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Mutual Risk Trade through Stochastic Cost Allocation in the Recovery Project from Disaster

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1. Introduction

When a disaster occurs, many infrastructures such as road networks, water supply systems will be damaged. Hanshin-Awaji Earthquake (1995) caused severe damage to highways, lifeline systems and buildings. Many people were obliged to live a hard time due to the infrastructure damage and the economic losses due to these damages affected not only the stricken area but also all of Japan and its neighboring countries. To minimize the negative effect caused by the disaster, we have to plan the recovery project and act immediately. It is noted that many agents (stakeholders) such as landowner, local government are involved in the project. In this sense, the recovery project is often implemented as a joint project.

One of the critical characteristics of the recovery project is that it is short-time-constrained. This is because we must plan and start the project as soon as possible. Thus it is difficult to predict the project cost precisely in very short time. Let suppose the land readjustment project after the disaster. In this project, some houses may be removed to create the open space such as road and park. To remove the houses, the agent who implements the project has to pay the compensation cost for removal to the house owners. If the house owners rebuild their houses, the compensation cost for removal is higher than the houses not rebuilt. How many house owners rebuild their houses is difficult to predict in a short time. This implies that how much the compensation cost is necessary is uncertain when we plan the project.

Once the project is planned, there are high sunk costs such as investigation. Thus, the agents have to commit to the project before the suck cost occurs. i.e., when the plan of the project begins to be made. Whether they commit to the project depends on how much cost is allocated. Thus project cost should be allocated when the agents commit to the project. At that time, however, the project cost is uncertain. So the cost allocation has to be a stochastic cost allocation (See Fig.1). Assuming that the project cost is uncertain, the allocated costs for all agents are also uncertain. This means that the agents are exposed to the risk of allocated project cost. If the risk is not acceptable for some agent, she will deviate from the project. This may incur the infeasibility of the project. If the project becomes infeasible, it gets more difficult for the

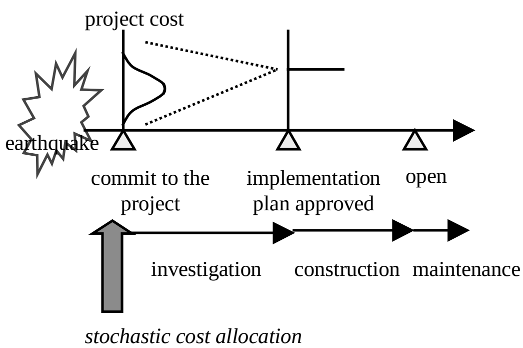

Fig.1 Process of Recovery Project

stricken area to recover. Thus this risk should be transferred by some ways.

If insurance is available and affordable, the agents can transfer their risk to the insurance company. If not, however, they will transfer the risk mutually. Specifically, we focus on that the risk can be traded in the stochastic cost allocation. We develop the stochastic cost allocation scheme in which the risk is mutually traded by taking the risk preference of the agents into accounted. We show that the efficient risk allocation through stochastic cost allocation can be derived. However, as the result of the efficient allocation, the risk of some agents may be higher than the risk allocation which does not consider the efficiency. Thus we introduce the risk bearing fee so that the efficient risk allocation is accepted by all agents. We call these two procedures, deriving the efficient risk allocation and introducing the risk bearing fee, as the risk trade.

2. Analytical Framework

2.1 Stochastic Cost Allocation Game

A joint project is a project by a coalition formed by all agents in the project. We can say that the project cost is a cost for the coalition. When forming the coalition, how to allocate the joint cost of the project becomes an issue. To discuss this issue, a cooperative game is an useful tool.

As a paper which dealt with stochastic cost game, we can find the work by Charnes and Granot[1]. They suggested to allocate stochastic cost in two stages. One stage is prior cost allocation to the players (= agents). In the second stage, the realization of the stochastic cost is awaited and a possibly non-feasible prior cost allocation has to be modified to this realization. This approach is followed by Charnes and Granot[2], Charnes and Granot[3] and Granot[4]. Suijs *et al.*[5][6] studied another way to deal with the stochastic cost allocation. The main difference between the approach introduced by Charnes *et al.*[1] and Suijs *et al.*[5][6] is that the latter explicitly considered the risk of allocated cost and incorporates the preferences of the risk of allocated cost. This paper also considers the risk of allocated cost and the players' preferences of the risk.

