Customizing Reinsurance and Cat Bonds for Natural Hazard Risks*

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Abstract:

This paper has the following two objectives: to examine how reinsurance coupled with new financial instruments can expand coverage to those residing in areas subject to catastrophic losses from natural disasters, and to show how reinsurance and the catastrophe-linked financial instruments can be combined so that the price of protection can be lowered from its current level.

To address these two questions we define the key stakeholders and their concerns with respect to catastrophic risks. We then construct a simple example to illustrate the relative advantages and disadvantages of catastrophe bonds and reinsurance in supporting a structure of payments contingent on certain events occurring (e.g. a severe flood in Poland or a major hurricane in Florida). On the basis of this comparison we suggest ways to combine these two instruments to expand coverage to those at risk and reduce the cost of protection. We suggest six principles for designing catastrophic risk transfer systems, and describe how they may be put into practice. The paper concludes by raising a set of questions for future research.

The unexpectedly large insured losses from Hurricane Andrew in the Miami, Florida area in 1992 ($15.5 billion) and the Northridge earthquake in California in 1994 ($13.5 billion) has forced the insurance industry to reevaluate whether it can provide coverage to all property in hazard-prone areas against catastrophic losses in the future. New institutions have been created such as windstorm pools in Florida and the California Earthquake Authority (CEA) to supplement or replace traditional reinsurance. At the same time the capital markets have developed new financial instruments such as Act-of God bonds to provide protection against these large losses from natural disasters.[1] To date, these new instruments have only made a small dent in the market for protection against the financial consequences of catastrophic events, although there is the expectation by many that they will play a larger role in the future.

Our approach is to examine whether the private market can offer ways to provide financial backing to deal with these risks. More specifically, the private market can provide hedges against catastrophic risks through catastrophe-linked securities, traditional excess-of-loss reinsurance and certain customized reinsurance coverage schemes. This paper has the following two objectives:

(1) to examine how reinsurance coupled with new financial instruments can expand coverage to those residing in areas subject to catastrophic losses from natural disasters, and

(2) to show how reinsurance and the new financial instruments can be combined so that the price of protection can be lowered from its current level.[2]

To address these two questions we begin our analysis by defining the key stakeholders and their concerns with respect to catastrophic risks. We then construct a

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[1] For more details on these new developments see Kunreuther and Roth, Sr. (1998).

[2] Our focus in this paper is *not* on determining the optimal price for a catastrophe bond, or evaluating the relative attractiveness to buyers of such a security compared to any other given financial instrument. Issues of "how much premium must investors in such bonds receive, to compensate them not only for the expected losses when catastrophes occur, but also for the variation in the return they receive on their securities?" and "what percentage of an investor's portfolio should be invested in such instruments?" will be subordinated to asking "What advantages and disadvantages might such securities have for the insurers who would be issuing them as alternatives to traditional or customized reinsurance contracts?"

simple example to illustrate the relative advantages and disadvantages of catastrophe-linked securities and reinsurance in supporting a structure of payments contingent on certain events occurring (e.g. a severe flood in Poland or a major hurricane in Florida). On the basis of this comparison we suggest ways to combine these two instruments to expand coverage to those at risk and reduce the cost of protection. We suggest six principles for designing catastrophic risk transfer systems, and describe how they may be put into practice. The paper concludes by raising a set of questions for future research.

I. Key Stakeholders and Their Concerns

Although each country faces a different set of hazards and has a different set of institutional arrangements, the common thread is how to offer greater protection to potential victims. There are a set of key stakeholders, each of whom has their own set of objectives and concerns, that need to be considered when designing a set of financial arrangements to provide them with protection after a major disaster occurs.

Homeowner and Businesses at Risk are concerned with having insurance to cover their losses because they are risk-averse. In particular, they are willing to pay a relatively small price today to protect themselves against a large decrease in wealth tomorrow from a natural disaster. The ultimate risk they face is insolvency or bankruptcy (or a dramatic forced reduction in consumption) and they are anxious to avoid this state of the world.

Insurers offer protection against these risks by taking advantage of the law of large numbers. In other words, if they have a large enough (and sufficiently diversified) portfolio they should be able to collect sufficient premiums to cover their losses even if worse-than-average periods occur. Even if the insurer is, on average, profitable, An unusually severe catastrophe can make even a well-capitalized insurer insolvent even if the insurer is, on average, profitable.[3] A natural disaster, such as an earthquake, raises problems for them because of the high correlation among the losses in their portfolio. This dependence among risks may require them to raise premiums and/or reduce the extent of their coverage in hazard-prone areas to keep their chances of insolvency to an acceptable level.

Reinsurers provide protection to private insurers in the same way that insurers cover the policyholder or property owner — that is, they provide coverage against unforeseen or extraordinary losses. In a reinsurance contract, one insurance company (the reinsurer, or assuming insurer) charges a premium to indemnify another insurance company (the ceding insurer) against all or part of the loss it may sustain under its policy or policies of insurance. For all but the largest insurance companies, reinsurance is almost a prerequisite for offering insurance against hazards where there is the potential for catastrophic damage. Reinsurers have similar concerns to those of the insurers and hence will limit their exposure in catastrophe-prone areas. They will follow the same options listed above for insurers to keep the chances of insolvency to an acceptable level.

Investors in Catastrophe Bonds will want to obtain a large enough return on their investment, in the form of higher than normal interest rates when no disaster occurs, to justify the risks of losing their principal and/or receiving a lower interest rate after a disaster.

Government is the reinsurer of last resort and has the ability to tax different stakeholders to raise money for providing financial payments when there is a catastrophic loss. A sovereign government which faces a hard debt constraint, however, may find borrowing additional funds difficult or impossible if a catastrophe were to strike.

II. An Illustrative Example

To better understand how these different stakeholders interact we construct a simple example consisting of two property owners, each of whom is subject to the probability (**p**) of a loss (**L**) from a natural disaster, such as a flood. There are thus four states of the world depending on whether or not each individual suffers a loss:

[3] That is, even if their revenues and investment gains exceed their administrative costs plus expected claims (*including* the costs of fully funding catastrophic losses if these losses were to occur in their expected proportions over a long period of time).

(0,0) ; **(0,L)** ; **(L, 0)** ; and **(L,L)**. The specific assumptions and analyses are detailed in the Appendix. Here we present the qualitative results; they hold more generally for the realistic case where there are many properties at risk.

Independent and Correlated Risks

Even in the simple case where the risks are independent, there is one state of the world where both individuals suffer a loss: state **(L,L)**, which occurs with probability $\mathbf{p^2}$. This state results in a net loss for the insurer unless it charges a premium much greater than the actuarial rate (See Table 1 in the Appendix). If the two homeowner risks were perfectly correlated (as would be the case if both properties were equally affected by a hurricane or earthquake), then there are only two states of the world: **(0,0)** and **(L,L)** and there is a much higher probability (**p**, rather than $\mathbf{p^2}$) that the insurer will suffer a net loss (See Table 2 in the Appendix). In reality, the damages to property from catastrophic events are partially correlated, so that the distribution of losses, insurer profits, and the probability of negative earnings fall somewhere between the two extremes.

Two generalizations follow from the above analysis. For independent risks, the probability of a large net loss (as a percentage of total premiums collected) in any given scenario falls as the number of insured increases, due to the law of large numbers. As the risks become more correlated, the chances of a large negative net profit in any given scenario increases for any given insured portfolio.

