# A Multicriteria Approach for the Choice of Remote Load Control Strategies

João Clímaco, Maria João Alves
Faculdade de Economia da Universidade de Coimbra and INESC Coimbra
and
Carlos Henggeler Antunes , Humberto Jorge, António Gomes Martins
Faculdade de Ciências e Tecnologia da Universidade de Coimbra and INESC Coimbra

Keywords: Load Management, Multicriteria Analysis

## 1. Introduction

Load management actions entail changing the regular working cycles of loads through the implementation of appropriate power curtailment actions. These actions consist of on/off patterns generally applied to groups of loads associated with energy services whose quality is not substantially affected by supply interruptions of short duration, such as electric water heaters and air conditioners in the residential sector. Since these are thermostatic loads, external changes to their working cycle have influence on their demand pattern in subsequent time periods. The use of these demand-side resources has been a current practice by electric utilities. The goals of these activities are associated with operational benefits, such as increasing load factor, reducing peak power demand, reliability concerns or loss and costs reduction. More recently, further attention has been paid to this kind of programs mainly due to the volatility of wholesale electricity prices and reliability concerns (transmission congestion and generation shortfalls). These programs, which include direct load control and voluntary load shedding, can be very attractive for a retailer dealing with volatile wholesale prices and fixed, over a certain time period, retail prices. However, load management programs can also give rise to undesirable effects, such as the payback effect (whenever power is restored simultaneously to loads thus eventually creating another peak) and possible reduction in revenues. The selection of load control actions requires the explicit consideration of multiple, incommensurate and conflicting evaluation criteria of the merit of alternative load shedding strategies, capable of reflecting economic, technical and quality of service aspects. Note that the quality of service dimension is crucial because consumers' acceptance is indispensable for the success of load management programs. The model presented in this paper is a slightly altered version of the model previously proposed by Jorge *et al*. (2000), taking into account the main concerns that have an important role in load management. The three criteria intend to minimize peak demand as perceived by the distribution network dispatch centre, to maximize utility profit corresponding to the energy services delivered by the controlled loads and to maximize quality of service in the context of load management.

The well known STEM method was used to analyse a case study using the tricriteria model referred to above. The quality of the solutions proposed in Jorge et al. (2000) is good, but due to the limitations of STEM method it was not possible to make a comprehensive search of the nondominated solution set. This is strongly recommendable namely because the a priori points of view of the electricity supply company and the points of view of the consumers are not compatible. So, the multicriteria interactive analysis should contribute to an indispensable learning process putting in evidence not only all the relevant profiles of well contrasted nondominated alternative options but also providing aids to achieve a satisfactory compromise solution for the various actors.

The software used to reach these goals is based on an interactive reference point method for multicriteria integer linear programming (MCILP) combining Chebychev scalarizing programs with branch and bound techniques (Alves and Clímaco, 2000), and a visualization approach to the indifference sets of reference points dedicated to tricriteria integer linear programming (Alves and Clímaco, 2001). Note that, as nondominated solutions are a discrete set in MCILP, there are multiple reference points that lead to the same solution, i.e. there are indifference sets on the reference point space. The sensitivity analysis procedure included in the method provides the foundations to calculate at least a good approximation of those areas with a small computational effort. The flexibility of the search and the visual interface constitute the main characteristics of the software tool to analyse the tricriteria integer programming model for aiding the definition of remote load control strategies.

In this work we describe in detail the analysis of the tricriteria load management model using the MCILP software package and we compare the results with those obtained using the STEM method.

## 2. Multiple objective model

The multiobjective model presented in this section is the model proposed by Jorge et al (2000) and does not depend on the type of controlled load, though it has been applied only to the control of electric water heaters groups (see section 4). The information on the electricity consumption of the load groups and on hot water temperatures with or without the application of the control strategies to the load groups is obtained with a physically-based load model (Gomes and Martins, 1995). Control strategies define the on/off schedule of load groups, during the period of time where maximum demand control is to be achieved.

**Mathematical model**

The network load diagram is assumed to be known by means of a load forecast procedure. The total control period (ΔT), that must be as long as needed in order to prevent a new peak demand caused by the payback phenomenon, is divided into *n* equal intervals (Δt=ΔT/*n*).

