Catastrophe Risks and Economic Valuation of Disaster Mitigation

Muneta YOKOMATSU*, Kiyoshi KOBAYASHI**

* Department of Social Systems Engineering, Tottori University
4-101, Koyama-mimami, Tottori, 680-8552, Japan
yoko@sse.tottori-u.ac.jp
** Department of Civil Engineering, Kyoto University
Yoshida-honmachi, Sakyo-ku, Kyoto, 606-8501, Japan
kkoba@psa2.kuciv.kyoto-u.ac.jp

Second EuroConference on
GLOBAL CHANGE AND CATASTROPHE RISK MANAGEMENT
Earthquake Risks in Europe
IIASA, A-2361 Laxenburg, Austria
July 6-9, 2000

Abstract

Catastrophe risks by natural disaster can be characterized by both individual risks and collective risks. The major result of this paper is: Catastrophe risks can be optimally, but not fully insured by Malinvaud=Arrow insurance, state-contingently composed of mutual insurance to mitigate individual losses across individual households as well as contingent securities to hedge collective risks. In this paper, we present a methodology to measure economic benefits of disaster mitigation investments, associated with the optimal allocation of disaster risks by Malinvaud=Arrow insurance.

1 Introduction

Thanks to the progress of disaster mitigation technologies, the frequencies that societies receive damages by earthquakes in developed countries have dramatically decreased, while catastrophe risks that a disastrous earthquake brings enormous economic loss have still increased owing to the accumulation of stocks in urban spaces. A large-scaled earthquake seldom occurs, but once it hits, a great number of people, firms and organizations are deprived of a large amount of wealth simultaneously. This study is concerned with the economic valuation of mitigation investment to reduce potential risks of large-scaled earthquakes, which may be termed as "catastrophe risks".

There has been an extensive literature on economic valuation of projects under uncertainty[1]. The previous studies, however, have commonly assumed random arrivals of small independent events, whose probabilities of occurences can be relevantly defined by aggregation over space and time. The insurance markets are supposed to fully function to cover the victims' losses caused by the events, presuming the law of large number sufficiently works. On the contrary, once a disastrous earthquake hits a society, a large number of victims may be seriously damaged at the same time. The catastrophic risks are characterized by their less frequent but bulky arrivals of damages. The traditional methods for economic valuation are incapable of dealing with such idiosyncrasies of catastrophe risks. We present an economic valuation methodology of disaster mitigation investment by taking these specific features of disasters explicitly into account.

The collective risks, in which arrivals of events are more or less mutually correlated, have been investigated in several fields. In the field of property-causality insurance theory, the Scandinavian school has carried forward formulating collectiveness of risks in reinsurance models[5][6][7]. Especially, Borch formulated a general equilibrium model[9] with reinsurance markets, which is based on the Arrow-Debreu type of general market equilibrium theory. In his model, Arrow's contingent securities (Arrow securities) are supposed to be transacted in a reinsurance market[8], and he proved that the Pareto efficient allocation of collective risks can be achieved by a decentralized way. Recently, Cass *et al.* extend Borch's work and to show that collective risks can be efficiently allocated by a new financial vehicle, combination of mutual insurance and Arrow securities[10]. While the basic structure of their model is very akin to the Borch model, it shows advantage to illustrates explicitly that the optimal combinations of mutual insurance and Arrow securities make it possible to spread efficiently the compound risks that are composed of individual risks and collective risks in markets.

There also exists another literature that proposes methods for evaluating benefits of avoiding the risks of irreversible events[11]. For example, a fatal accident of a nuclear plant might destroy a society completely. The death of human being cannot be substituted by other things. In this tradition, the benefits of avoiding arrivals of 'doomsday' is defined by those of avoiding the deaths of the households comprising the society. Hence, there remain no rooms for discussing about risk allocation of damage among potential victims. Over the decades, the disaster insurance and reinsurance markets have been remarkably developed. Reflecting this wave, there are an increasing number of papers related to disaster insurance, most of which have focused upon the calculation of insurance premium rates[12]. To the authors' knowledge, there are no previous attempts of applying disaster insurance theory to market valuation of disaster mitigation projects.