If the insurer cannot rely on other sources of funds (e.g. reinsurance, cat bonds, or government bailout) for protection against the catastrophic loss, then it has two ways of protecting itself: (1) it can have surplus capital available to pay for these losses should they occur, or (2) it can raise premiums to the point where insolvency is impossible even in the worst case.

Insolvency risk occurs when an insurer with a limited capital base covers a book of business that includes the possibility of losses exceeding the insurer's ability to pay. The insurer is most likely to declare insolvency in an **(L,L)** state and thus pay off only a portion of its claims, precisely when the property owners are most in need of capital.

More specifically, the risk-averse homeowners will be denied the beneficial option of offloading their risks and stabilizing their own wealth levels.

If a catastrophe will create insurer insolvency, and policyholders will not be fully paid, the expected value of the insurance policy decreases and hence the policyholders' willingness to pay for coverage. If the insurer charges lower premiums to generate demand, then its chance of insolvency increases even further, thus reducing insurance demand even further. This downward spiraling of premiums and upward spiraling of insolvency risk may eventually produce a situation where the insurer would prefer not to offer this type of coverage at all because it cannot cover its marketing and administrative costs. The market will thus fail to clear, as an indirect consequence of the insolvency risk, leaving consumers uninsured against moderate-level risks as well as catastrophes.

Role of Reinsurance, Financial Instruments and the Government

Suppose the insurer would like to protect itself against losses that exceed its premiums by turning to reinsurance or catastrophe-linked bonds, and that it is willing to pay at least a small premium over actuarial cost to do so. What are the relative merits and challenges in each of these forms of protection against a catastrophic loss (i.e., the event **(L,L)** for this example)?

Excess of Loss Reinsurance

Until recently a standard reinsurance contract was an excess-of-loss policy whereby the reinsurer would pay a claim to the insurer if the insurer's total loss were to exceed some fixed amount (e.g., $1 billion) called an *attachment point*. The reinsurer would reimburse the insurer up to 100% of its loss in excess of the attachment point.

By purchasing this type of protection, the insurer's distribution of profits is smoother than before, because it has received an extra layer of coverage from the reinsurer at the time of a catastrophic loss. In other words, by paying premiums to the reinsurer when it suffers no losses or small losses, the insurer avoids a large loss in state **(L,L)**. On the other hand, this type of reinsurance requires a constant premium payment, independent of the types of normal (non-catastrophic) losses that are incurred.

If the reinsurer provides a large amount of coverage to insurers, it faces the possibility of a net loss when the state of the world is **(L,L) –** perhaps so large that it must be concerned with insolvency just as the insurer was (see Table 3a of the Appendix).[4] This *credit risk* will impact the insurer's demand for reinsurance coverage in a similar way to the policyholders' reduced demand for primary insurance above. Let **q** represent the probability that the reinsurer becomes insolvent. The higher the value of **q**, the less interested the insurer will be in an excess-of-loss contract, and hence the lower the premium it will be willing to pay for this coverage. This may lead to a breakdown in the insurance-reinsurance relationship -- a similar problem to the failure of the primary insurance market. To reduce credit risk is thus a key objective in developing more effective policies for transferring catastrophic risk.

After a catastrophe, it is desirable for policyholders that claims be paid as quickly as possible. Traditional reinsurance requires a claims settlement and audit process to occur before payments are forthcoming. This would mean a time delay of weeks or months before the cash flows from these claims become available to policyholders. If it were to pay major claims in advance of reimbursement, an insurer would likely become technically insolvent. The short-term cost of capital, both to the insurer and to the policyholders, may be very high immediately following a catastrophe.[5] Compared to the benefits from immediate payment, the economic cost to policyholders of having the payments delayed by three months may be equivalent to the economic costs of delaying payments for years in normal circumstances.

The reinsurer, on the other hand, may have some concern that the insurer will create greater risks than it otherwise would, now that the insurer knows it is protected against large losses. This moral hazard problem has been analyzed in some detail by Bohn and Hall (1996) and Doherty (1997). There are two principal ways that the insurer

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[4] Even with reinsurance, the insurer will still face the possibility of a loss in state **(L,L)** as shown in Table 3b in the Appendix.

[5] Consider the widely anticipated shortage of liquidity surrounding January 1, 2000. Global short-term rates, as measured by the three-month eurodollar interest rate futures contract, have been anticipated to be disturbed by as much as 0.3% -- almost a 10% proportional change in the cost of capital (Commerzbank, 1999). Given that this "catastrophe" is widely anticipated and its timing perfectly foreseen, we can forecast that the disturbance in short-term costs of capital would be even greater if it resulted from an *unforeseen* catastrophic loss.

could behave to increase the reinsurer's risk. With reinsurance protection the insurer could write more policies in the hazard-prone area, since it will collect premiums when there are no losses and only be responsible for a portion of the losses should a disaster occur. The insurer could also shirk in its claims adjustment and payment process by being more lenient than it otherwise would be and devoting less time to analyze the losses, knowing it will only be partially responsible for covering them.

State-contingent Reinsurance

A more flexible form of reinsurance is one in which the payment to the insurer is a function of the different states of the world. In other words, rather than providing a fixed excess-of-loss contract with stable premiums, the reinsurer could make its payments contingent on the severity of the catastrophic event itself, rather than on the monetary loss that the insurer suffered. Conflicting estimates of the severity of the disaster itself by the insurer and reinsurer can be turned over to a disinterested third party for arbitration. The advantage of using the catastrophic event itself, rather than the insurer's losses from it, to determine payments by the reinsurer is that it eliminates shirking and other forms of moral hazard.

This extra degree of freedom over an excess-of-loss contract enables the reinsurer to provide a stream of payments that can meet the insurer's needs more precisely, while receiving a large enough return, in the form of reinsurance premiums, to find the contract attractive. In order to compare the performance of this type of state-contingent reinsurance policy with a standard excess-of-loss contract, we consider a payment structure in our example in the Appendix which satisfies the following three conditions:

Condition 1: In states of the world where net insurer profits would otherwise be negative, the reinsurance provides sufficient payments to the insurer so that net insurer profits are zero.

Condition 2: In the state of the world where net insurer profits are highest, the reinsurance contract requires payment from the insurer.

Condition 3: The reinsurer is provided with a positive expected net return exactly equal to the expected net profit obtained by an excess-of-loss reinsurance contract.

This type of reinsurance contract still has similar problems to the standard excess-of-loss contract in that there is a credit risk associated with insolvency of the reinsurer, time delay of payments by the reinsurer, and moral hazard costs associated with the behavior of the insurer. The relative magnitude of each of these types of risks depends on the nature of the contract. (See Tables 4a and 4b in the Appendix.)

Catastrophe Bonds for Insurers

Rather than turning to the reinsurance market for protection against catastrophic losses, the insurer may want to utilize catastrophe-linked bonds (henceforth referred to as *cat bonds*) for protection. A cat bond requires the investor to put money up front, which could be used to pay for claims if some type of triggering event were to occur. This event could be a specific loss reported by the insurer (e.g. $1 billion), in which case a cat bond would be very similar to a reinsurance contract, having time delay risks to the insurer and moral hazard risks to the investors. In return for this commitment of funds, the investor receives a higher-than-normal interest rate in time periods during which no disaster occurs. The impact of two types of cat bonds on insurer's and investor's profitability in different states of the world are presented in the Appendix (See Tables 5-8).

