Application of EDGE software and Monte Carlo methods for pricing catastrophe bonds

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Program of presentation

1. Concept of catastrophe bonds
2. Features of Integrated Catastrophe Management Model
3. Catastrophe bonds pricing problem
4. Examples of applications
5. Future developments

Catastrophe bonds

Catastrophe bond (in abbreviation: *cat bond*) is relatively new, fast developing financial instrument. It is similar to normal bond, issued by a government or an enterprise, but its structure of payments depends on the occurrence of specified type of natural catastrophe (ie. flood, earthquake) in the established region and time interval. Such event is called *triggering point*.

Example of catastrophe bond: A–1 USAA

- The A–1 bond was initiated in April 1997 by USAA
- The payment from the bond equalled LIBOR plus 288 bp.
- The *triggering point* was cumulated loss amount from catastrophe: if there had been a hurricane on the east coast of USA between July 1997 and December 1997, the coupon of the A–1 would have been lost
- The principal was always guaranteed

Important feature of catastrophe bonds

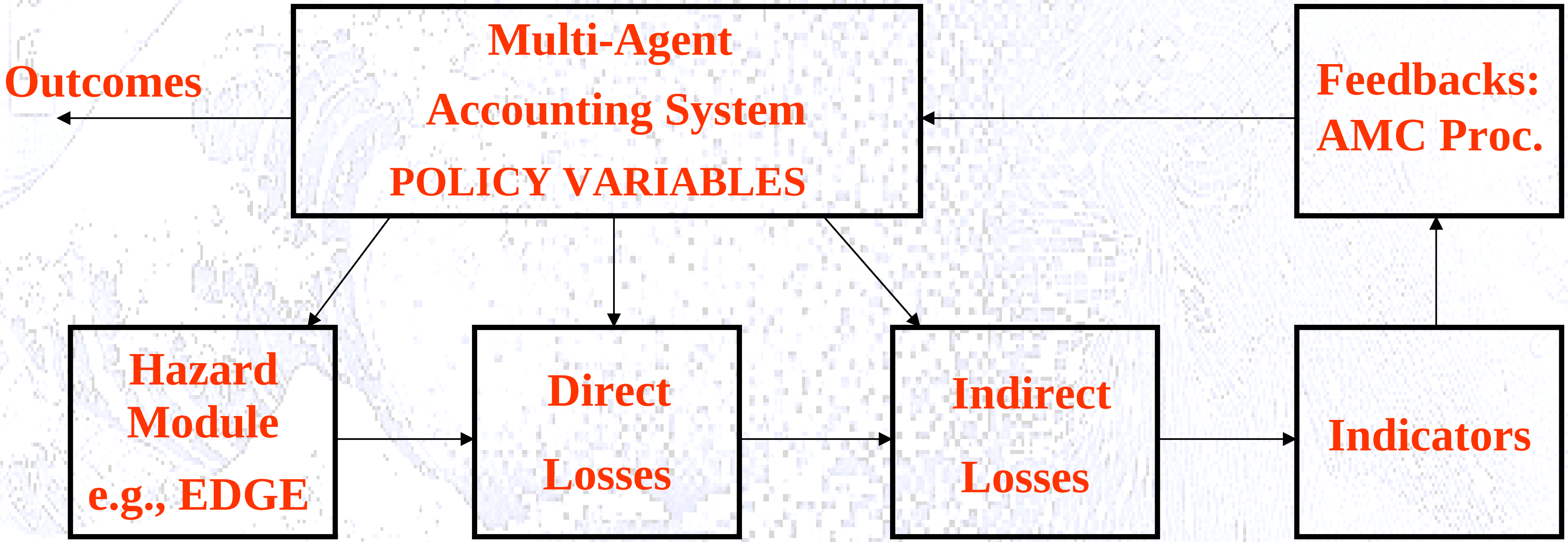
The payments from catastrophe bonds depend on two variables:

