**Title**

Analysis of Close-to-optimal Zones in LP Decision-support Models

**Authors**

Smirnov A, Wagner F, Rovenskaya E

**Affiliations list**

IIASA, IIASA, IIASA

**Abstract**

This project combines the latest insights of different strands of knowledge in order to support environmental policy decision making. Concretely, we use highly efficient computational methods based on mathematical insights of optimization problems to describe all feasible solutions of a linear programming problem that are not optimal but lie “within epsilon” of the optimal solution. These feasible solutions all have some properties in common (not only that they are within a certain cost range, but also that certain functions defined on the solution space (e.g., environmental impact indicators) also lie within certain ranges). In order to identify relevant invariants we project the feasible solutions into two-dimensional planes to make them accessible for direct scrutiny by human eyes. As an example we have applied this method to the GAINS optimization module that is used, inter alia, by European policy makers to design air pollution policies. The method we have developed can be used to efficiently estimate the additional cost for tightening or relaxing environmental constraints. The results generated so far allow us to specifically describe the trade-offs between global and local cost considerations, between different environmental objectives, or between preferences of different regions or countries. The method can also be used to identify directions of the solution spaces that are particularly “flat” with respect to the optimum.

# Analysis of close-to-optimal zones in LP decision-support models

Alexey Smirnov[12*], Fabian Wagner[1], Elena Rovenskaya[12]

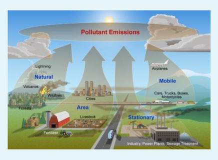
## Motivation

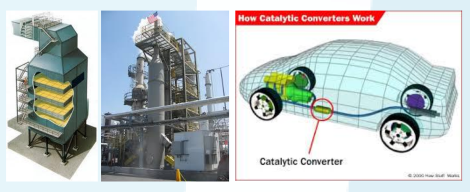
**Example:** reduce air pollution in Europe by 20%

**Flexibility in:**

Location 

Pollutants 


Sectors 

Technology 


With so many possible solutions, which option should be preferred?

**Standard way:** use optimization techniques ("linear programming (LP)") to identify a single – best – solution

**BUT**: we lose **all** information on other feasible solutions !

**Key research questions**

1. What can learn about the many solutions that are "close" to the optimum in large-scale linear programming problems?
2. How can we characterize these zones?

## Method

Consider the following LP problem

(1) Minimize $(c,x)$ subject to $Ax \le b$.

Let $x_0$ be an optimal solution to (1).

Very often besides the optimal vector $x_0$ we are interested in in calculation of various indexes for the value $x_0$, say $I_1(x_0)$ and $I_2(x_0)$, where $I_1$ and $I_2$ are given **linear** functions.

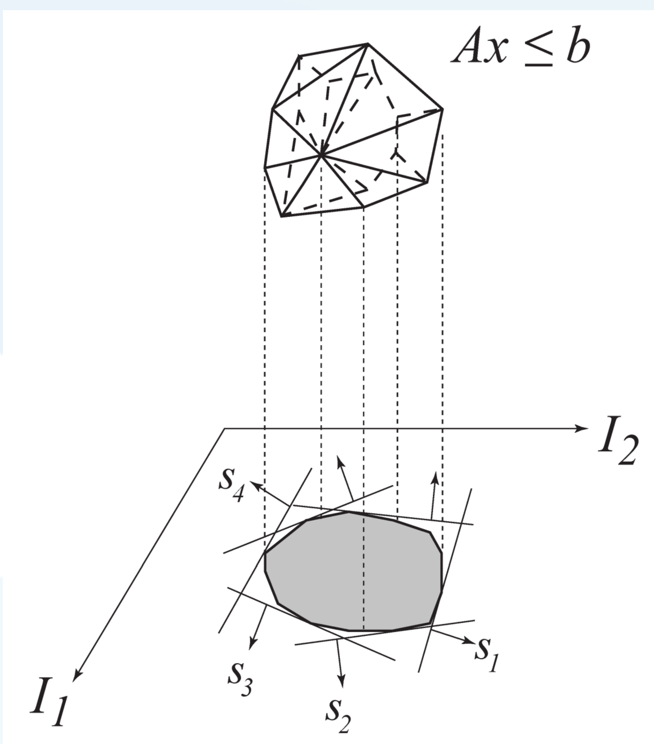
The idea of the proposed project is to look wider on the system (1) and instead of getting a single optimal solution point to investigate the **whole range of possible values** for the indexes $I_1$ and $I_2$.

Actually the goal is very straightforward, we take all feasible points $x$ which satisfy the constraint $Ax \le b$, calculate indexes $I_1$ and $I_2$, and put the resulting point on the plane $(I_1, I_2)$. As a result we will get a closed convex set on the plane. We can name it the **attainability zone** for the LP problem (1).