There is one major difference between the two instruments: the insurer does not face any credit risk from the cat bond because the money to pay for the losses is already in hand (usually deposited in escrow, and invested in short-term liquid securities). The first cat bond issued by USAA in June 1997 to protect itself against cat losses from hurricanes in Florida is an interesting example of this type of instrument. In this case, there were two tranches, geared towards different types of investors. Tranche one paid only a modest interest-rate premium above LIBOR but investors would lose only their interest payments if USAA suffered hurricane losses during a 15 month period which exceeded $1 billion. Tranche 2 offered a higher premium over LIBOR but the investors' entire principal was at risk in case of severe hurricane losses by USAA.[6]

Most of the cat bonds which have been issued since the USAA offering are tied to a loss index (e.g., total insured losses from an earthquake in California) or to a disaster

[6] For more detail about USAA's financing decision, see Froot and Seasholes (1997). Bantwal and Kunreuther (1999) discuss pricing structure of recent cat bonds.

severity index (e.g., paying amounts for earthquake damages based on the Richter-scale measurements at specific locations in Japan) rather than to the insurer's losses.[7] If the index is independent of actual losses (as in the case of a disaster severity index), the insurer cannot manipulate the claims. This eliminates the moral hazard issue and the requirement for an audit and adjustment process. Hence claims to insurers can be made immediately after the disaster rather than being subject to a time delay as in the case of reinsurance.

On the other hand, such a cat bond may create **basis risk**. Basis risk refers to an imperfect correlation between the actual losses caused to the insurer and the payments received from the cat bond. Traditional excess-of-loss reinsurance has zero basis risk because there is a direct relationship between the loss and the payment delivered by the reinsurance instrument. A cat bond based on some verifiable, non-manipulable index (e.g., aggregate insurance industry losses, the Richter or Saffir-Simpson scales, total rainfall in Rangoon during August) is subject to basis risk. In other words, the insurer's book of business may not be accurately represented by the index, and therefore the insurer's losses will not be perfectly correlated with the actual payments from the cat bond triggered by the index.[8]

In choosing whether cat bonds or reinsurance provides the better hedge against catastrophe one needs to make the following comparisons:

- the extent of the cat bond's basis risk in relation to the reinsurer's credit risk;
- the more immediate payoffs from cat bonds compared to the delayed timing of payments from reinsurance;
- the costs of implementing each financial protective measure;
- the costs of each instrument (i.e., the premium for reinsurance and higher interest rates for cat bonds) required to provide protection and deal with possible moral hazard problems from the insurer.

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[7] For more details on the structure of recent bonds see Insurance Services Office (1999).

[8] See Major (1996) for a more detailed description of basis risk, and the effect of basis risk on insurers' ability to address catastrophic losses.

Catastrophe Bonds for Reinsurers

Reinsurers can also turn to cat bonds as a way of coping with its losses in state **(L,L)** and reducing its credit risk. As in the case of insurers, this added protection should smooth the returns to the reinsurer. Reinsurers will have incentives to invest in differently-structured cat bonds than those desired by insurers. Reinsurers are concerned only with losses when a catastrophic disaster occurs, whereas insurers may desire protection even in non-catastrophe states of the world. If the payments from bond investors to the reinsurer are directly tied to its losses, then there are timing-of-payment and moral hazard problems but no basis risk. If, on the other hand, these payments are tied to a general index, then the reinsurer faces basis risks. An example of the impact of a cat bond to reinsurers and investors is presented in the Appendix (Tables 9 and 10).

Government Reinsurance

It may even be necessary for a governmental entity to assume the responsibility of "insurer of last resort." A well-financed and liquid government may be able to "bail out" otherwise insolvent homeowners, insurers, reinsurers, or investors. Claims can be reimbursed immediately and then the required funds to replenish the Treasury can be collected via taxes during later periods. Although nobody enjoys paying taxes, they are less painful when their assessment does not coincide with the natural disaster.

A "cat tax" could easily be devised which requires payments by citizens during non-catastrophe periods, the proceeds for which would fund a subsidy during times of catastrophe – similar to a state-contingent cat bond.[9] Consider, for example, a tax (imposed on the homeowner/policyholders) collected only in state (**0,0**) -- when no claims (catastrophic or otherwise) have been incurred. If this tax can be applied to "bail out" an otherwise insolvent part of the insurance system in the same period in which it is collected, for example, it may serve as an important component of a hedging system.

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[9] One major benefit of such a plan is that, for a well-capitalized sovereign government with access to the capital markets after a catastrophe, claims can be reimbursed immediately and then the principal value collected via taxes during later periods. For sovereign governments whose access to the capital markets may be impaired (or prohibitively expensive) after a catastrophe, there may be some merit to considering reinsurance or *ex ante* catastrophe-tied financing at the sovereign level (Croson, Richter, and Kleindorfer, 1999).

Without actively determining prices in the insurance or reinsurance market, the government may thus play an important role in smoothing out the insolvency risks faced by all parties.

III. Principles of System Design for Cat Risk Transfer

Some combination of insurance, reinsurance and cat bonds with government reinsurance as the financial instrument of last resort can meet the objectives of the different stakeholders. There are some guiding principles that can be utilized for an optimal system design which are discussed below. We then propose a combination of risk-bearing instruments for providing protection against losses from natural disasters.

The following six principles should be followed in designing any system of protection against the financial consequences of catastrophic risk: *utilize scientific risk estimates, develop incentives to reduce moral hazard, expedite settlement for catastrophic claims, link premium payments to non-catastrophic claims periods, employ capital-market instruments to reduce credit risk*, and *customize risk transfer instruments to address basis risk.*

Principle 1: Utilize Scientific Risk Estimates

New scientific studies, extensive engineering analyses of built-up areas and advances in information technology (IT), offer an opportunity to estimate the probabilities of, and potential losses from, future disasters significantly more accurately than in the past. More sophisticated risk assessments have reduced the uncertainty associated with estimating the probabilities that earthquakes and hurricanes of different intensities and magnitudes will occur in specific regions.

Engineering studies, building on the experience of past disasters, have provided new information on how structures perform under the stress of natural forces. The development of faster and more powerful computers enables one to combine these data in ways that were impossible even five years ago. These advances should give a more complete picture of property at risk, be able to simulate potential damages from hypothetical future disasters, and thereby support a common understanding among buyer and seller of catastrophic risk-transfer instruments of the risks to be handled.

Suppose one wished to develop appropriate prices (whether based on willingness-to-pay to reduce risk or on willingness-to-accept to assume responsibility for it) for each layer of protection. A stakeholder concerned with cat-risk transfer can utilize these risk estimates to specify both the expected losses as well as the chances of a net loss for any given portfolio. By putting uncertainty bounds on these estimates, we can construct confidence intervals around these point estimates to examine "worst case" scenarios. The Wharton Managing Catastrophic Risk project has utilized data from three leading modeling firms (Applied Insurance Research, EQE and Risk Management Solutions) for constructing three model cities (Miami/Dade County, FL; Long Beach, CA; and Oakland, CA). The project has examined the financial risks to a set of hypothetical insurance companies under a wide variety of different scenarios regarding different choices of reinsurance and loss mitigation. (For more details see Kleindorfer and Kunreuther, 1999).

Principle 2: Develop Incentives for Reducing Moral Hazard

Consider two parties who engage in a contract. Moral hazard is created because of asymmetry of information about the other party's actions, and divergence of incentives between these two parties. The catastrophic-risk context of moral hazard takes the form of a classic principal-agent problem. The entity who provides protection (the principal), such as a reinsurer, cannot completely monitor the behavior (selection of underwriting risks, effort invested in accurate claims adjustment, investment in mitigation, etc.) of the entity that receives protection (the agent), such as a primary insurer.