Under load control, demand at each elementary interval is given by:

$$X_i = \sum_j \sum_k (c_{ijk} x_{jk}) + L_i \qquad i = 1, \ldots, n \qquad (1)$$

Notation:

*i* Elementary time interval index (i = 1, …, n)

*j* Load group index (j = 1, …, m)

*k* Control strategy index (k = 1, …, q)

$X_i$ Average network demand at interval *i* with load control

$c_{ijk}$ Difference at interval *i* between load group *j* demand when control strategy *k* is applied to it and load group *j* demand without any control action.

$x_{jk}$ Binary decision variable that assumes the value 1 if control strategy *k* is selected to be applied to group *j,* and 0 otherwise

$L_i$ Average forecasted network demand at interval *i* without load control

The first objective function consists in minimizing peak demand, or, equivalently, in minimizing the maximum network controlled demand, that is: $Min\ Max\ \{X_1, X_2, \ldots, X_n\}$

This objective can be formulated in an alternative way, maximizing the peak demand reduction:

$$Max\ Min\ \{P\text{-}X_1, P\text{-}X_2, \ldots, P\text{-}X_n\}$$

where $P$ is the forecasted network peak demand without control.

This objective is not linear, but it can be achieved solving the following linear problem:

$$Max\ r$$

subject to:

$$P - X_i - r \geq 0 \qquad (i = 1, \ldots, n) \qquad (2)$$

$$r \geq 0$$

where $r$ is a decision variable that represents the network peak demand reduction

The second objective consists in optimizing the profit. This is equivalent to maximizing the revenue variation caused by the electricity consumption variation achieved with the application of control strategies.

Thus, maximizing profit may be stated as:

$$Max \sum_{j} \sum_{k} (R_{jk} x_{jk}) \tag{3}$$

whit $R_{jk}$ given by:

$$R_{jk} = \frac{\Delta t}{60} \sum_{i} c_{ijk} m_{ij} \tag{4}$$

Where $R_{jk}$ is the profit variation corresponding to the consumption variation in group $j$ when subject to control strategy $k$ and $m_{ij}$ is the net revenue perceived by the utility per kWh at interval $i$ by selling energy to group $j$.

The third objective function consists in minimizing the consumers' discomfort. Measuring discomfort caused by control actions is based on the number of loads for which the minimum comfort threshold has been violated.

Minimizing discomfort caused to consumers corresponds to minimizing the following function:

$$Min \sum_{j} \sum_{k} D_{jk} x_{jk} \tag{5}$$

where $D_{jk}$ is given by:

$$D_{jk} = \alpha_A A_{jk} + \alpha_B B_{jk} \tag{6}$$

$\alpha_A$ and $\beta_B$ are coefficients of importance with respect to the accumulated value ($A_{jk}$) and the maximum number ($B_{jk}$) of loads in group $j$ that violate the minimum comfort threshold, when subject to control strategy $k$, and $D_{jk}$ is a measure of discomfort defined as a function of $A_{jk}$ and $B_{jk}$.

The constraints of the model are the following:

a. One control strategy can be applied, at most, to each load group:

$$\sum_{k} x_{jk} \leq 1 \qquad (j = 1, ..., m) \tag{7}$$

b. There is a maximum number of loads that are allowed to violate the minimum comfort threshold. Control strategies that lead to a higher number of loads in such situation are rejected.

$$\sum_k B_{jk}\, x_{jk} \;\le b_j \qquad (j = 1, ..., m) \qquad (8)$$

where $b_j$ is the maximum number of loads in group $j$ that are allowed to simultaneously violate the minimum comfort threshold.

## 3. Analysis of the load management model using the MCILP software package

The study of the above mentioned model using the well-known STEM method, for a problem corresponding to 4000 water heaters aggregated into 8 groups, was reported in Jorge et al. (2000). This problem includes 65 variables (64 binary variables $x_{jk}$ plus variable $r$). In that study, the payoff table (table 1) and five nondominated solutions were computed (table 2).