1. Trajectory of some underlying asset
2. Possible scenario of catastrophe

These variables may be modelled by adequate stochastic processes

IIASA Integrated Catastrophe Management MODEL: Adaptive Monte Carlo simulation

Ex - ante and ex - post decisions

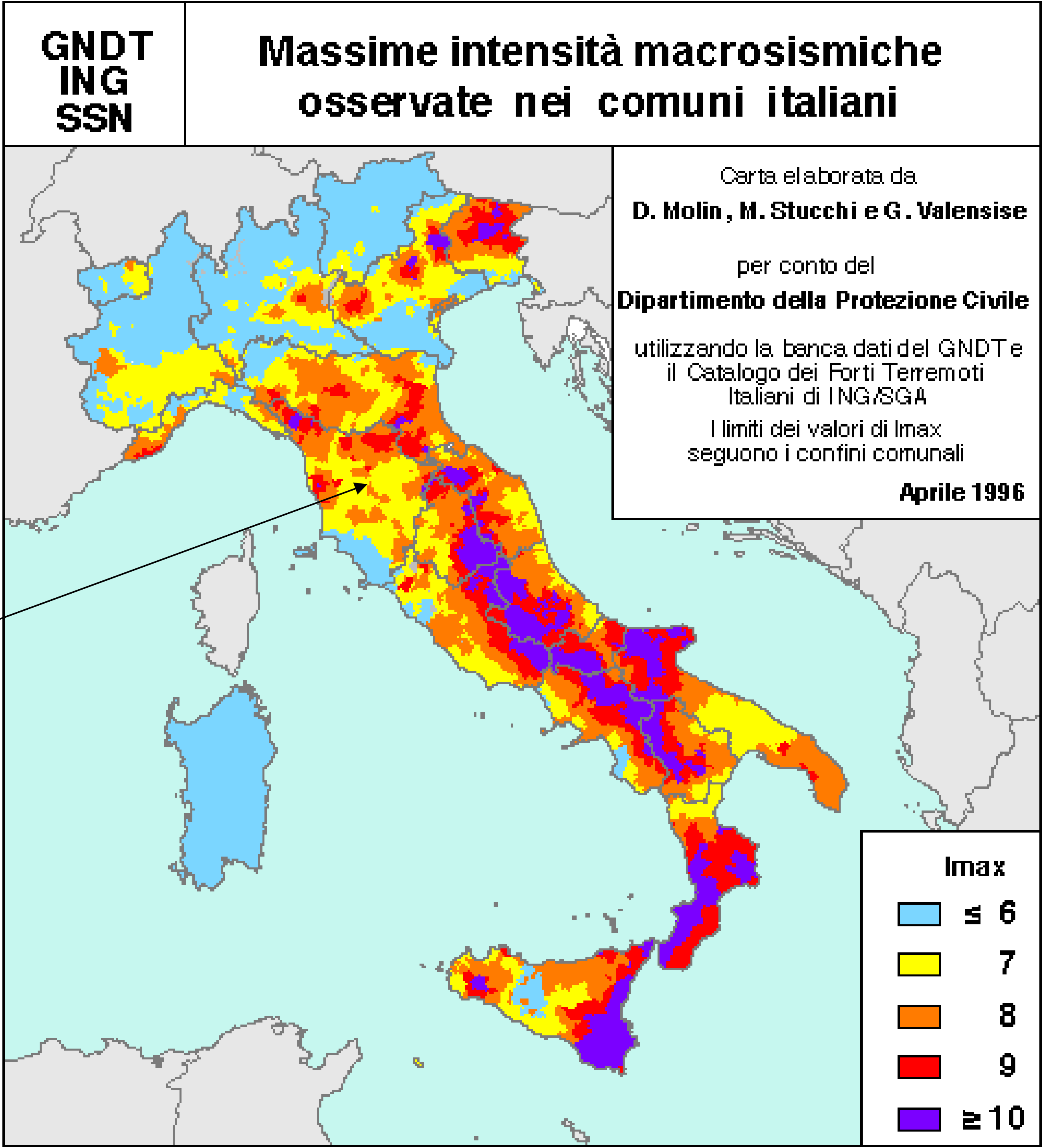


Reduced versions of SPATIAL DYNAMIC MODELS

EVENTS in SPACE and TIME

CASCADES of losses, MACRO - and MICRO - economic MODELS, SPATIAL and DYNAMIC aspects