2.2 Notation

We explain the notation used in this paper:

- i: a player in the game. Each agent in the joint project is a player.
- $N=\{1,2,..,n\}$: a set of players
- S ($\subset N$): subset of N, sub-coalition
- $C(S)$: cost function
- $|S|$: cardinality of sub-coalition
- x_i: allocated cost for player

We can call N a grand coalition and $\{i\}$ a singleton coalition. A grand coalition is a coalition realized in a joint project. Sub-coalition is a possible coalition to be formed by a subset of players. We denote $C(S)$ for stochastic value and $\overline{C}(S)$ for deterministic value.

Let denote the expectation and the variance of $C(S)$ by $\mu(S)$, $\sigma(S)^2$ respectively. We represent the covariance of $C(S)$ and $C(T)$ by $\sigma(S,T)$ and the correlation coefficient by $\rho_{S,T}$.

2.3 Cost Allocation Method

We will let the Generalized Shapley value [7] be taken as a cost allocation method. The formulation is shown by :

$$x_i = \sum_{S\subseteq N, i\in S} p(S, S\setminus\{i\})[\overline{C}(S) - \overline{C}(S\setminus\{i\})] \quad (1)$$

where $p(S,S\setminus\{i\})$ has the interpretation as the weight that i joins $S\setminus\{i\}$, given that $S\setminus\{i\}$ has formed as a coalition. While standard Shapley value gives fixed weight, the Generalized Shapley value can give an arbitrary weight. We give the weight so as to attain the efficiency of the risk allocation. How to give this weight is explained later. As equation (1) shows, the Generalized Shapley value gives a cost allocation based on the certain cost. We apply this cost allocation method even if the cost is stochastic such that:

$$x_i = \sum_{S\subseteq N, i\in S} p(S, S\setminus\{i\})[C(S) - C(S\setminus\{i\})] \quad (2)$$

As this equation shows, an allocated cost is given by a combination of stochastic costs for all coalitions. This means that allocated cost is al-so stochastic. The solution by the Generalized Shapley value to each player i is the weight-ed average of her marginal cost, $(C(S)-C(S\setminus\{i\}))$ to all possible coalitions and sequences. Theoretically, the number of possible sequences is $n!$. Fig.2 shows that possible sequence of coalition formation leading to the grand coalition in 3-person game. Let denote the vector of weight of sequence by $p = (p_1, p_2,\ldots, p_{n!})$. Note that weight is normalized such that $\Sigma_k\ p_k=1$. $p(S,S\setminus\{i\})$ is derived by p.

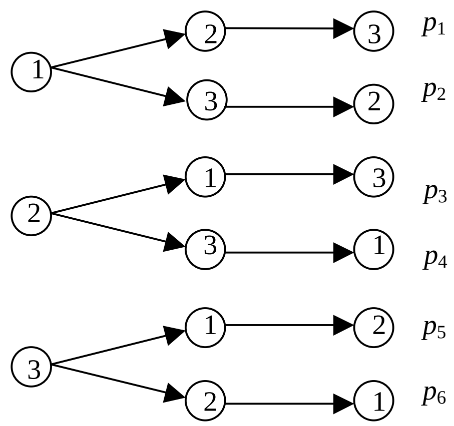

Fig.2 Possible sequence of grand coalition formation

The allocated cost for a player given by Shapley value in a 2-person game ($N=\{1,2\}$) is shown by:

$$x_1 = p_1C(1) + p_2(C(N)–C(2))$$

$$x_2 = p_2C(2) + p_1(C(N)–C(1)) \quad (3)$$

where p_1, p_2 is a weight when the player 1,2 participates in the coalition first respectively. Note that $C(\{i\})$ is shown as $C(i)$ for simplicity. It holds that $p(\{1\},\phi) = p(N,\{1\}) = p_1$, $p(\{2\},\phi) = p(N,\{2\}) = p_2$. In standard Shapley value, p_1 and p_2 are given as 1/2. Generally, it gives $p_k = 1/n!$.