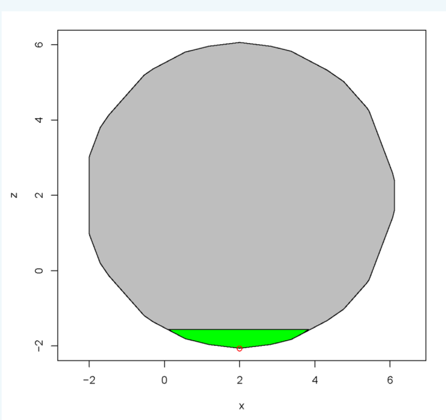
Unfortunately it is impossible technically to implement such a straightforward procedure even in the low dimensional cases. To get the result we must iterate too much points!

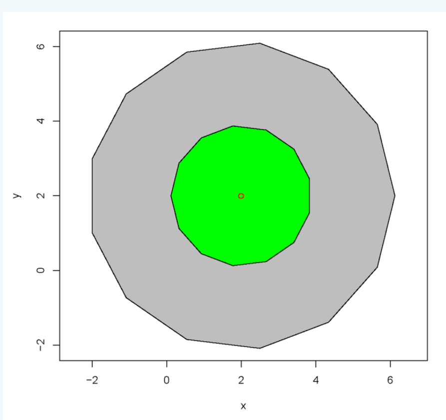
However there is a constructive approach which need **drastically less** number of iterations and constructs only the **boundary** of the attainability zone on the plane $(I_1, I_2)$.

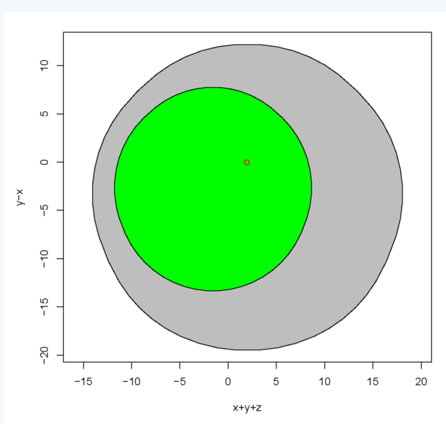
The set of all feasible points $Ax \le b$ is a given convex set of points in a high dimensional space. The two linear indexes $I_1$ and $I_2$ define a projection of this set into two-dimensional plane. The projected set will be also convex. To get the boundary points of this set we iterate unit vectors $s_i$ from two-dimensional sphere and solve the special LP problem close to (1). In this way we get a polygon which gives the outside approximation of the attainability zone. The more vectors $s_i$ we take the better approximation we get.