RUIN, BENEFITS, PROFITS, COSTS (Future), STABILITY



Petrini et al.
CNR 1995
Tuscany

$$1 - F(I) = \exp[-\alpha(I - I^C)] \quad (1)$$

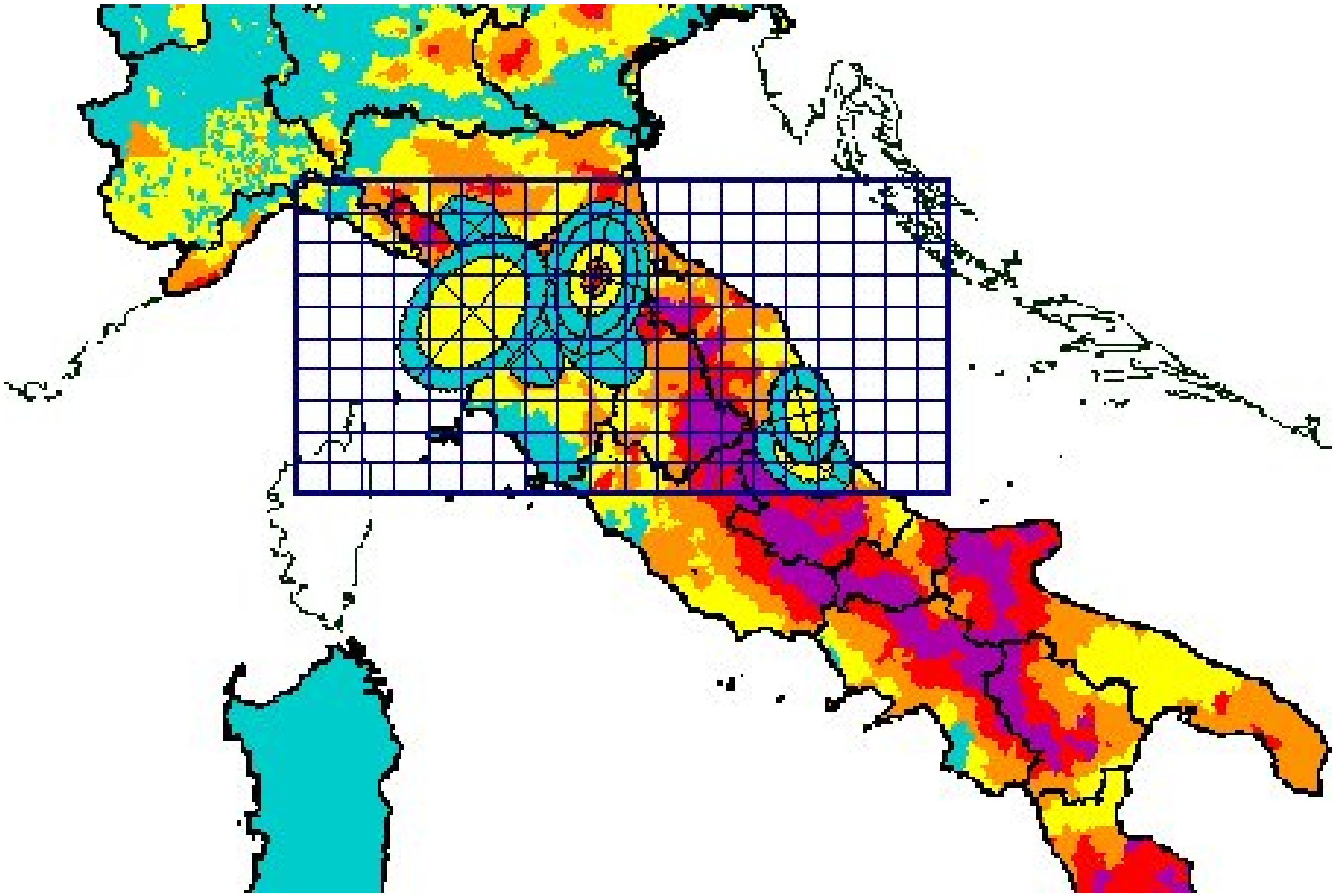
$$1 - F(I) = \exp[\exp(\alpha I^C) - \exp(\alpha I)] \quad (2)$$