2.4 Player's Preference for the Risk

All players in a game are assumed to be risk averse. We describe this preference by a player's utility function $u(y)=a_1+a_2e^{by}$, $(a_i,b < 0)$ where y is a payoff. For this utility function, the measure of risk aversion for a player is constant. Assuming expected utility maximizers based on this utility function, Suijs *et al.*[5] proved that for the games in this subclass the preference $\{\succeq_i\}_{i\in N}$ are such that for each player $i\in N$ there exists a function m_i satisfying:

1. for all stochastic variables $y, z : y \succeq_i z$ if and only if $m_i(y) \succeq_i m_i(z)$,
2. for all deterministic variable d : $m_i(d) = d$,
3. for all stochastic variable y : $m_i(y - m_i(y)) = 0$,
4. for all stochastic variable y and all deterministic variables $d, d'(d < d')$: $m_i(d + y) < m_i(d' + y)$

The amount of $m_i(y)$ is called the certainty equivalent of y. Thus we mainly discuss efficiency of risk allocation based on the certainty equivalent rather than the expected utility.

3. Stochastic Cost Allocation

3.1 Individual Rational Allocation

Before discussing cost allocation, we should study whether a player prefers working together with all the other players or not. Let denote the sub-additive game satisfying:

$$\overline{C}(S) + \overline{C}(T) \geq \overline{C}(S \cup T), (S \cap T = \phi) \quad (4)$$

where ϕ is an empty set, and S and T are any two disjoint subsets of grand coalition N. Sub-additivity shows that the larger the coalition is formed the more cost reduction can be attained holds if the cost is certain. The sub-additive game is that all players have incentives to form a coalition. Assuming the sub-additivity, the Generalized Shapley value is individually rational, which means the Shapley value is less than the cost for a singleton coalition [8]. This implies that all players are better off in the joint project than a project by herself so long as the cost is certain. In a stochastic cost allocation, however, cost is uncertain. Intuitively all players have incentives to join a grand coalition even if the cost is uncertain because they will be better off in the grand coalition for any state in the future. This can be shown easily in the following. Let denote any state in the future by ω. For all ω, next equation holds.

$$\overline{x}_i(\omega) \leq \overline{C}(\{i\})(\omega) \quad (5)$$

By monotonicity of utility function, the next equation can be derived.

$$u(\overline{x}_i(\omega)) \geq U(C(\{i\})(\omega)) \quad (6)$$

Let denote the probability of state ω by π(ω). Then the next equation holds.

$$\pi(\omega)U(\overline{x}_i(\omega)) \geq \pi(\omega)U(\overline{C}(\{i\})(\omega)) \quad (7)$$

By integrating all states, we have the next equation.

$$\begin{aligned} &\pi(\omega)U(\overline{x}_i(\omega)) \\ &\geq \int_\omega \pi(\omega)U(\overline{C}(\{i\})(\omega))d\omega = EU(C(\{i\})) \end{aligned} \quad (8)$$

where EU is the expected utility. This equation shows that the expected utility when a player belongs to the grand coalition is more than if she was to form singleton coalition. This is a condition that all players should join a grand coalitions as long as they all can accept the risk. In this paper, we assume the sub-additive game is satisfied.

3.2 Certain Equivalent of Allocated Cost

We discuss risk allocation not based on utility function but certain equivalent. Certain equivalent for allocated cost (X_i) is approximated as:

$$X_i = E[x_i] + \frac{1}{2} r_i Var[x_i] \quad (9)$$

where $E[x_i]$ and $Var[x_i]$ are expectation and variance of x_i respectively. r_i is the measure of risk aversion for player i. It is noted that $r_i\, Var[x_i]/2$ is a risk premium.

Because allocated cost x_i is a function of weights vector p, the certain equivalent X_i is also a function of p; the risk of allocated cost depends on the weights.

3.3 Efficient Risk Allocation

As mentioned above, certain equivalent differs from each weight. We consider determining the weights p^* so as to attain the efficiency. That is formulated as:

$$p^* = \arg[\sum_{i\in N} X_i \to \min] \quad (10)$$

Because it holds, $\sum_{i\in N} E[x_i] = m(N)$ = const, eq.(10) is reduced to risk premium minimization. By considering constraint $\sum_{k=1}^{n!} p_k = 1$, Lagrange function L is derived as:

$$L = \sum_{i\in N} \frac{1}{2} r_i Var[x_i] + \lambda\left(1 - \sum_{k=1}^{n!} p_k\right) \quad (11)$$

First condition for it is shown as:

$$\frac{\sum_{i\in N} \frac{1}{2} r_i Var[x_i]}{p_k} - \lambda = 0 (1 \le k \le n!)$$

$$\sum_{k=1}^{n!} p_k = 1 \quad (12)$$

We assume the standard Shapley value, in which weight is given as p_k=const as a risk allocation method which does not consider the efficiency of risk allocation.