There are several ways that the design of a protection system can reduce the burden of moral hazard. For one thing, the principal can require the agent to share part of the risk so the agent will have an incentive to behave more carefully (e.g., invest more in mitigation, cautious underwriting, or accurate claims adjustment). In the context of insurance arrangements, *deductibles* and *coinsurance* can be introduced into the insurance contract. A sufficiently large *deductible* can act as an incentive for the insureds (the agents) to continue to behave carefully after purchasing coverage because they will be responsible for covering a portion of their loss before the insurance (or reinsurance) begins to pay. All small losses (those below the deductible) therefore end up entirely the responsibility of the agent, who thus has strong incentives to ensure that these small losses do not occur.[10]

With *coinsurance* the insurer (principal) and the insured (agent) share the loss in some pre-negotiated proportions. An 80 percent coinsurance clause in an insurance policy means that the insurer pays 80 percent of the loss (above a deductible), and the insured pays the other 20 percent. As with a deductible, this type of risk-sharing arrangement encourages more responsible behavior because the insured party wants to avoid having to pay for some of its losses. Coinsurance encourages the agent to consider the expected cost of future disasters, including losses from catastrophic events, in determining whether to take protective actions. However, the agent does not have nearly as strong an incentive to eliminate small losses as with deductibles. She incurs only 20% of the benefit from eliminating them but bears 100% of the costs of doing so. (For more details about the impact of moral hazard on the provision of insurance coverage, see Pauly, 1974.)

Another way of avoiding moral hazard is to place restrictions on the agent's actions, or to structure the contract to shape the agent's incentives to perform such actions, once the agent has entered a contract. Reinsurers, who sell excess-of-loss reinsurance to insurers in catastrophe-prone areas, may explicitly prohibit insurers from writing more policies in that area. Alternatively, such reinsurers may insist on basing their contracts on insurers' current policies in force at the time of the contract writing. This will discourage insurers from aggressively marketing new policies in a hazard-prone area to collect full premiums and incur only partial losses in case of a disaster.

Finally, and perhaps most importantly, one can rely on long-term repeated relationships and the importance of trust to discourage the agent from taking advantage of the principal. For example, an insurer (agent) who overstates its losses so it can collect from the reinsurer (principal) will have a problem getting protection in the future if this

[10] This incentive for the agent to prevent small losses may come at the expense of making large losses more likely than they otherwise would have been. Such induced moral hazard bears further study.

action is discovered. This threat of non-protection may be enough to discourage it from cheating if the following three conditions are satisfied:

(1) the probability of discovery is sufficiently high;

(2) the benefits from maintaining the ongoing relationship are significant;

(3) the difference between the payments under truthful statement of losses and the payments from overstated losses is modest.

Condition 1 can be addressed by inspecting the damage before paying claims (the role of the audit process). Condition 2 is likely to be met if the pricing contract between the two parties produces large enough profits to both parties that they have incentives to continue their relationship. Condition 3 can be satisfied using customized contracts or securities which closely track the insurer's portfolio, thereby eliminating both basis risk and the incentive to misrepresent actual claims.

Principle 3: Expedite Settlement for Catastrophic Claims

Consumers of primary insurance (e.g., homeowners subjected to flood risk) experience a sharp decrease in their wealth immediately after a natural disaster. They thus place the highest marginal value on reimbursement from their insurance company precisely when the damage to their property is the greatest. Any change in the system of catastrophic risk transfer that leads to expedited delivery of payments to homeowners (and their primary insurers) will increase the consumers' willingness to pay for primary insurance and increase the value of reinsurance to an insurer -- thereby potentially paying for itself.

Principle 4: Link Premium Payments to Non-Catastrophic Claims Periods

To smooth out their own incomes, insured consumers want to pay their premiums during times when they are not otherwise suffering losses. A catastrophe-linked security (or a customized reinsurance policy with similar terms and conditions) could separate the timing of cash-flow events (payment of premium and settlement of claims) from the provision of economic value through insurance. An economic benefit can be created simply by matching these cash flows to the appropriate times: non-loss states for premium payments, and immediately upon catastrophic events for reimbursement.[11] Traditional reinsurance requires a steady inflow of premiums regardless of whether normal (non-catastrophic) losses are incurred, and also does a less-than-perfect job at providing immediate liquidity to primary insurers so that these emergency cash flows can be provided when needed.

Principle 5: Employ Capital-Market Instruments to Reduce Credit Risk

The emergence of cat bonds for providing protection against large-scale losses offers the best opportunity to reduce credit risk. Because the principal amount of these bonds is held in escrow until the time period covered by the cat bond has elapsed, the funds are always available when needed. In other words, the credit risk from these source of funds is *zero.* In contrast, reinsurers could become insolvent if they are faced with potential claims after a catastrophic loss that exceeds their surplus. The challenge will be to find a price for protection that is simultaneously attractive to the investor (principal) as well as the insurer or reinsurer (agent) who is protected by the bond.

Principle 6: Customize Risk Transfer Instruments to Address Basis Risk

There has recently been an emergence of modeling firms who employ mathematical techniques to evaluate the risks associated with different insurance and reinsurance portfolios. These firms employ simulation tools to quantify risks, project the effects of future disastrous events on an insurer's portfolio at risk, and construct an estimated probability distribution of future insured losses.

As these models become more accurate representations of insurers' actual risks, they offer an opportunity to design customized contracts to reduce the basis risk associated with any given form of protection (whether reinsurance or cat bonds) without necessarily increasing credit risk or moral hazard costs. More specifically, it should be

[11] This desirable property is certainly not unique to cat bonds; a custom reinsurance policy with an appropriate waiver-of-premium rider can mimic the premium and payout structure of each of our proposed cat bond structures in the Appendix.

possible to design contractual relationships between principals and agents where an index of loss is constructed around the estimates of these models. This would represent an improvement over the existing indices, such as the Richter scale intensity for earthquakes in Japan or flood height of the River Oder in Poland.

IV. Elements of A System Designed According to Risk-Transfer Principles

We now turn to the design of an integrated system of insurance, reinsurance and cat bonds based on the six principles outlined above. Each country, with its own set of institutional arrangements, will develop their own package of risk-bearing instruments from the private and public sectors; these principles ought to be incorporated into the design of any such set of arrangements.