$$1 - F(I) = \exp[\exp(\alpha I^C + \beta) - \exp(\alpha I + \beta)], \quad (3)$$

where α and β are parameters from the Table 1, I - simulated intensity, and I^C is a critical intensity.

Class of tectonic plate	α	β	**Distribution of intensities**
1	0.8385	-5,2678	(3)
2	0.9691	-6.6969	(3)
3	0.74447	-5.0468	(3)
4	0.17129	----	(2)
5	0.36373	-1.4810	(3)
6	0.21167	----	(2)
7	0.20078	----	(2)
8	0.05491	2.2900	(3)

Table 1. Relations between the geo-tectonic structure, intensity distribution, and parameters α and β [21].



S. Baranov et.al. - IIASA - YSSP 1999

Ermoliev Y., Ermolieva T.Y et al. IIASA reports IR-97-028, IR-97-068, IR-98-056.

Ermoliev Y. et al. (2000) : “*A systems approach to catastrophe management*”. -
- European Journal of Operational Research, 122 452-460

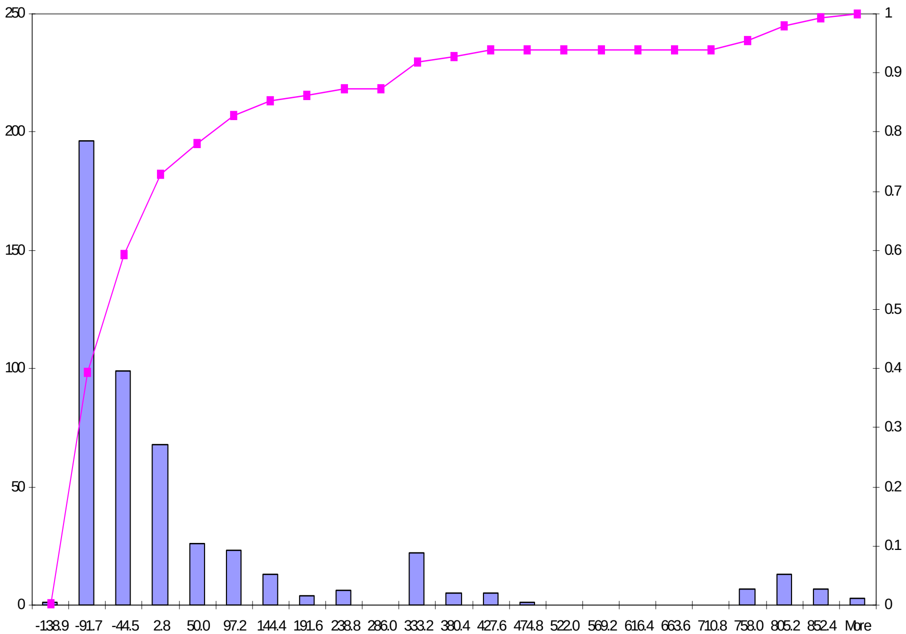
Amendola A. et al. (2000) "*A Systems Approach to Modeling Catastrophic Risk and Insurability* ” –
- Natural Hazards -Volume 21, Issue2/3, May 2000, pp 379/391