3.4 Analysis for Two-person Game

We derive the weights for the two-person game. Let denote the weight for sequence of grand coalition formation in which player 1,2 joins first by p_1, p_2 respectively. Then the certain equivalent of allocated cost for player i (X_i) is shown as:

$$X_1 = p_1\mu(1) + p_2(\mu(N) - \mu(2)) + \frac{1}{2} Var[x_1]$$

$$X_2 = p_2\mu(2) + p_1(\mu(N) - \mu(1)) + \frac{1}{2} r_2 Var[x] \quad (13)$$

First condition (12) for this problem is shown as:

$$\begin{pmatrix} f_1 & -f_3 & -1 \\ -f_3 & f_2 & -1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (14)$$

Let denote the determinant for the 3×3 matrix by D. D>0 is the second condition for this problem. Assuming the inner solution, it is shown as:

$$p_1^* = \frac{f_3 + f_2}{D}$$

$$p_2^* = \frac{f_3 + f_1}{D}$$

$$\lambda^* = \frac{f_1 f_2 - f_3^2}{D}$$

$$D = 2f_3 + f_2 + f_1 \quad (15)$$

where $f_1 \sim f_3$ is:

$$f_1 = r_1\sigma(1)^2 + r_2\left(\sigma(1)^2 - 2\sigma(1,N) + \sigma(N)^2\right)$$

$$f_2 = r_1\left(\sigma(2)^2 - 2\sigma(2,N) + \sigma(N)^2\right) + r_2\sigma(2)^2$$

$$f_3 = r_1(\sigma(1,2) - \sigma(1,N)) + r_2(\sigma(1,2) - \sigma(2,N)) \quad (16)$$

Corner solution is either (p_1, p_2)=(1,0), (0,1).

3.5 Characteristics of the obtained risk allocation

(1) Measure of risk aversion and weight

Applying the efficient risk allocation scheme, risk is transferred from higher risk averse players to lower risk averse players. This would be examined by deriving the weight variation for the measure of risk aversion variation.

$$\frac{\partial p_1^*}{\partial r_1} = \frac{\frac{\partial(f_3 + f_2)}{\partial r_1} D - \frac{\partial D}{\partial r_1}(f_3 + f_2)}{D^2} \quad (17)$$

$$= \frac{1}{D^2}\left(\frac{\partial(f_3 + f_2)}{\partial r_1}, \frac{\partial(f_3 + f_2)}{\partial r_1}\right)\begin{pmatrix} f_3 + f_1 \\ -f_3 - f_2 \end{pmatrix}$$

It is noted that the next equations hold.

$$\frac{\partial(f_3 + f_2)}{\partial r_1} = \sigma(1,2) - \sigma(1,N) + \sigma(2)^2$$

By the assumption of the inner solution, $f_3 + f_1, f_3 + f_2 \ge 0$ holds.

$\sigma(2)^2 - 2\sigma(2,N) + \sigma(N)^2$ is $Var[C(N) - C(2)]$ and $\sigma(1)^2$ is $Var[C(1)]$. The former is the variance when player 1 is the last participant, the latter is when she is the first participant. If $Var[C(N) - C(2)] \le Var[C(1)]$ holds, her variance is lower when she participates as the last player. If the measure of risk aversion for player 1 becomes greater, therefore, the efficiency holds by decreasing the weight p. That is to say, if $\partial(f_3 + f_2)/\partial r_1$ is small and $\partial(f_3 + f_1)/\partial r_1$ is great, it may hold $\partial p_1^*/\partial r_1 \le 0$. If $Var[C(N) - C(2)] \ge Var[C(1)]$, however, $\partial p_1^*/\partial r_1 \ge 0$ may hold. Same result can be derived for the variation of r_i.

(2) Variance and weight

Variation of p_1 for the variation of σ(1) as:

$$\frac{\partial p_1^*}{\partial\sigma(1)}$$

$$= \frac{1}{D^2}\left(\frac{\partial(f_3 + f_2)}{\partial\sigma(1)}, \frac{\partial(f_3 + f_1)}{\partial\sigma(1)}\right)\begin{pmatrix} f_3 + f_1 \\ -f_3 - f_2 \end{pmatrix} \quad (19)$$

Note that the next equations hold.