*Table 1. Solutions that optimize individually each objective function*

| | Max. F1 (KW) | Max. F2 (PTE) | Min. F3 | Control Strategies | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Optimal solution F1 | **514** | 8108 | 4420 | 1 | 2 | 8 | 3 | 2 | 2 | 2 | 2 |
| Optimal solution F2 | 293 | **15402** | 7360 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| Optimal solution F3 | 17 | 2976 | **980** | 6 | 3 | 3 | 6 | 6 | 3 | 6 | 7 |

The analysis of the model using the STEM method proceeded as follows: Firstly, solution STEM-1 was computed. Then, the decision maker decided to relax F2 by Δ2=3500 in order to improve the other objective functions, and the STEM method computed solution STEM-2. The interactive process continued in the same way until the decision maker considered solution STEM-5 a satisfactory compromise solution.

*Table 2. Solutions computed by STEM interactive method*

| | Max. F1 (KW) | Max. F2 (PTE) | Min. F3 | Control Strategies | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Solution STEM-1 | 125 | 12607 | 4030 | 7 | 8 | 8 | 2 | 8 | 8 | 1 | 1 |
| Solution STEM-2 | 364 | 9265 | 2450 | 7 | 4 | 6 | 4 | 2 | 8 | 1 | 2 |
| Solution STEM-3 | 293 | 8072 | 1910 | 7 | 3 | 2 | 4 | 6 | 8 | 5 | 1 |
| Solution STEM-4 | 310 | 8001 | 2100 | 2 | 4 | 4 | 2 | 2 | 8 | 1 | 1 |
| Solution STEM-5 | 319 | 10255 | 2810 | 7 | 7 | 7 | 2 | 8 | 7 | 1 | 1 |

In our study we have considered a slight different version of that model. The differences between the two models are the following: 1) The coefficients of the variables were rounded to integer values, in order to enable the application of a sensitivity analysis tool that computes indifference regions on the reference point space. 2) The group of constraints (7) is replaced by $\sum_k x_{jk} = 1$ ($j$ =1,...,$m$). Note that constraints $\sum_k x_{jk} = 1$ ($j$=1,...,$m$) force that one strategy control is selected for each group. If they were of type "≤", the optimal solution for F3 (which minimizes the discomfort caused to consumers) would not establish any control strategy, leading to (F1, F2, F3) =(0, 0, 0). Although Jorge et al. (2000) have considered constraints "≤", the authors also imposed in the computation of solutions that the variation of profit might be positive, which in practice leads to the same outcome as constraints of type "=" (the type we consider in the model proposed herein).

Tables 1 and 2 show solution values for the altered model (to be consistent with the next results). Nevertheless, they encompass the same control strategies as the solutions presented by Jorge et al. (2000) for the original model, presenting irrelevant differences on the objective function values (differences not over than two units)

In tables 1 and 2, the Control Strategies are identified by a number that represents the control strategy applied in each group (from 1 to 8). For instance, the solution that optimizes F1 (the one that most reduces the peak demand in KW) consists in applying strategy 1 to group 1, strategy 2 to group 2, strategy 8 to group 3, and so on.

Indeed, we believe that solution STEM-5 can be a satisfactory compromise solution, but due to the limitations of STEM method, it was not possible to make a comprehensive search for the nondominated solution set.

The software we have used to study this problem is based on an interactive reference point procedure for multicriteria integer (and mixed-integer) linear programming (MCILP), which combines Chebychev scalarizing programs with branch-and-bound techniques (Alves and Clímaco, 2000). It is mainly devoted to perform directional searches, for which the decision maker only has to specify the criterion he wants to improve with respect to the previous nondominated solution.

This software also provides the visualization of indifference sets of reference points for tricriteria (or bicriteria) integer programs (Alves and Clímaco, 2001). As nondominated solutions are a discrete set in multiobjective integer programming, there are multiple reference points that lead to the same solution, i.e. there are *indifference regions* on the reference point space. A sensitivity analysis procedure provides the foundations to calculate approximations of those regions with a small computational effort. They are only approximations because they can represent a subset of reference points that lead to the same nondominated solution, instead of the whole indifference set.