State-Contingent Payment Structures

Theoretically, the construction of a financial instrument (whether a cat bond or a customized reinsurance contract) whose returns depend on future states of nature is straightforward -- provided that all of these future states can be identified, described, and have consequences attached to them. The *number* of distinct states is irrelevant to this analysis (limited only by the ability of the contracting parties to collect and assimilate data.)[12]

Customized Indices

Suppose such an index of all possible outcomes could be constructed ahead of a catastrophe, mapping observable inputs into (possibly probabilistic) outputs. A catastrophe-linked security based on this index could then be constructed, with its payouts contingent on the state of the world observed *ex post*. Here are some of the

[12] Although a theoretical model could support a continuous index variable, we plan to model the conditions on which such catastrophe-linked instruments may be designed via an actual set of data measurements, which as a practical matter must be finite in number -- leading technically to a discrete distribution of possible outcomes, even though the number of data points may be very large and the differences between them small.

features of this type of index that will make it attractive to both the buyer and seller of the catastrophe-linked security:

(1) The proper index would be agreed on ahead of time by both buyer and seller;

(2) both sides are convinced that the value of the index could be reliably determined and reported without risk of moral hazard or manipulation;

(3) the index chosen correlates highly with the anticipated damages (claims against the insurer's book of business) which would occur if a catastrophe happened. (This feature is especially significant for the insurer, who would like to gain a substitute for reinsurance coverage through issuing this instrument.)[13]

In practice it may be difficult to construct a complete contingent list of outcomes, which buyers and sellers agree comprises all possible future states – the required condition for a catastrophe-linked security to be a perfect substitute for a fully customized reinsurance policy. A more realistic alternative is to utilize the predictions of the consequences of catastrophes of different magnitudes, from one or more of the modeling firms, to generate the index on which the claims will be based. Claims are then paid based on the *predicted* outcome, rather than the actual outcome. The residual risk between the amount paid based on the model's outputs and the actual claims suffered by the insurer are either retained by the insurer or hedged with a customized reinsurance policy. The benefit of this refinement is that paying claims based on an existing index-style policy solves a significant moral hazard problem (that insurers may not exercise full care in adjusting claims) without forcing insurers to use an index which does not match their book of business (a source of basis risk).[14]

The motivation for such a potential solution is based on the idea that an existing model of a catastrophic event may represent both sides' best available information at the

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[13] Major (1996) notes that a ZIP-based model accounts for 99.5% of the actual variation in claims after Florida hurricanes, leaving model basis risk of only 0.5%.

[14] Compare the intuition behind this system to that of the two-stage least squares (2SLS) process of estimating a regression equation in econometrics. If the instrumental variables (the model) are highly correlated with the actual data (the disaster), the fit from the two-stage process will be good (in the cat-risk sense, basis risk will be low). By using the model rather than the actual data (losses), we avoid a problem (in econometrics, endogeneity of independent variables; in cat-risk transfer, moral hazard.)

time that a hedging decision must be made. If both sides are using Company XYZ's modeling expertise to determine their own exposures, the data generated by XYZ analyses represents their best estimate of damages. Both sides plan to use these outputs in constructing a reinsurance contract or security. It seems a natural extension for the ceder and acceptor to agree in advance that the added sophistication of the model could be used *after* a catastrophe to resolve the amount of damages to be paid, thus saving costly claims verification and accelerating payouts.

The two sides could, for example, agree to run the model after a catastrophe, using as parameters whatever readily measurable model inputs could be reliably collected (*e.g.*, wind speed at several predesignated locations, temperature, angle of incidence of a tropical storm with the coastline, etc.). They would then base the amount of funds transferred after a disaster on the model's simulated amount of damage – a predictable and verifiable figure. For such an arrangement to work, it is critical that both buyer and seller of coverage could verify that the inputs which generated these figures reflected the characteristics of the actual catastrophe.[15] Buyer and seller could then use the model (or, in the case of proprietary models owned by a third party, could commission a simulation to be done on their behalf) to support their decisions about how to structure and price their agreement.[16]

Intertemporal Basis Risk

Even using a customized index, an insurance company employing this technique will be left with "intertemporal basis risk." This refers to the risk associated with changes in the book of business from the time when the model was used to price the policy. Note that claims will be paid based on the *initial* book of business. Presumably,

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[15] Also important is that the model's prediction after the catastrophe would have been the same regardless of whether the specific inputs represented a *hypothetical* catastrophe (from a model run before signing the contract) or an *actual* recent catastrophe (the output of the model after signing the contract.)

[16] If the model employed Monte Carlo simulation, two runs of the model, even with the exact same measurably inputs, would not produce exactly the same results. A predeterminedly large sample size would reduce the width of the confidence interval around the mean estimate to an arbitrarily small size. Computational requirements would be lessened because such a large sample size would not need to be computed *ex ante* for any possible scenario, but only *ex post* using one specific set of parameters measured after a disaster.

however, the insurance company can always alert the reinsurer to changes in composition of its book of business and have its policy adjusted at any time before a catastrophe.

Certainly, there may be some transactions costs for updating the reinsurance policy. There may also be a problem of adverse selection, in which insurers selectively update their "book of business of record" with the reinsurer only when such changes are favorable to them, and neglect to update the model otherwise. If the insurer's goal is truly to reduce its basis risk, rather than speculate on whether the model is pricing particular risks correctly, this adverse selection effect should be minor. In addition, giving the insurer any incentive to update their contract with the reinsurer, no matter how selectively, can only improve on the current situation, where the insurer has no incentive to update the contract at all.

Model Basis Risk

Using a customized index any systematic deviation of the actual catastrophe from the model's prediction would not be insured – an example of "model basis risk." It should be noted that this risk is not currently insured through traditional or custom reinsurance, nor through catastrophe bonds. Both buyers and sellers use acknowledgedly imperfect models on which to base their contracts with one another, and are thus exposed to errors in these models. Thus, any treatment of this model basis risk, no matter how minor, will improve on the status quo where the primary insurer bears all of this risk.

The impact of this model basis risk can be alleviated through diversification. Much as an investment portfolio can reduce the variance of its return by diversifying among different assets, so an insurer can create a portfolio of individual cat bond (or customized reinsurance) investments to diversify basis risk. A custom bond or reinsurance arrangement could also be designed to generate payouts conditioned on a weighted combination of different companies' models. This "meta-model" would reduce the impact on the insurer of the deviation of actual claims from a particular model's predicted claims.[17] Such cross-model diversification reduces basis risk and comes at no

[17] Such an instrument would be the cat-risk equivalent of an "asset allocation" fund, a mutual fund designed to provide automatic diversification across different asset classes. For example, assume that Model ABC and Model XYZ have similar (and uncorrelated) levels of basis risk. Consider an insurer

additional cost of moral hazard, as the marginal incentives for the insurer (to underwrite accurately, invest fully in judging claims, and so on) remain unchanged.

Customized reinsurance may also be used to deal with model basis risk. By definition, determining the exact deviations between model and actual claims will be impossible until after the catastrophe has occurred. A custom "model gap" reinsurance policy, which would reimburse the insurer for any shortfall between what the cat bond pays and actual claims, could combine with the cat bond to effectively provide synthetic 100% excess-of-loss reinsurance. The reinsurer will have very little information about how to price such coverage (all of the model's information having been captured in the design of the cat bond, before the reinsurer is asked to absorb the residual risk not explained by the model). Due to the uncertainty about the probabilities of loss and the distribution of loss severity, a custom reinsurer will thus need to charge a very high "ambiguity premium" for such a policy, especially in the early stages of market development before the extent of model basis risk is fully known.[18]

How large will this risk covered by the "model gap" reinsurance policy be? This policy will be responsible for paying for insurer's claims which are not reimbursed by the custom cat bond. Assuming that (a) the overall correlation of the model with the actual claims will be quite high, and (b) that this custom policy need pay only when the model *underpays* a claim (which presumably happens approximately 50% of the time), the pure risk-bearing portion of this customized reinsurance policy will be low -- perhaps as low

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concerned with basis risk replacing $1 billion of 0%-coinsured coverage based on Model ABC's predicted losses with either

(1) $1 billion of 50%-coinsured coverage based on Model ABC's predicted losses and $1 billion of 50%-coinsured coverage based on Model XYZ's predicted losses, or

(2) $1 billion of 0%-coinsured coverage based on a 50%-50% weighted average of models ABC and XYZ's respective predicted losses.

Option (1) represents constructing a diversified portfolio of individual cat bonds; Option (2) represents a customized instrument embedding a meta-model. Either combination yields the same expected amount of coverage, and the insurer will have diversified the basis risk of Model ABC in either case.