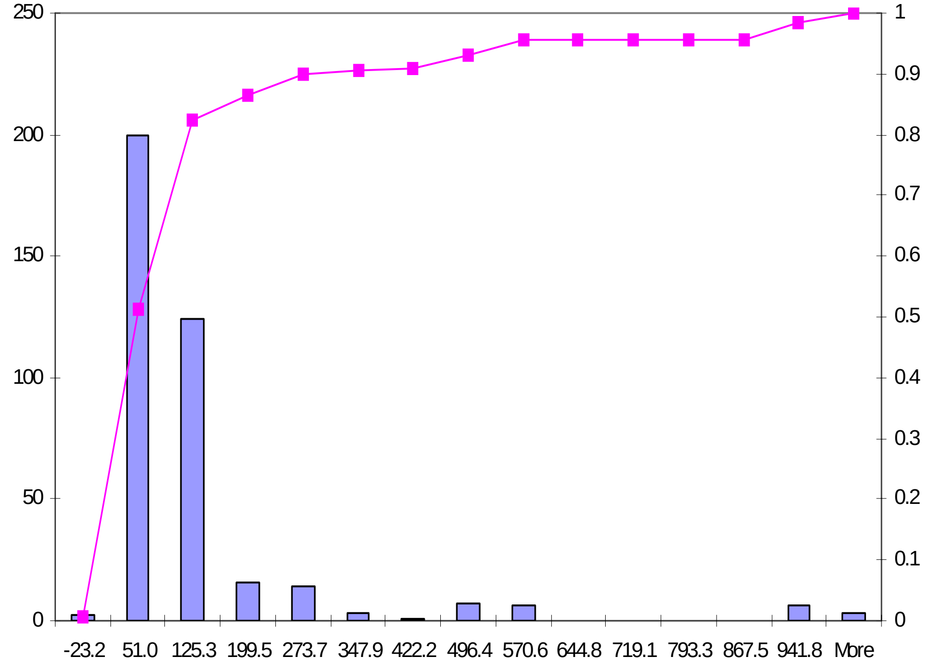
Risks Pricing Scenarios

Scenario 1: Premiums calculated based on the average damage over all municipalities - “egalitarian”

Scenario 2: Location-specific premiums calculated based on average damage in the location -” risk based”

Scenario 3: Premiums calculated according to stochastic optimization procedure “minimizing insurance exposure and minimizing risk of premium overpayment.





Catastrophe bonds pricing problem

EDGE software and financial mathematics tools may be used to solve some problems connected with issuing new catastrophe bonds or evaluating previously introduced ones:

- Pricing problem – finding *present value* for future catastrophe bond cash flows
- Volume problem – finding sufficient amount of issued bonds, assuming that eg. 95% possible losses will be covered
- And others...

Simulation of underlying asset trajectory

The very important task is to simulate trajectory of the underlying asset (eg. index, share). One may use appropriate stochastic equations to model the behaviour of the considered market. After applying equivalent martingale measure method and Monte Carlo simulations, one can generate trajectories of the primary instrument prices.

Examples of stochastic equations

Price of the underlying asset may be modelled by geometrical Brownian motion with a constant drift:

$$S_t = S_0 \exp(\mu t + \sigma \mathcal{W}_t)$$

S_t – price of the underlying asset (eg. share) at time moment t

μ – drift of the underlying asset

σ – volatility of the underlying asset

$\mathcal{W}_t$ – standard Brownian motion

Examples of stochastic equations

After applying the equivalent martingale measure we acquire the formula:

$$S_t = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right) t + \sigma \mathcal{W}_t\right)$$

r – risk–free banking yield

And appropriate equation for Monte Carlo simulations (ie. Euler scheme):

$$S_i = S_{i-1} \exp\left(\left(r - \frac{1}{2}\sigma^2\right) \Delta t + \sigma \sqrt{\Delta t} \epsilon_i\right)$$