Let us briefly explain the concept of *indifference region* on the reference point space. Consider that a reference point for a problem with $k$ objective functions is denoted by $(q_1, q_2, ..., q_k)$. This point belongs to the $k$-dimensional real space, the same as the objective functions' space. Thus, for a tricriteria problem, a reference point can be denoted by $(q_1, q_2, q_3)$ belonging to $\Re^3$. We say that a reference point $q^a=\left(q_1^a, q_2^a, ..., q_3^a\right)$ *leads* to the nondominated solution $\bar{x}$ if $\bar{x}$ optimizes the scalarizing program (9) with $q^a$ the vector of parameters.

$$\min \alpha - \varepsilon_i \sum_{i=1}^{k} f_i(x) \tag{9}$$

$$\text{s.t.} \quad f_i(x) + \alpha \geq q_i \qquad i=1,...,k$$

$$x \in X$$

(where $X$ is the feasible region of the multiobjective problem).

Thus, if both $q^a=\left(q_1^a, q_2^a, ..., q_3^a\right)$ and $q^b=\left(q_1^b, q_2^b, ..., q_3^b\right)$ *lead* to $\bar{x}$, then both $q^a$ and $q^b$ belong to the indifference region of $\bar{x}$. Moreover, and independently of the problem, it is known that $(q_1, q_2, q_3)$ and $(q_1+\delta, q_2+\delta, q_3+\delta)$, $\forall\delta\in\Re$, lead to the same nondominated solution. So, if an appropriate constant $S$ is chosen, such that $q_1+q_2+q_3=S$, the 3D graph of indifference regions can be visualized on a 2D graph, which improves its legibility. This can be followed by an appropriate selection of the origin of the graph (assuring that any indifference region has a representation on this graph) and considering, for instance $q_1$ on the $x$-axis and $q_2$ on the $y$-axis. In this representation, $q_3 = S-q_1-q_2$.

To analyze the load management problem, the MCILP software package started by computing the three solutions that optimize individually each objective function, presented above in table 1 (solutions 1, 2 and 3, respectively) and the corresponding indifference regions – figure 1. It should be noted that the graphical representation of indifference regions in figure 1 considers the system of points $\tilde{q} = (\tilde{q}_1, \tilde{q}_2, \tilde{q}_3) = (q_1 \times 10,\ q_2, -q_3)$. Since F1 and F2 are maximizing functions and F3 is a minimizing function, the symmetric of F3 is considered for scalarizing purposes. Further, F1 has been rescaled to F1×10 in order to put the three objective function values on comparative scales. Hence, an original reference point $q=(q_1, q_2, q_3)$ is transformed into $\tilde{q} = (\tilde{q}_1, \tilde{q}_2, \tilde{q}_3) = (q_1 \times 10,\ q_2, -q_3)$ to be used in the scalarizing program and also in the representation of reference point indifference regions. For this representation we have adopted $S$=44 414, with $\tilde{q}_1$ varying from 5140 to 29992 and $\tilde{q}_2$ from 15402 to 40254 (for details on the determination of these bounds, see Alves and Clímaco, 2001).

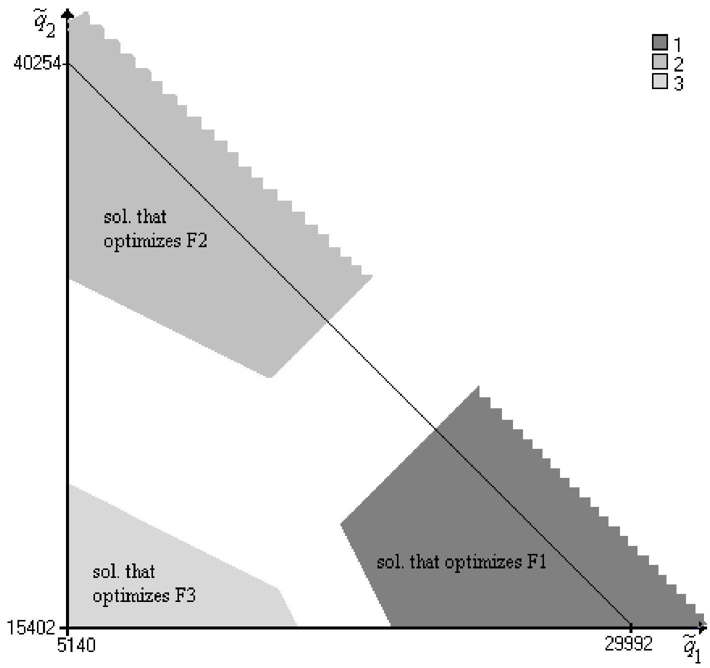


*Figure 1 – Indifference regions for the solutions that optimize individually each objective function*

