frac{2}{3}\right) \quad (34b)$$

$$V(c) = w'\left(\frac{1}{3}\right)v'\left(\{1M\} \mid \frac{1}{3}\right) + w'\left(\frac{2}{3}\right)v'\left(\{0,1M\} \mid \frac{2}{3}\right) \quad (34c)$$

$$V(d) = w'\left(\frac{1}{3}\right)v'\left(\{0\} \mid \frac{1}{3}\right) + w'\left(\frac{2}{3}\right)v'\left(\{1M\} \mid \frac{2}{3}\right). \quad (34d)$$

In the set Θ let x^0 and x^* be the worst consequence and the best consequence, respectively, then

$$x^0 = 0, \quad x^* = 1M. \quad (35)$$

Therefore, we obtain

$$v'(\{0\} \mid \mu) = 0, \quad v'(\{1M\} \mid \mu) = 1, \quad \forall \mu. \quad (36)$$

Let an index of optimism be $\alpha = \alpha(0,1M)$, then

$$a \succ b \Rightarrow V(a) > V(b) \quad (37a)$$

$$\Rightarrow w'\left(\frac{1}{3}\right) > w'\left(\frac{2}{3}\right)v'\left(\{0,1M\} \mid \frac{2}{3}\right) \quad (37b)$$

$$\Rightarrow w'\left(\frac{1}{3}\right) > \alpha w'\left(\frac{2}{3}\right) \quad (37c)$$

$$d \succ c \Rightarrow V(d) > V(c) \quad (38a)$$

$$\Rightarrow w'\left(\frac{2}{3}\right) > w'\left(\frac{1}{3}\right) + w'\left(\frac{2}{3}\right)v'\left(\{0,1M\} \mid \frac{2}{3}\right) \quad (38b)$$

$$\Rightarrow w'\left(\frac{1}{3}\right) < (1-\alpha)w'\left(\frac{2}{3}\right). \quad (38c)$$

To hold these preference relation we need to have $\alpha = \alpha(0,1M)$ such that

$$\frac{w'\left(\frac{1}{3}\right)}{w'\left(\frac{2}{3}\right)} > \alpha, \quad \frac{w'\left(\frac{2}{3}\right) - w'\left(\frac{1}{3}\right)}{w'\left(\frac{2}{3}\right)} > \alpha. \quad (39)$$

If $\alpha = \alpha(0,1M) < 0.5$, Eq. (39) holds. This situation shows that, in general, one is pessimistic about events with unknown probability. The Ellsberg paradox is

resolved by the descriptive model $f^*(\Omega, \mu)$ of a value function under uncertainty.

3.4. Potential applicability to modeling public sector decision problems

Recent increase of carbon dioxide concentration around the globe is getting serious and it is said that the resulting greenhouse effect and global warming may cause serious damages in our life. Therefore, we need to restrict the emission of carbon dioxide and other greenhouse gas somehow. By using a value function under uncertainty shown in 3.2 we could deal with a set element of like $\{d_1, d_2\}$, where

d_1 : get damage caused by unusual weather due to global warming.

d_2 : get damage caused by unusual weather which is not related with global warming.

Actually, when we get damage caused by unusual weather, we do not know whether it is due to global warming or not. Basic probability could be assigned to such a set element $\{d_1, d_2\}$. Then, we could construct a measurable value function under uncertainty for evaluating the alternative policies to decrease the emission of carbon dioxide for avoiding global warming. The measurable value function under uncertainty could also evaluate the preference of various type of decision makers; ordinary, pessimistic or optimistic ones.

4. Concluding remarks

A behavioral model of decision analysis in individual decision making under uncertainty is described. In this model basic probability for a set of events is known but probability of occurring each event is not known. It is shown that Ellsberg paradox is consistently resolved by using this model. Potential applicability to global environmental problem is also mentioned.

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