$\epsilon_1, \ldots, \epsilon_n$ – iid random variables from $N(0, 1)$ distribution

Examples of stochastic equations

Example of more sophisticated stochastic process (with two kinds of jumps):

$$S_t = S_0 \exp\left(\mu t + k_1 \hat{N}_t^{\varkappa_1} + k_2 \hat{N}_t^{\varkappa_2} + \sigma \mathcal{W}_t + \tilde{N}_t^{\varkappa}(\rho)\right) ,$$

where

$$\tilde{N}_t^{\varkappa}(\rho) = \sum_{i=1}^{N_t^{\varkappa}} \xi_i ,$$

and Brownian motion $\mathcal{W}_t$, compensated Poisson processes $\hat{N}_t^{\varkappa_1}$ with intensity $\varkappa_1 > 0$ and $\hat{N}_t^{\varkappa_2}$ with intensity $\varkappa_2 > 0$, and Poisson process $N_t^{\varkappa}$ with intensity $\varkappa$ are independent

Examples of applications: Price of the cat bond

Catastrophe bond connected with earthquake occurrence of magnitude beyond R degrees in Richter's scale:

$\mathcal{S}_t$ – geometrical Brownian motion with volatility σ

r – risk–free yield rate

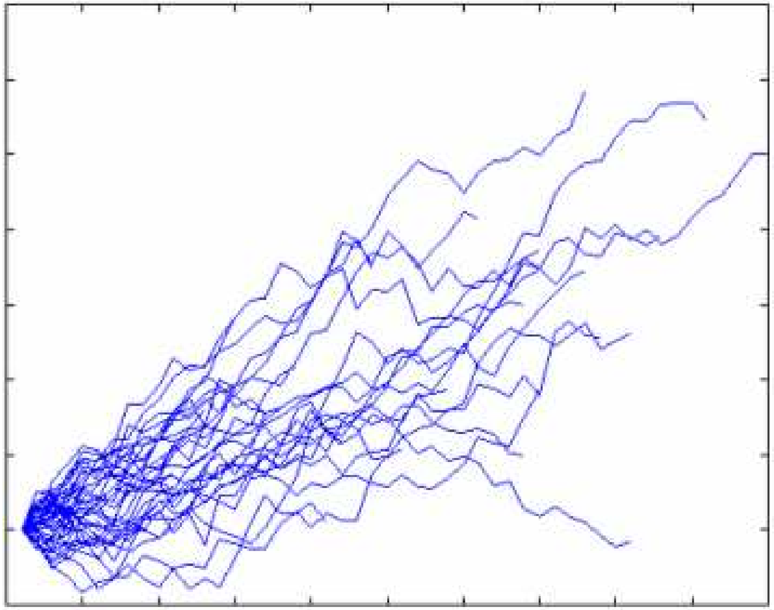
T – termination date of the bond

$\mathcal{X}$ – time moment of the *triggering point*

Payment function $f(\mathcal{S}_t, \mathcal{X})$ for this bond is given by:

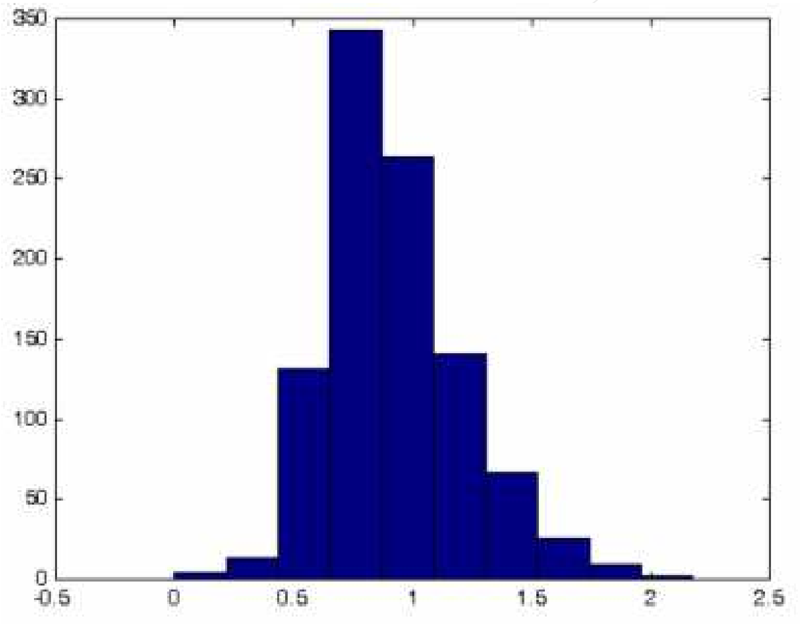
$$f(\mathcal{S}_t, \mathcal{X}) = \begin{cases} \mathcal{S}_T & \text{for } \mathcal{X} > T \\ 0 & \text{for } \mathcal{X} \leq T \end{cases}$$

Examples of applications: Price of the cat bond



Example of simulated pairs: trajectories of the underlying asset prices $\mathcal{S}_t$ stopped in moments of the earthquake occurrence $\mathcal{X}$

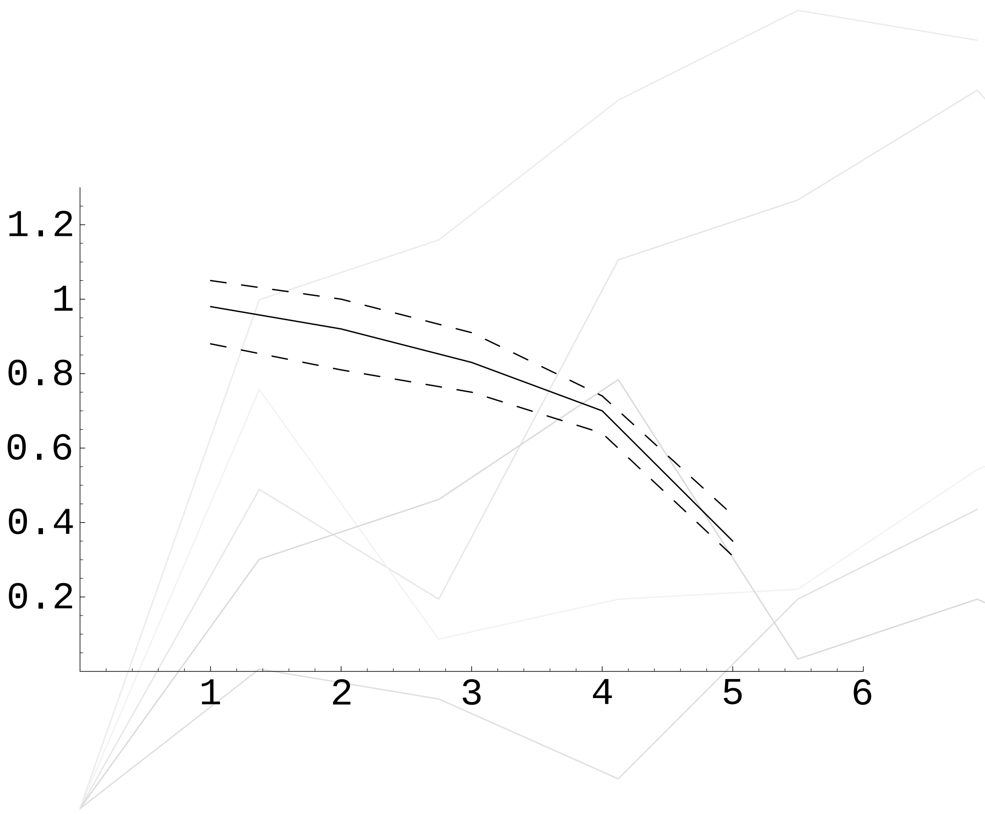
Examples of applications: Price of the cat bond