## Results from a toy LP model

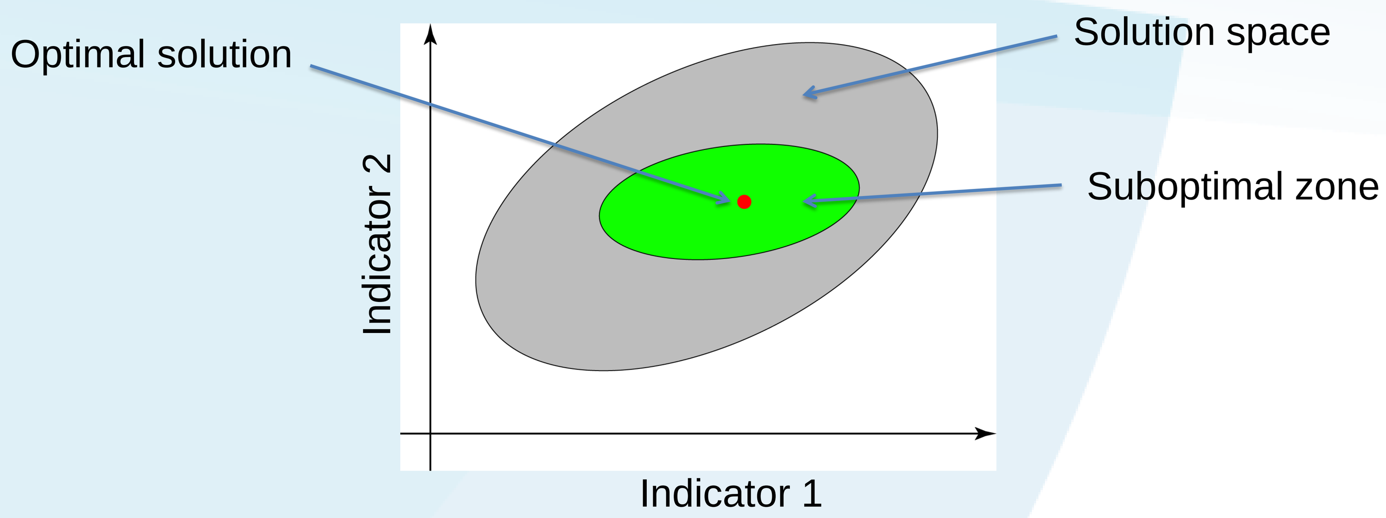






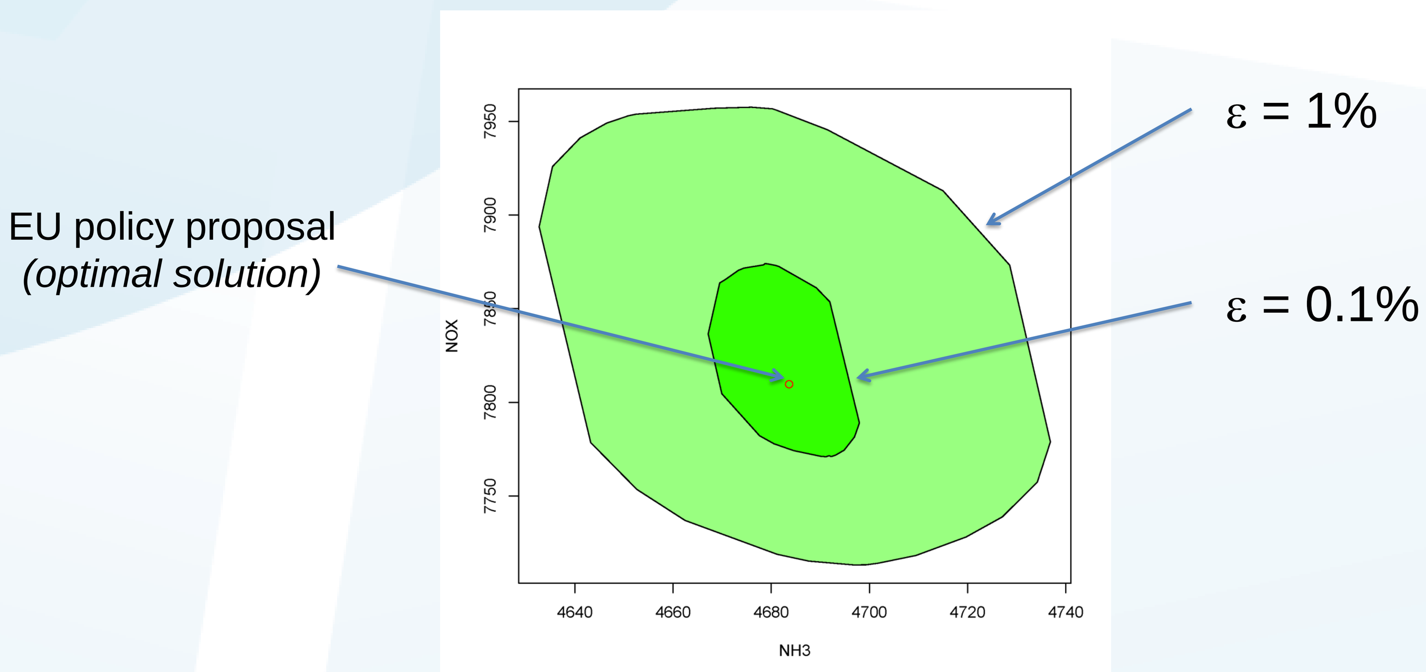
## Methodology

1. The project develops a methodology of computation and visualization of sets of suboptimal (i.e. with allowed gap for the objective function compare to optimal value) solutions in large-scale LP optimization models.
   - Focus on zones rather than individual points