$$\frac{\partial(f_3 + f_2)}{\partial\sigma(1)} = \rho(1,2)\sigma(2)(r_1 + r_2)$$

$$\frac{\partial(f_3 + f_1)}{\partial\sigma(1)} = \rho(1,2)\sigma(r_1 + r_2) \quad (20)$$

$$+ 2(\sigma(1)(r_1 + r_2) - r_2\rho(1,N)\sigma(N))$$

From these equations, if i)greater $\sigma(1)$ is, ii)smaller r_2 is, iii)smaller $\rho(1,N)$ is, $\partial p_1^*/\partial\sigma(1) \le 0$ may hold. If $\sigma(1)$ is greater, the risk is more dominant for the summation of risk for all players. Thus variation of p_1 is varied more so as to cancel the increase of its risk if $\sigma(1)$ is greater. This is the interpretation of i). ii) shows the variation of p_1 must be allowed by player 2. If r_2 is smaller, she is tolerant for her increase of risk. iii) shows that p_1 can be varied unless it incurs an increase of risk for grand coalition. The result is the same for $\partial p_1^* / \partial\sigma(2)$.

(3) Covariance and weight Variation of p_1^* for the variation of $\sigma(1,2)$ is shown as:

$$\frac{\partial p_1^*}{\partial\sigma(1,2)} = \frac{1}{D^2}(r_1+r_2, r_1+r_2)\begin{pmatrix} f_3+f_1 \\ -f_3-f_2 \end{pmatrix} \tag{21}$$

From these equations, $\partial p_1^* / \partial\sigma(1,2) \geq (\leq) 0$ holds if and only if $f_1 - f_2 \geq (\leq) 0$ Let consider f_1, f_2 here. Because $f_1 = r_1\sigma(1)^2 + r_2(\sigma(N)-\sigma(1))^2$, f_1 is total variance for both players weighted by a measure of risk aversion when player 1 joins the coalition first. f_2 is one when player 2 joins first. If the total variance when player 1 joins the coalition first is more than the one when player 2 joins first, p_1^* increases with the increase of $\sigma(1,2)$. This implies that the efficiency holds only by increasing the total variance if $\sigma(1,2)$ increases. This result shows that increase of covariance is not desirable for the mutual risk trade.

For the relationship between weights and covariance, what results if the variation of covariance is large? If covariance is large, f_3 becomes large. Assuming that f_3 is large enough so that D becomes positive, solution is the inner solution. Then p_1^*, p_2^* can be rewritten as:

$$p_1^* = \frac{1+\frac{f_2}{f_3}}{2+\frac{f_2+f_1}{f_3}}, \quad p_2^* = \frac{1+\frac{f_1}{f_3}}{2+\frac{f_2+f_1}{f_3}} \tag{22}$$

By these equations, when f_3 is larger, p_1^*, p_2^* approach 1/2, which is the weight of the standard Shapley value [9]. This is where the weights are given as $p_k=1/n!$, $(1\leq k\leq n!)$. While we can derive this property in a two-person game due to symmetry, unfortunately it is not easy to extend the result to n-person game ($n\geq 3$).

3.6 Relationship between Standard Shapley value and Generalized Shapley value with efficient risk allocation

We obtain the next lemma.

Lemma.1: If the next equations hold, the weights derived by equation (10) is equivalent to the one by standard Shapley value.

$$\sigma(S)^2 = \sigma(T)^2, (|S| = |T|; \forall S, T \subset N)$$
$$\rho_{S,T} = \rho_{S'T'} \tag{23}$$
$$(|S| = |S'|; |T| = |T'|; \forall S, S', T, T' \subset N)$$
$$r_i = r_j, (\forall i, j \in N)$$

These equations shows that cost function is symmetric in terms of variance, covariance and measure of risk aversion. That is to say, the variance and covariance of cost function for coalition are determined by the number of players in the coalition.

Proof:

Let define g such that:

$g:\{\mu(S)\}_{S\subseteq N} \rightarrow \{\mu'(S)|\ \mu'(S)=h(|S|)\}_{S\subseteq N}$

h is an arbitrary function. The operation g converts the expectation of cost function $\mu(S)$ to $\mu'(S)$ which gives the expectation of cost only dependent on the number of players in the coalition.

Operating g to cost function under the equations (23) each sequence of grand coalition formation is symmetric. Therefore we obtain $p_k = p_{k'}, (k\neq k')$. By $\Sigma_k p_k=1$, we have $p_k=1/n!$. Because the equation (10) equivalent to the risk premium minimization problem, the solution by equation (10) is independent on the expectation of cost function. Thus the solution by (10) and (10) operating g is equivalent. Thus if the equations (23) hold, the solution by (10) is equivalent to the solution by standard Shapley value.(Q.E.D)

By this result, we can say that weight given by the standard Shapley value is a particular case of the risk allocation developed in this study.