Histogram plotted for the considered type of catastrophe bond with parameters: $r = 0.5$, $\sigma = 0.2$, $S_0 = 1$, $T = 5$, $R = 7$

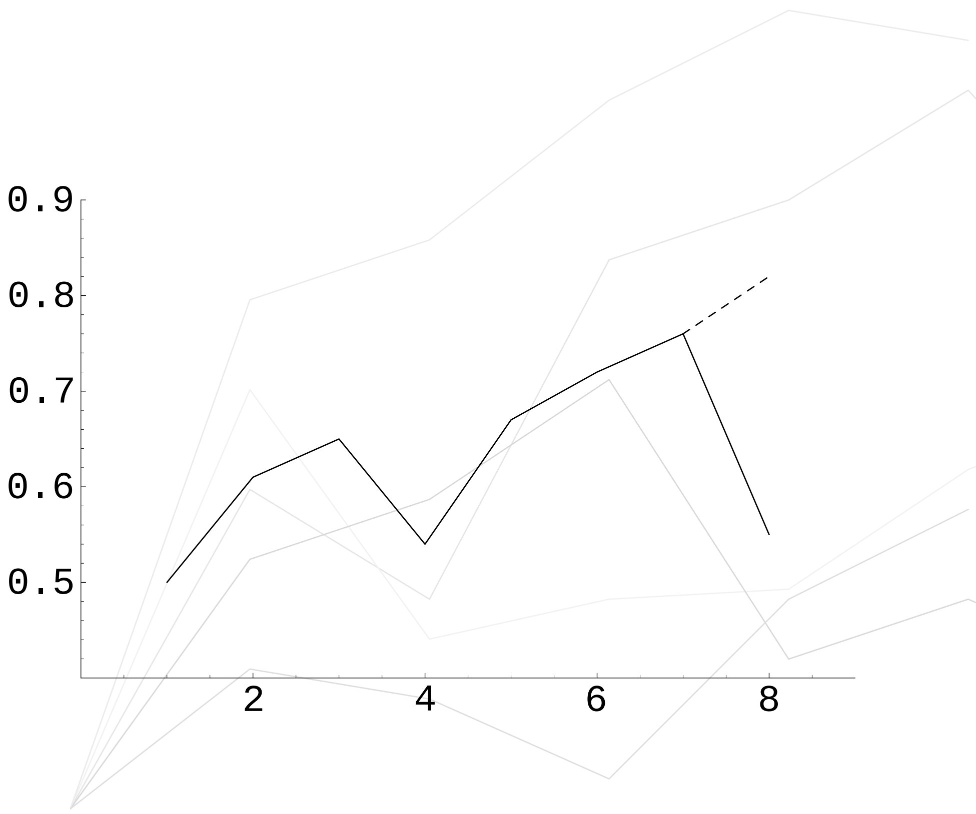
Estimators of cat bond price: 0.9202 (average), 0.8767 (median)

Examples of applications: Multi – Histogram



Prices of a cat bond depended on various values of the one parameter (eg. earthquake magnitude, volatility) with appropriate confidence intervals may be simultaneously plotted on one graph

Examples of applications: Jumps of the asset



The occurrence of catastrophe may have impact on price of the underlying asset – using simulations we may easily generate suitably changed trajectories and calculate cat bond prices in this case

Future developments

- Decision making system – a software which generates possible sets of catastrophe and market scenarios to compare them with various types of insurance instruments, thus finding "optimal" kind of insurance system for fixed confidence level
- Volume problem – finding sufficient amount of issued catastrophe bonds, assuming that eg. 95% possible losses from catastrophe will be covered