After computing the first 3 solutions, a directional search was performed starting from solution 2 and choosing F3 to be improved (to diminish the consumers' discomfort), since this was a very sacrificed criterion in solution 2. Twenty-eight nondominated solutions were computed throughout this directional search. These solutions (from 4 to 31) are shown in table 3. We have decided to change the direction because most of these solutions present very low values of reduction on the peak demand (F1).

*Table 3. Solutions computed in the first directional search*

| | Max. F1 (KW) | Max. F2 (PTE) | Min. F3 | Control Strategies | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Directional search starting from solution 2 to improve F3:* | | | | | | | | | | | |
| Solution 4 | 183 | 14652 | 7240 | 8 | 8 | 8 | 8 | 8 | 7 | 8 | 8 |
| Solution 5 | 94 | 14294 | 6930 | 8 | 8 | 8 | 8 | 7 | 8 | 8 | 8 |
| Solution 6 | 273 | 14281 | 6430 | 8 | 8 | 8 | 3 | 8 | 8 | 8 | 8 |
| Solution 7 | 269 | 14034 | 5570 | 8 | 8 | 8 | 1 | 8 | 8 | 8 | 8 |
| Solution 8 | 44 | 13778 | 5180 | 8 | 8 | 8 | 2 | 8 | 6 | 2 | 8 |
| Solution 9 | 279 | 13371 | 5150 | 3 | 8 | 8 | 1 | 8 | 8 | 8 | 8 |
| Solution 10 | 11 | 13228 | 4870 | 8 | 8 | 8 | 2 | 2 | 6 | 2 | 8 |
| Solution 11 | 51 | 13133 | 4430 | 4 | 8 | 8 | 1 | 8 | 6 | 8 | 8 |
| Solution 12 | 23 | 12947 | 3870 | 7 | 8 | 8 | 6 | 8 | 6 | 1 | 8 |
| Solution 13 | 40 | 12784 | 3770 | 7 | 8 | 8 | 2 | 2 | 6 | 1 | 8 |
| Solution 14 | 40 | 12372 | 3700 | 7 | 8 | 8 | 2 | 4 | 6 | 1 | 8 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Solution 15 | 38 | 12175 | 3650 | 7 | 8 | 8 | 1 | 2 | 6 | 5 | 8 |
| Solution 16 | 18 | 12141 | 3390 | 7 | 7 | 6 | 1 | 8 | 6 | 8 | 8 |
| Solution 17 | 48 | 11866 | 3300 | 7 | 7 | 8 | 1 | 8 | 6 | 2 | 8 |
| Solution 18 | 35 | 11732 | 2980 | 7 | 7 | 7 | 2 | 8 | 6 | 1 | 8 |
| Solution 19 | 23 | 11355 | 2960 | 7 | 1 | 6 | 4 | 8 | 6 | 1 | 8 |
| Solution 20 | 44 | 11345 | 2770 | 7 | 7 | 7 | 6 | 8 | 6 | 1 | 8 |
| Solution 21 | 44 | 11196 | 2610 | 7 | 7 | 1 | 6 | 8 | 6 | 1 | 8 |
| Solution 22 | 44 | 11033 | 2510 | 7 | 7 | 1 | 2 | 2 | 6 | 1 | 8 |
| Solution 23 | 44 | 10621 | 2440 | 7 | 7 | 1 | 2 | 4 | 6 | 1 | 8 |
| Solution 24 | 16 | 10463 | 2320 | 7 | 3 | 1 | 6 | 8 | 6 | 3 | 8 |
| Solution 25 | 28 | 10371 | 2300 | 7 | 7 | 1 | 2 | 6 | 6 | 1 | 8 |
| Solution 26 | 16 | 10140 | 2270 | 7 | 3 | 1 | 6 | 8 | 6 | 1 | 8 |
| Solution 27 | 44 | 10088 | 2180 | 7 | 7 | 1 | 6 | 7 | 6 | 1 | 8 |
| Solution 28 | 126 | 10069 | 2170 | 7 | 3 | 1 | 4 | 2 | 6 | 3 | 8 |
| Solution 29 | 16 | 10068 | 2140 | 7 | 3 | 1 | 6 | 8 | 6 | 5 | 8 |
| Solution 30 | 16 | 9977 | 2110 | 7 | 4 | 6 | 4 | 3 | 6 | 1 | 8 |
| Solution 31 | 126 | 9770 | 1990 | 7 | 3 | 1 | 4 | 3 | 6 | 1 | 8 |