4. Fee of Risk Bearing

As we see, the amount of allocated risk is different for each player. It is natural to think that every player that bears high risk may claim the payment for accepting the risk. In this section, we determine the fee of risk bearing on the condition that risk allocation remains efficient.

Let denote the risk bearing fee which is paid by player i to j by ν_{ij}. Note that ν_{ii}.=0, ν_{ij}.= $-\nu_{ji}$ hold. The second equation shows the payment and receipt for risk bearing only occurs among the players in the game. The certain equivalent for player i after paying risk bearing ν_{ij} is shown as:

$$X_i^{**} = X_i^{*} + \sum_{j\in N} \nu_{ij} \tag{24}$$

From $\Sigma i \Sigma j \nu_{ij}=0$, X_i^{**} is also efficient. Let denote the certain equivalent without risk trade by X_i^0. We assume X_i^0 is given by the standard Shapley value. The saving certain equivalent of player i by trading risk among the players can be represented by $X_i^0 - X_i^{**}$. Using this amount, risk bearing fee ν_{ij} is determined by:

$$\{\nu_{ij}\} = \arg[\varepsilon \to \max]$$

$$\text{s.t. } X_i^0 - X_i^{**} \ge \varepsilon \tag{25}$$

This equation shows that risk bearing fee is determined so as to maximize the minimal certain equivalent saving.

In a two-person game, ν_{12} is represented by:

$$\nu_{12} = \frac{X_1^0 - X_1^* - X_2^0 + X_2{}^*}{2} \tag{26}$$

From this equation, the more player who saves the risk, the more she has to pay. It is meaningful to show that $\varepsilon \ge 0$ in (25) because all players are better off by efficient risk allocation than risk allocation without trade.

At first, we prove the lemma.

Lemma.2: $\sum_{i\in N} X_i^{**} \le \sum_{i\in N} X_i^0$ hold

Proof. $\sum_{i\in N} X_i^{**} = \sum_{i\in N} X_i^*$ hold. This lemma is equivalent

to $\sum_{i\in N} X_i^* \le \sum_{i\in N} X_i^0$ .Assuming $\sum_{i\in N} X_i^* > \sum_{i\in N} X_i^0$ .This is clearly contradiction to the definition of X_i^* which minimize the certain equivalent. Hence lemma is satisfied.(Q.E.D)

Lemma.3: $\varepsilon \ge 0$ holds for (25)

Let assume $\varepsilon < 0$. Then there exists a player i^- where $X_{i^-}^0 - X_{i^-}^{**} = -M < 0$ .By lemma2, $\sum_{i\in N}\left(X_i^0 - X_i^{**}\right) \ge 0$,we can find the set of players S where $X_i^0 - X_i^{**} > 0 (i \in S)$ and $\sum_{i\in N}\left(X_i^0 - X_i^{**}\right) = M + dM, (dM \ge 0)$ For this set of player S, let define $X_i^{***} = X_i^{**} + d\nu_{ii^-}, (i \in S)$ which satisfies $X_i^0 - X_i^{***} \ge 0$ and $\sum_{i\in S} d\nu_{ii^-} = M$. Because the payment and receipt of risk bearing is only among the players, $d\nu_{ij} = -d\nu_{ji}$ must hold. Hence, $X_{i^-}^{***} = X_{i^-}^{**} - \sum_{i\in S} d\nu_{ii^-} = X_{i^-}^{**} - M$ holds. For $X_{i^-}^{***}, X_{i^-}^0 - X_{-i}^{***} = -M + M$ holds. By applying the same operation to the players satisfying $X_i^0 - X_{-i}^{**} < 0$, we can derive ε so that $\varepsilon \ge 0$. This is a contradiction to $\varepsilon < 0$. Henceforth $\varepsilon \ge 0$ holds.(Q.E.D)

5. Numerical Examples

Let us suppose that the area is stricken by an earthquake and most houses are heavily damaged. As one the of recover projects, a land readjustment project is proposed. The local government of the area and two developers are thinking of participating in the project. The local government is exposed to the risk due to the compensation stated above and each developer is exposed to the risk of demand for residential houses and commercial buildings respectively. If the demand is lower than they predicted, they will suffer the more maintenance cost and debt. These risks cause the uncertainty of project cost. They can reduce the project cost by working together because of collecting the investigation data jointly, for instance. We call the developer for residential houses as player 1, the one for commercial buildings as player 2 and the local government as player 3 respectively.