The payouts from the latter combination will have lower variance, however, and thus investors in this combination will require less of a premium than on the "pure" ABC or XYZ instruments -- leading to a lower cost to the insurer. Indeed, it is possible for the total basis risk of the portfolio to decline even if Model XYZ has higher basis risk than Model ABC. If the two models' predictions are very highly positively correlated, however, the gains from this diversification strategy will be modest.

[18] Given the difficulty of a reinsurer's accurately modeling this risk, one would expect that such a policy would be priced according to the primary insurer's willingness to pay, rather than directly based on a reinsurer's expected costs of providing such coverage.

as 1% or less of the total covered amount. Presumably, given that this custom reinsurance policy would not bear a particularly large amount of the disaster risk, the correlation between claims on the "model gap" reinsurance policy and claims on the cat bond should be low.

This custom reinsurance policy will also, however, concentrate all the moral hazard problems from all layers of insurance coverage in its own very small coverage layer.[19] Inasmuch as the purpose of a fully indexed catastrophic instrument based on predicted claims is to eliminate all basis risk except for the model basis risk, without introducing costs of moral hazard -- as close to a first-best solution as is technologically feasible -- such an instrument should *not* be combined with custom reinsurance unless 100% indemnification is absolutely necessary. Reintroducing a customized reinsurance policy to completely eliminate the model basis risk brings back all of the traditional moral hazard costs; the reinsurer's requirement of being paid for taking on these costs of moral hazard will greatly reduce the cost benefit to the insurer.[20]

IV. Conclusions and Suggestions for Future Research

The principal purpose of this paper has been to understand more fully the tradeoffs between different ways of financing catastrophic risk using the private sector. There are several areas on which more research is needed to more realistically characterize the problems facing different stakeholders concerned with catastrophic risks.

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[19] These costs may turn out to be prohibitive unless bundled with a sufficiently *large* amount of pure insurance, priced at above expected losses. The combination implied by such a customized reinsurance policy of a small amount of pure insurance, along with a large amount of concentrated moral hazard costs, may make it impossible for the insurance market to organize such a policy. Compare this insurance market failure to the similar problem caused by adverse selection in dread-disease health insurance markets (Thatcher, 1998).

[20] In particular, the insurer will once again have the incentive to report claims in excess of what the customized-index model predicted, because they will now receive incremental reimbursement from their custom reinsurance policy -- whereas before their incremental reimbursement as a result of their overstatement was zero. The insurer will also have an incentive to over-write policies in the catastrophe-prone area, because the custom reinsurance policy will cover their losses even when the model discounts them. Finally, the insurer will under-invest in adjusting and monitoring claims, because the custom reinsurance program fully indemnifies them against losses. A more severe collection of moral hazard risks brought on by a smaller reduction in risk is difficult to imagine.

Nature of the Risk

There is imperfect information on the probability and losses associated with the risks from natural hazards. This will play a role in influencing the premium required by insurers and reinsurers as well as the return required by investors. Improvement in simulation models of catastrophic events will not only reduce basis risk for insurers, but also reduce reinsurer ambiguity about how to price customized reinsurance policies. Some open research questions relating to the nature of the risk are:

(1) Can more accurate models significantly reduce basis risk?

(2) To what extent can customized contracts for disasters achieve substantial reductions in basis risk?

(3) What information do insurers, reinsurers, and capital market participants need to better assess risks, and how do these stakeholders anticipate obtaining this information in the near future?

Capacity Constraints

There are often constraints on how much reinsurance, on an absolute dollar basis, can be issued due to capacity restrictions triggered by a concern with insolvency by the reinsurer. Following major disasters there is normally a shortage of reinsurance so that insurers are seeking other sources of funds. If cat bonds are attractive enough to insurers relative to reinsurance, or if reinsurers can employ cat bonds themselves, then this capacity problem may be eased considerably. Some open questions are:

(1) How much extra effective capacity does the nascent use of catastrophe-linked bonds by primary insurers generate for the reinsurance industry?

(2) How should reinsurers use cat bonds to most effectively increase their own capacity?

(3) What, if any, are the limits on the capacity of reinsurers to provide customized reinsurance, including policies to cover residual basis risks?

Determinants of Insurer Adoption of Customized Instruments

Foppert (1993) (quoted in Major, 1996) points out that basis risk generates "significant practical and philosophical barriers" to insurers' employing index instruments to hedge catastrophic exposure. If insurers are able to eliminate most of the basis risk via a model-indexed custom policy, they may be willing to hold the residual model basis risk. By taking this step, they will effectively avoid paying not only the risk-bearing cost of reisnurance, but also the extra charge for moral hazard.

Given the magnitude of the moral-hazard component identified in Section III of the price of custom reinsurance, the willingness by insurers to retain the residual model basis risk induced by custom instruments will be a key step towards theit widespread adoption. We hypothesize that insurer adoption of these arrangements will rise when the following four conditions occur:

(1) insurers become more confident that the predictions of existing models have reasonable correlations with future catastrophic claims;

(2) the price of custom-indexed cat-bonds or state-contingent reinsurance coverage falls, relative to the price of traditional excess-of-loss reinsurance;

(3) the magnitude of the residual model basis risk, as a percentage of the insurer's total capital, falls;

(4) the tendency increases for insurance companies to think about their long-term expected return, rather than focusing on a single catastrophic event.

Role of Government

The government, in its combined role as "reinsurer of last resort" and possessor of the power to impose taxes on property owners, may be able to supplement a cat bond/reinsurer combination as suggested in the last section to deliver an even better hedge. The optimal timing of such a tax, its terms, and the method of distributing the proceeds needs further investigation.

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APPENDIX

An Simple Example Illustrating The Role of Reinsurance and Cat Bonds for Dealing with Natural Hazard Risks

Nature of Assumptions

To determine the types of interactions between the different stakeholders we will first make a set of simplifying assumptions which can then be relaxed in future analyses:

- *All stakeholders have perfect information on the probability and loss distribution.*
- *There is no moral hazard problem. In other words, all parties will not behave more carelessly than normal because they are now protected and will tell the truth about their losses.*
- *There are no transaction costs or administrative expenses when making contracts between parties*

Independent and Correlated Risks

To illustrate the tradeoffs between reinsurance and cat bonds, consider two homeowners who reside in an area both subject to the same risk where the risks are independent of each other

There is a probability $p = 0.5$ that each individual suffers a loss (**L**) of 1
$p = 0.5$ that each individual suffers a loss (**L**) of 0

There are thus four states of the world depending on whether or not each individual suffers a loss: (**0,0**) ; (**0,L**) ; (**L,0**) ; (**L,L**)

Each homeowner's expected loss = 0.5 which is the actuarially fair premium for an insurer to charge them (assuming zero deductible.)