Thus, the search was redirected to improve F1 (starting from solution 31). Solution 28 (in table 3) was obtained again and new solutions were then computed (from solution 32 to 36 – see table 4). Solution 36 is identical to solution STEM-2: (F1, F2, F3)=(364, 9265, 2450).

*Table 4. Solutions computed in second and following directional searches*

| | Max. F1 (KW) | Max. F2 (PTE) | Min. F3 | Control Strategies | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Directional search starting from solution 31 to improve F1:* | | | | | | | | | | | |
| Solution 32 | 211 | 9644 | 2250 | 7 | 7 | 1 | 2 | 6 | 8 | 1 | 1 |
| Solution 33 | 262 | 9776 | 2500 | 7 | 4 | 6 | 4 | 8 | 8 | 1 | 1 |
| Solution 34 | 293 | 9497 | 2530 | 7 | 7 | 1 | 1 | 8 | 7 | 5 | 1 |
| Solution 35 | 293 | 9679 | 2610 | 7 | 3 | 2 | 4 | 8 | 8 | 3 | 1 |
| Solution 36 =Sol. STEM-2 | 364 | 9265 | 2450 | 7 | 4 | 6 | 4 | 2 | 8 | 1 | 2 |
| *changing direction to improve F2, considering F1≥300:* | | | | | | | | | | | |
| Solution 37 | 331 | 9646 | 2690 | 7 | 7 | 7 | 1 | 8 | 7 | 5 | 1 |
| Solution 38 | 319 | 9744 | 2760 | 7 | 7 | 7 | 2 | 2 | 7 | 1 | 2 |
| Solution 39=Sol. STEM-5 | 319 | 10255 | 2810 | 7 | 7 | 7 | 2 | 8 | 7 | 1 | 1 |
| Solution 40 | 300 | 10288 | 2960 | 7 | 7 | 7 | 2 | 8 | 8 | 4 | 1 |
| Solution 41 | 362 | 10415 | 3130 | 7 | 7 | 7 | 1 | 8 | 6 | 5 | 2 |

At this stage we considered that F1 had a reasonable value. Besides, values for F1 that satisfy F1≥ 300 would also be acceptable. Therefore, this constraint was imposed and a new search was performed in order to improve profits (F2). Solution 37 was computed (331, 9646, 2690) and new solutions – from 38 to 41 – followed it (see table 4). As expected, all of them present a peak reduction of at least 300KW and growing values on profits. Solution 39 coincides with solution STEM-5. As this had been considered before a satisfactory compromise solution, we decided to make a local analysis around the last solutions obtained.

The value of F3 was considered too high in solution 41 (362, 10415, 3130), so the following bound was imposed: F3≤3000 (which is not satisfied by solution 41). Also, it was imposed F2≥10200, keeping up F1≥300. The nondominated solution closest to the last reference point (which led to solution 41) that satisfies these additional constraints is solution 40 (300, 10288, 2930), already known.

The improvement of F1, satisfying the same constraints, leads to solution 39 (319, 10255, 2810). Figure 2 shows bar graphs for the last three solutions presented to the decision maker (41, 40 and 39, respectively) and the numerical information of solution 39. Relaxing the bound on F2 to F2≥10000, and keeping the other bounds, the outcome for the same reference point is also solution 39. Moreover, if F1 is chosen to be improved from this point, and maintaining the previous bounds, the procedure announces that no better value for F1 is possible. Identical message is shown if F3 is chosen to be improved. Therefore, this region of nondominated solutions was completely explored.