$$\begin{aligned}
&C(1) \sim (5.0, 1.00)\\
&C(2) \sim (6.0, 1.44)\\
&C(3) \sim (7.0, 1.69)\\
&C(12) \sim (10.0, 3.61)\\
&C(13) \sim (11.1, 4.41)\\
&C(23) \sim (12.2, 5.29)\\
&C(N) \sim (16.8, 9.00)
\end{aligned} \tag{27}$$

Let assume that the correlation coefficient matrix is given as:

$$\begin{pmatrix}
1.00 & 0.30 & 0.30 & 0.50 & 0.60 & 0.30 & 0.40\\
0.30 & 1.00 & 0.30 & 0.60 & 0.50 & 0.60 & 0.50\\
0.30 & 0.30 & 1.00 & 0.50 & 0.60 & 0.70 & 0.60\\
0.50 & 0.60 & 0.50 & 1.00 & 0.40 & 0.50 & 0.60\\
0.60 & 0.50 & 0.60 & 0.40 & 1.00 & 0.60 & 0.70\\
0.30 & 0.60 & 0.70 & 0.50 & 0.60 & 1.00 & 0.80\\
0.40 & 0.50 & 0.60 & 0.60 & 0.70 & 0.80 & 1.00
\end{pmatrix} \tag{28}$$

It is noted that we number the coalition $(\{1\},\{2\},\{3\},\{12\},\{13\},\{23\},N)$ as $(1,2,3,\cdots)$.

The *i-j* component in the matrix shows the correlation coefficient for *i*-th coalition and *j*-th coalition.

The risk aversion measure for the players is given by $(r_1, r_2, r_3) = (4.0,1.0,1.2)$. This implies that player 1 is the most risk averse. Given these numerical data, the weights derived by risk allocation stated above is shown as:

$$p^*=(0.087,0.245,0.283,0.004,0.229,0.154) \quad (29)$$

The risk premium given by these weights are (1.899,0.993,1.341), which is for player 1, 2 and 3 respectively. The risk premium given by the risk allocation without risk trade, by standard Shapley value, is (2.257,0.823,1.349). The total 0.196 risk premium is reduced by risk allocation with risk trade. For the certain equivalent, (6.348,6.642,8.043) for risk trade with risk trade and (6.807,6.423,7.999) for one without trade.

Next, we compare the variance of allocated cost for all players. While $(Var(x_1), Var(x_2), Var(x_3))$ for with risk trade is (0.475, 0.993, 1.118), it is (0.564,0.823,1.124) without it. This result shows that total risk premium reduction by risk allocation with risk trade is realized by increasing the variance of allocated cost for player 2 who is the least risk averse.

Then we introduce the risk bearing fee. The fee paid by player 1 for the other players, $\nu_{12} + \nu_{13}$, is calculated as 0.394 by equation (25). Similarly, the ones by player 2 and 3 are –0.284, –0.109 respectively. Then the certain equivalent savings for all players is calculated as 0.065. By the risk bearing fee, the certain equivalent for each player is derived as (6.742, 6.358, 7.934).

This result shows that the certain equivalent can be reduced for all players by mutual risk trade. This implies that the all players are more willing to participate in the joint project by applying the allocation scheme proposed in this study than not applying it. In this sense, the feasibility of the joint project which can be implemented in lower cost than separating each other may be improved.

6. Conclusion

We study risk allocation in a joint project in which all agents are exposed to the risk of allocated cost. To allocate project risk efficiently, we develop a stochastic cost allocation. Because efficiency is always satisfied for grand coalition under sub-additivity game in cooperative game in which cost is certain, cost allocation is also efficient. When cost is uncertain, however, efficiency depends on risk (cost with uncertainty) allocation itself. This is the main difference between the deterministic cooperative game and the stochastic cooperative game in this study. Once given the efficient risk allocation, we show the fee of risk bearing can be introduced on the condition that efficiency remains. Using this risk allocation, we can conclude that efficient risk allocation can be obtained so that all agents are better off than risk allocation without risk trade. To apply the cost allocation scheme proposed in this study, the feasibility of the project will improve; that contributes to the immediate recovery in the stricken area.

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