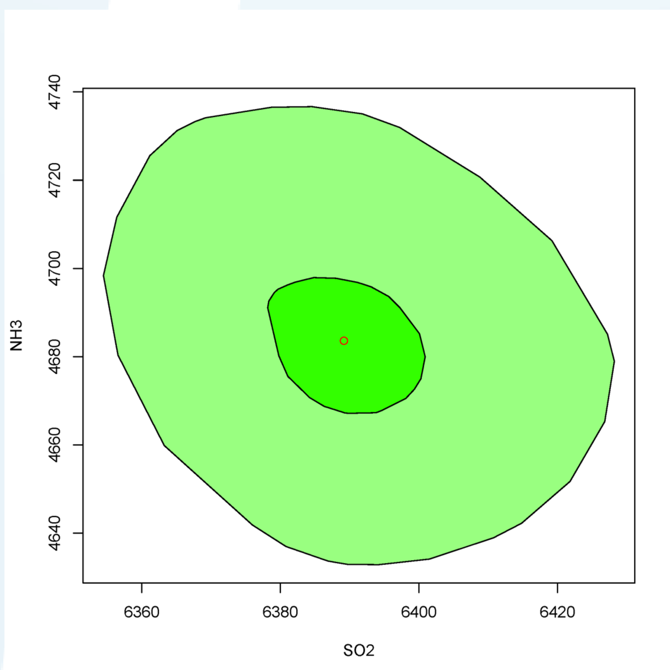


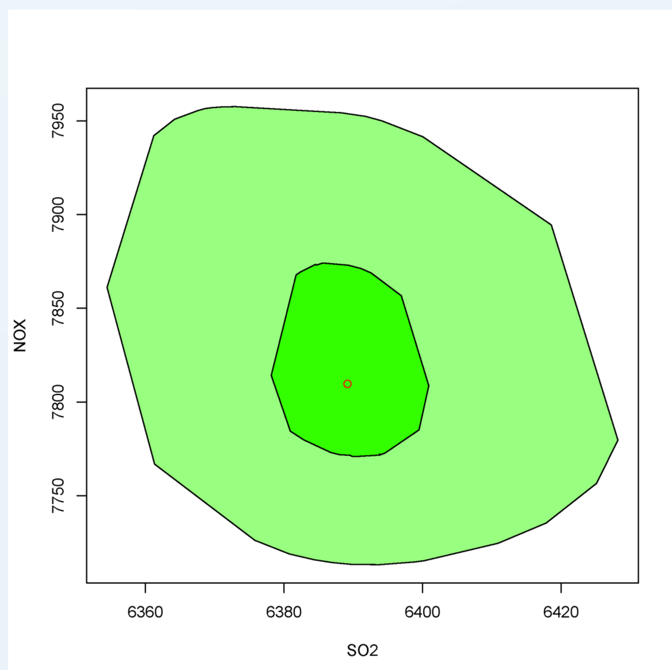

2. The key idea of visualization approach is that in many cases researches analyze the model outcome based on values of a number of indicators calculated for the optimal solution.
3. The proposed method gives a computationally efficient way to generate 2D graphs of values of a pair of indicators for the set of model outcomes close to optimal solution.

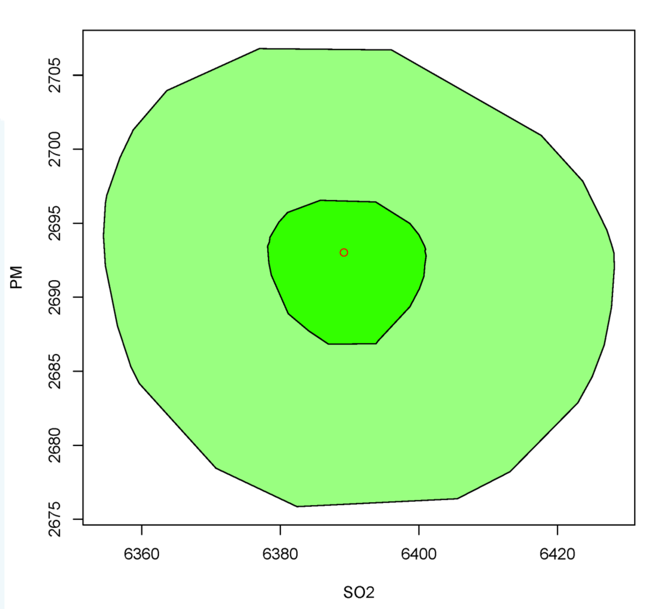
## Results from GAINS

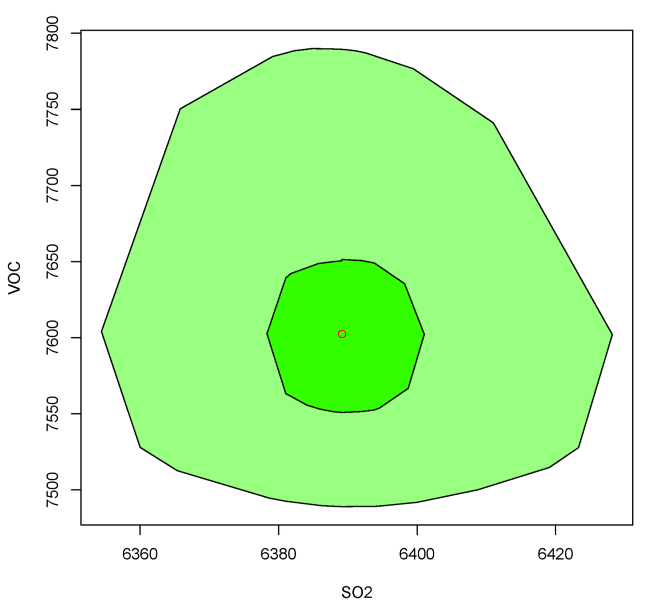



**Trade-offs between 5 Pollutants: $SO_2$, NOx, PM, $NH_3$, VOC**




[1] International Institute for Applied Systems Analysis, Laxenburg, Austria; [2] Lomonosov Moscow State University, Moscow, Russia

* Contact e-mail: smirnov@iiasa.ac.at