The indifference regions on the reference point space for the nondominated solutions computed are shown in figure 3.

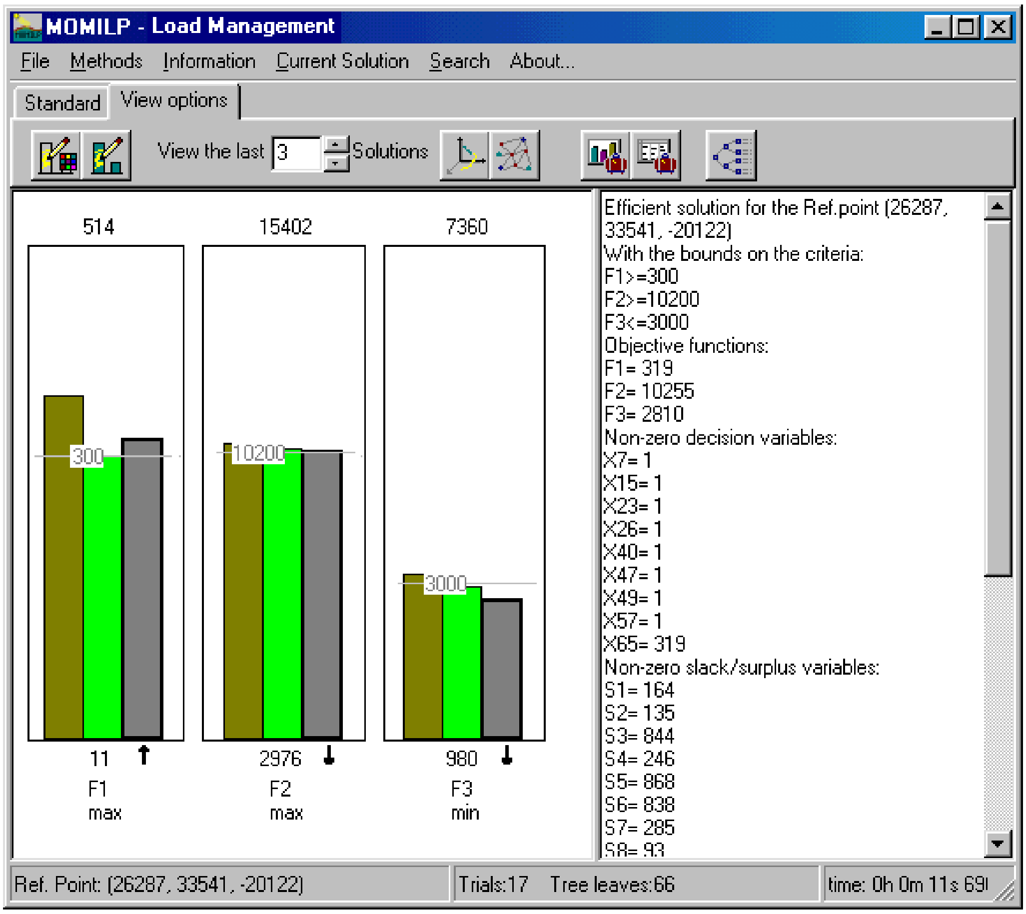


*Figure 2 – Graphs of solutions 41, 40 and 39, respectively, and  and numerical information of solution 39.*

This analysis of the load management problem is just an example of a sequence of directional searches that could be followed. Since the software enables a free exploration of the problem, many other search strategies could be performed, providing more information about the problem. We may further expect that the nondominated set contains a large number of solutions, as suggested by the unfilled areas in graph of figure 3. In fact, from a deeper analysis of this problem, we have realized that more than 200 nondominated solutions exist for this problem. As a curiosity, we can mention that among those 200 solutions only 39 are nondominated supported solutions. In what concerns the search for 41 solutions described above, 12 of them are supported and 29 are unsupported solutions.