Each homeowner is concerned with the possibility of a loss of 1. If he is risk averse then he is willing to pay more than 0.5 for insurance against this event. Suppose that he is willing to pay 0.6 for full coverage and that the insurer charges this amount. (We assume that administration costs, reinsurance premiums, and all other expenses must be paid out of the extra 0.1 of premium over actuarially expected losses to homeowners.) The insurer's expected profit for each homeowner insured is 0.6-0.5=0.1, so that the insurer's total expected profit is 2(0.1)= 0.2. The distribution of profits and losses for the insurer is depicted in Table 1 when no reinsurance is available:

Table 1 : Distribution of Profits and Losses for Insurer With No Reinsurance
(Uncorrelated Risks)

State of the World	(**0,0**)	(**0,L**)	(**L,0**)	(**L,L**)
Probability	0.25	0.25	0.25	0.25
Indiv. 1	0.00	0.00	-1	-1
Indiv. 2	0	-1	0	-1
Total Loss	*0*	*-1*	*-1*	*-2*
Premium	1.2	1.2	1.2	1.2
Net Profit	1.2	.2	.2	**-0.8**

Note that even in the simple case where the risks are independent, there is one state of the world where both individuals suffer a loss [(**L,L**)] and the insurer will have a negative net profit even if it charges a premium greater than the actuarial rate.

Suppose the risks are perfectly correlated between homeowners as would be the case if both properties were equally affected by a hurricane or earthquake. Now there are only two states of the world: (**0,0**) and (**L,L**) and the insurer's profit and loss distribution is shown in Table 2:

Table 2 : Distribution of Profits and Losses to Insurer With No Reinsurance
(Correlated Risks)

State of the World	(**0,0**)	(**L,L**)
Probability	0.5	0.5
Indiv. 1	0	-1
Indiv. 2	0	-1
Total Loss	0	-2
Premium	1.2	1.2

Net Profit	1.2	**-0.8**

The total expected profit for the insurer is still 0.2, but now there is a much higher probability (double) that the insurer will suffer a net loss.

Role of Reinsurance and Financial Instruments

Suppose the insurer would like to protect itself against losses which are greater than 1 by turning to reinsurance or financial instruments (e.g., cat bonds), and that it is willing to pay at least a small premium over actuarial cost to do so. Paying a reinsurance premium or creating a catastrophe-linked financial instrument attempts to reduce the risk of insolvency by distributing the potential losses from catastrophe across multiple stakeholders. The average amount of damage done by the catastrophe will remain unchanged, but its burden upon the various stakeholders in the insurance value chain (consumers, primary insurers, reinsurers, and investors) will be reduced. What are the relative merits of these forms of financing against a catastrophic loss (i.e. the event (**L,L**) for this example)? We will analyze this problem for the case where the risks are independent.

Excess-of-Loss Reinsurance A reinsurer would be asked to provide an excess-of-loss policy wherein the reinsurer would contract to pay a claim to the insurer if the insurer's total loss were to exceed K. Let TL_i equal the total insured loss from event **i** . We assume that, in the process of determining what an actuarially fair rate would be for this coverage, a reinsurer would undertake the following steps:

1. determine those of events **j** leading up to claims where $TL_j > K$,
2. compute the probabilities of each of these events occurring (p_j)
3. determine the resulting loss ($TL_j - K$).
4. determine the expected loss for the reinsurer: $\sum_j p_j \left(\mathrm{TL}_j - K\right)$

For the above example of independent risks, assume the insurer would like protection if TL_j exceeds 1. The only event where this can happen is if TL_j=2 and the reinsurer's claim payment to the insurer would be TL_j-1 = 1. This event occurs with p=0.25 so the expected loss to the reinsurer would be (0.25)(1) = 0.25 for this type of excess loss coverage. Suppose that the reinsurer charged a premium of 0.35 so that its expected profits were (0.35-0.25) = 0.10. We can now compute the distribution of losses and profits for the reinsurer as shown in Table 3a:

Table 3a : Distribution of Profits and Losses to Reinsurer
(Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability .	0.25	0.25	0.25	0.25
Total Insurer Loss	0	-1	-1	-2
Total Reinsurer Loss	0	0	0	-1
Reinsurer Premium	0.35	0.35	0.35	0.35
Net Reinsurer Profit	0.35	0.35	0.35	-0.65

The reinsurer faces the possibility of a net loss of -0.65 when the state of the world is (**L,L**) with a probability of 0.25, so that if its net capital is less than 0.65 it must be concerned with insolvency just as the insurer was. In our model the reinsurer cannot influence the state of the world or the claims resulting from disasters (for example, by requiring mitigation) in any way. As will be seen, it can take steps to alter its own financial structure to reduce the chances of insolvency and reduce the variability of its earnings by issuing cat bonds.

The insurer's distribution of losses and profits with excess loss coverage from the reinsurer is shown in Table 3b:

Table 3b: Distribution of Profits and Losses to Insurer With Reinsurance
(Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability	*0.25*	*0.25*	*0.25*	*0.25*
Premium to Insurer	1.20	1.20	1.20	1.20
Reinsurer Premium	-0.35	- 0.35	-0.35	-0.35
Total Insurer Loss	0	-1	-1	-2
Reinsurance Claim	0	0	0	1
Net Insurer's Profit	0.85	-0.15	-0.15	-0.15

Now the insurer's expected profit is 0.1, which is lower than its expected profit of 0.2 without reinsurance. This decrease is due to the reinsurer premium of 0.3 which is 0.1 (33%) above the actuarial loss. On the other hand, the insurer's distribution of profits is smoother than before, because it has received an extra layer of protection from the reinsurer. It now suffers small losses in the last 3 states of the world but avoids a large loss in state (**L,L**). The variance of the insurer's profit is now 0.1875, rather than 0.5 in the no-reinsurance scenario. Note that the insurer could now avoid losses in any state of the world by charging each homeowner a premium of 0.675 rather than 0.6 -- a premium increase likely to be significantly more palatable to these consumers than increasing it to 1.0, which was the premium previously required to eliminate the risk of default.

State-Contingent Reinsurance A reinsurer might instead be asked to provide a customized, state-contingent policy such that the reinsurer would contract to pay a fixed amount to the insurer depending on the state of the world. Let PMT_j equal the total paid by the reinsurer to the insurer in state **j** . We assume that, in the process of determining what an actuarially fair rate would be for this coverage, a reinsurer would undertake the following steps:

1. compute the probabilities of each of these states occurring (p_j)

2. calculate the resulting payment to the insurer (PMT_j).
3. determine the expected loss for the reinsurer: $\sum_j p_j \left(PMT_j\right)$

It should be noted that the excess-of-loss contract described above is a special case of this state-contingent reinsurance, where $PMT_j = TL_j - K$ for each state.

For the above example of independent risks, assume the insurer would like protection if TL_j exceeds 1. If the reinsurer were to write a state-contingent contract which paid 1 in the state **(L,L)** and charged a premium of 0.35 for this contract, the state-contingent contract would mimic the excess-of-loss contract described above.

Suppose instead we wished to construct a contract that minimized the insurer's variance of profit, subject to the constraints that the reinsurer does not ever pay out more than 1 in any state and that the reinsurer would make an expected profit of exactly 0.1.

Table 4a: Distribution of Profits and Losses to Reinsurer
(Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability .	0.25	0.25	0.25	0.25
Total Insurer Loss	0	-1	-1	-2
Total Reinsurer Loss	0	0	0	-1
Reinsurer Premium	1.10	0.10	0.10	0.10
Net Reinsurer Profit	1.10	0.10	0.10	-0.90

The insurer's distribution of losses and profits with excess loss coverage from the reinsurer is shown in Table 4:

Table 4b : Distribution of Profits and Losses to Insurer With Reinsurance
(Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability .	*0.25*	*0.25*	*0.25*	*0.25*
Premium to Insurer	1.20	1.20	1.20	1.20
Reinsurer Premium	-1.10	- 0.10	-0.10	-0.10
Total Insurer Loss	0	-1	-1	-2
Reinsurance Claim	0	0	0	1
Net Insurer's Profit	0.10	0.10	0.10	0.10

Now the variance of the insurer's profit is zero (rather than 0.5 in the excess-of-loss reinsurance arrangement), while the amount paid to the reinsurer is the same as in the excess-of-loss example, above. Provided that the reinsurer has a capital base of at least 0.9, it need not worry about insolvency.