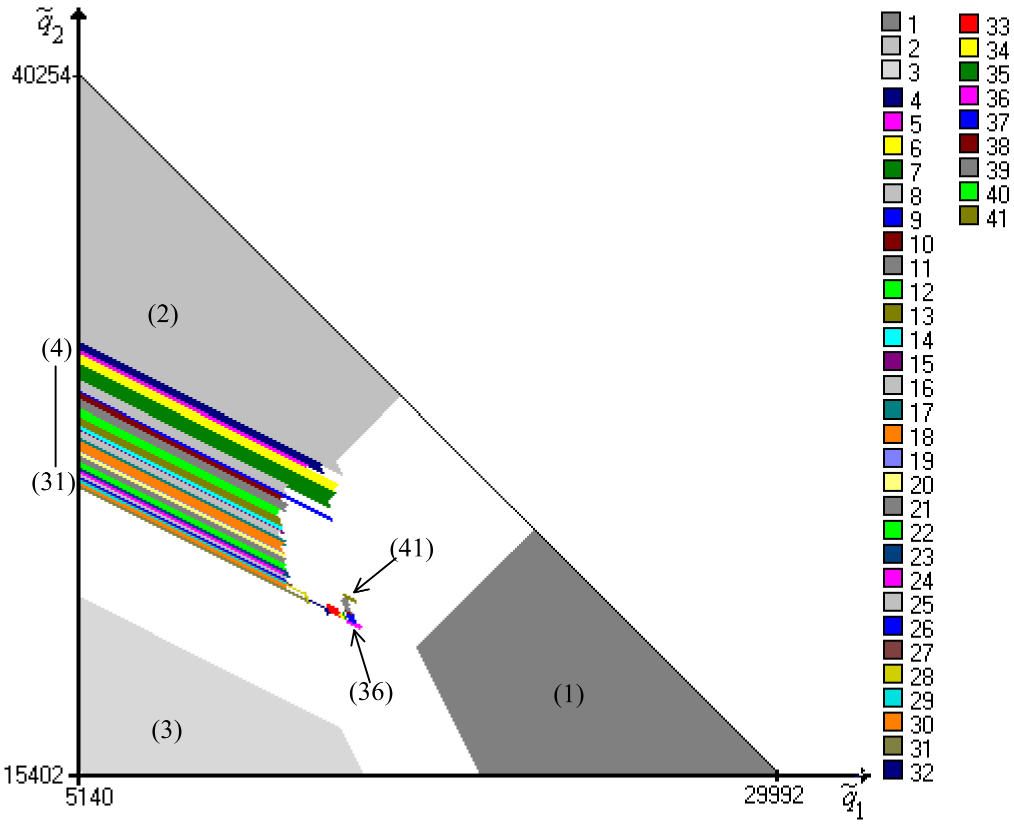


*Figure 3 – Indifference regions for all the nondominated solutions computed*

## 4. Conclusions

The use of this type of search, combined with the graphical display of indifference regions for the nondominated solutions, can provide important information to the decision maker. When compared with the STEM method some advantages can be pointed out. In the STEM method, the decision maker specifies a relaxation quantity for one objective function, in each interaction, in order to improve the other objectives.

However, the decision maker has not the possibility of selecting a criterion he would desire to be privileged in the following computation(s).

Instead, following directional searches the decision maker can choose the objective function he wants to improve at each moment, having also the possibility of imposing constraints on the objective function values. These constraints may be revised whenever the decision maker wants, by relaxing or tightening the bounds. If a local analysis is made (which may be useful in a final phase of the decision process) the decision maker can easy conclude whether a region of interest is completely explored or not.

Moreover, successive solutions obtained throughout directional searches can give a good perception of the geometry of the problem. There exist "consecutive" solutions that present similar values for all the objective functions, but there are others that present very different values in one (or more) objective(s). The decision maker can perceive these asymmetries. An example of the first situation is the transition from solution 21 to 22 (see table 3): F1 maintains its value, F2 diminishes 1.5% and F3 improves 3.8%. Thus, these solutions have close values for all the objective functions. An example of the second situation is the transition from solution 27 to 28: F1 improves 186% just sacrificing F2 on 0.12%, and F3 improves 0.46%. These are "consecutive" solutions with similar values on F2 and F3, but a very different value on F1. So, it can be easily considered that solution 28 is superior to solution 27.

The indifference regions give also an idea of the stability degrees of the nondominated solutions with respect to the intention of improving one criterion.

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