Cat Bonds for Insurers Suppose the insurer, rather than purchasing reinsurance as above, issues a "cat bond" -- a security whose returns are linked to the incidence or absence of catastrophic events -- whose payment structure satisfies the following conditions:

Condition 1: In states of the world where net insurer profits would otherwise be negative, the bond provides sufficient payments to the insurer so that net insurer profits are zero.

Condition 2: In the state of the world where net insurer profits are *highest*, the bond provides a positive return to the investor (and requires payment from the insurer.)

Condition 3: The investor is provided with a positive expected net return exactly equal to the expected net profit obtained by the reinsurer.

We will construct several cat bonds with varying properties to show how this risk transfer can be effected.

Cat Bond 1: For the above example of independent risks, we turn to Table 1 for a distribution of insurer profits and losses.

- Condition 1 implies that Cat Bond 1 would provide 0.8 to the insurer in state (**L,L**).

- Condition 2 implies that the bond would provide a positive return to the investor in state (**0,0**).

- Condition 3 implies that the return from the bond in the (**0,0**) state has to be sufficiently high that the net expected return across all four possible states equals 0.1. Let B= the positive expected return in state (**0,0**) so that net expected investor returns = 0.10. Since the probability of being in (**L,L**) is 0.25, the expected loss to the investor from this event is 0.25(.8) = 0.2. The probability of being in state (**0,0**) is also 0.25, so that .25 (B) -0.2 =0.1 and thus we calculate B=1.2.

The impact of the cat bond on the distribution of losses and profits to the insurer is depicted in Table 5:

Table 5 : Distribution of Profits and Losses to Insurer With Cat Bonds (Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability	. *0.25*	*0.25*	*0.25*	*0.25*
Premium to Insurer	1.2	1.2	1.2	1.2
Payment to(-) or from (+) Investor	-1.2	0	0	+0.8
Total Insurer Loss	0	-1	-1	-2
Net Insurer's Profit	0	+0.2	+0.2	0

Note that, by design, the insurer's expected net profit is 0.1 and the variance of its profits is 0.01 -- exactly the same as if it had purchased reinsurance, rather than used Cat Bond 1, to hedge its losses.

The investor's distribution of profits and losses which gives him a net expected return of 0.1 is shown in Table 6:

Table 6 : Distribution of Profits and Losses to Investor in Cat Bond 1
(Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability	. *0.25*	*0.25*	*0.25*	*0.25*
Net Investor's Profit	1.2	0	0	-0.8

The expected return to the investor is 0.1 and the variance of this return is 0.52. This very simple cat bond shows the basic power of catastrophic risk securitization to completely eliminate the insurer's risk of insolvency while at the same time creating a positive expected-value investment opportunity for the investor. The basic structure can certainly be improved -- insurers may wish to smooth out their profits even further, and investors value reducing the variance of their returns.

Cat Bond 2: Had the insurer wanted to make the same (positive) level of profits in every state of the world, it could have designed an improved bond (Cat Bond 2) which yielded the same expected return to the investor as Cat Bond 1, but offered a different distribution of payouts across states of the world.[21] More specifically, there would be a smaller positive return to the investor in state (**0,0**), a positive return in states (**0,L**) and (**L,0**), and the investor would pay out more to the insurer in state (**L,L**). Table 7 illustrates the design of such a bond:

[21] It is conceivable that the managers of a stock-based insurance company would prefer a smooth and predictable earnings stream in addition to assuring solvency, for example, whereas those of a mutual insurance company would be solely concerned with assuring solvency. Other than smoothing earnings, there is no particular reason to prefer Cat Bond 2 to Cat Bond 1 for the firm (and, as will be seen, investors will also slightly prefer Cat Bond 2 because of its lower variance.)

Table 7: Distribution of Profits and Losses to Insurer With Cat Bond 2
(Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability	. *0.25*	*0.25*	*0.25*	*0.25*
Premium to Insurer	1.2	1.2	1.2	1.2
Payment to(-) or from (+) Investor	-1.1	-0.1	-0.1	+0.9
Total Insurer Loss	0	-1	-1	-2
Net Insurer's Profit	0.1	0.1	0.1	0.1

The variance of the insurer's profit is obviously zero. The investor's distribution of profits and losses which gives him a net expected return of 0.1 from Cat Bond 2 is shown in Table 8:

Table 8: Distribution of Profits and Losses to Investor in Cat Bond 2
(Uncorrelated Risks)

State of the World	**(0,0)**	**(0,L)**	**(L,0)**	**(L,L)**
Probability	*0.25*	*0.25*	*0.25*	*0.25*
Net Investor's Profit	1.1	0.1	0.1	-0.9

The variance of the investor's profit is 0.51 – slightly less than for Cat Bond 1. Cat Bond 2 illustrates the flexibility of using this instrument to satisfy both investor and insurer needs. The insurer now has the same expected profit as in Cat Bond 1 but it is constant across all periods. The investor's net expected return from Cat Bond 2 is the same as Cat Bond 1 but now he receives a positive expected return in the first 3 periods rather than in just period 1, and the investor's overall variance of return is reduced in the bargain. Note the similarity of Cat Bond 2 to state-contingent reinsurance from the insurer's point of view, offering identical payoffs in every state but avoiding the credit risk of insurer insolvency.

Cat Bonds for Reinsurers The reinsurer can also turn to financial instruments as a way of coping with its losses in state (**L,L**). Suppose the reinsurer wants to issue a cat bond (Cat Bond 3) to cover its losses in state (**L,L**). The cat bond will be designed to yield a net return to investors of 0.05 so that both reinsurers and investors will make identical expected profits, splitting the 0.1 surplus equally between themselves. Suppose that Cat Bond 3 is designed in a similar fashion to Cat Bond 2 with the reinsurer making the same profit in all states of the world. Table 9 depicts the impact of such a bond on the reinsurer:

Table 9 : Distribution of Profits and Losses to Reinsurer
(Uncorrelated Risks)

State of the World	(**0,0**)	(**0,L**)	(**L,0**)	(**L,L**)
Probability .	0.25	0.25	0.25	0.25
Reinsurer Premium	0.35	0.35	0.35	0.35
Payment to(-) or from (+) Investor	-0.30	-0.30	-0.30	0.70
Total Reinsurer Loss	0	0	0	-1
Net Reinsurer Profit	0.05	0.05	0.05	0.05

The investor's distribution of profits and losses which gives him a net expected return of 0.05 from Cat Bond 3 is shown in Table 10:

Table 10 : Distribution of Profits and Losses to Investor in Cat Bond 3
(Uncorrelated Risks)

State of the World	(**0,0**)	(**0,L**)	(**L,0**)	(**L,L**)
Probability .	*0.25*	*0.25*	*0.25*	*0.25*
Net Investor's Profit	0.3	0.3	0.3	-0.7

Cat Bond 3 may still be attractive to risk-averse investors because they will now lose less money in state (**L,L**) than if they purchased Cat Bond 1 or 2 and will receive a larger return in states 2 and 3. The variance of the return to the investor in Cat Bond 3 is 0.19, which is somewhat lower than the other two cat bonds; the payout to investors is also only half as great, however.