The catastrophic risks are characterized by their less frequent but bulky arrivals of damages. The traditional methods for economic valuation are incapable of dealing with such idiosyncrasies of catastrophe risks. We present an economic valuation methodology of disaster mitigation investment by taking these specific features of disasters explicitly into account. In tradition, the economic benefits of disaster risk mitigation have usually been calculated based upon the amount of expected loss reduction driven by the policy means in concern. As is discussed later on, evaluation based upon an expected-losses-reduction paradigm implicitly presumes that the society is always equipped with an ideal insurance system by which the victims' losses could be fully and instantly covered by insurance money without any friction. This is not apparently the cases with catastrophe risks. The catastrophe risks cannot be fully decentralized in disaster insurance markets. The losses of the victims can only be partially covered by insurance money. Hence, in economic valuation under catastrophe risks, the economic loss caused by partial coverage of disaster insurance should be explicitly considered.

This paper applies the Cass model[10] to proposes a new method to evaluate the economic benefits of catastrophe risks mitigation by disaster prevention investment. The original Cass model describes the direct contracts of the mutual insurance and direct transactions of the Arrow securities respectively among households, while we construct the disaster insurance that is equipped with both functions and formulate the disaster insurance market. Further we introduce the willingness-to-pay for mitigation of the risks in the disaster insurance market, and examine how it is affected by the improvement of the financial market.In what follows, Section 2 discusses the social optimal allocation of catastrophe risks, while section 3 formulates the insurance system which attains the optimal allocation in decentralized ways. Section 4 proposes the method for evaluating economic benefits of disaster mitigation investment, and some numerical examples are illustrated in section 5.

2 Social Optimal Allocation of Risks

2.1 The Assumptions

Suppose a single country which comprises hazardous regions as well as safe regions. The households can be classified into multiple categories, each of which is characterized by the homogeneous preferences, endowment, and risk perceptions. Regions could be regarded as household types. The households in the same region are assumed to be (ex ante) faced with homogeneous risks. Assume that households cannot change their types; migrations among regions are forbidden. The households in hazardous region may be deprived of part of their wealth if a large-scaled earthquake hits the region. The disaster insurance is available in the market. The government can partly control the collective risk by mitigation investment.

Let us characterize catastrophe risk by combinations of collective and individual risks. The collective risk corresponds to the distribution of aggregated losses that the society may suffer as a whole when an earthquake hits it. Given a state of collective risk, individual risk is described by a lottery of how the losses could be allocated among potential victims in the society. The catastrophe risk can be represented by a 'two-stage compound lottery'[13][14][10]. Let there be N_h households of type h, indexed by $h = 1, \cdots, H$. The total population is $N = \sum_h N_h$. The individual risk is characterized by $S + 1$ distinct states, denoted by s $(s = 0, 1, \cdots, S)$, each of which is defined by the amount of losses that the respective household receives when an earthquake occurs. The individual risk is represented by

$$\xi_h(s) = \begin{cases} 0 & \text{for } s = 0 \\ L(s)\ (> 0) & \text{otherwise,} \end{cases} \tag{1}$$

where $L(s)$ is the loss of wealth when individual state s occurs. The collective risk can be characterized by the distribution of losses aggregated over the population as a whole. A complete description of the states of the economy, called a social state, is given by a full set of individual states over the population. By definition, we have $\sum_{h=1}^{H}(S+1)^{N_h}$ distinct social states. A statistical description of the economy, called a collective state and indexed by t, is a statement of the sub-population in each individual state. Clearly, many social states map into a distinct collective state t. A collective state t is described by a vector $\boldsymbol{n}_t = (\boldsymbol{n}_{1t}, \cdots, \boldsymbol{n}_{Ht})$, where $\boldsymbol{n}_{ht} = \{n_{ht}^j\ (j = 0, \cdots, S)$ and n_{ht}^j denotes the number of households in type h being in individual state j. By definition, there holds $\sum_{s=0}^{S} n_{ht}^s = N_h$. The case where the economy suffers no loss, as one of collective states, is represented by the vector $\boldsymbol{n}_0 = (\boldsymbol{n}_{10}, \cdots, \hat{\boldsymbol{n}}_{H0})$ where $\boldsymbol{n}_{h0} = (N_h, 0, \cdots, 0)$. Denote the probability that collective state t occurs by $\pi(t)$. By definition, we see that the collective states are mutually exclusive, and there holds $\sum_{t=0}^{T} \pi(t) = 1$. Let us also denote the conditional probability that given households type h and collective state t, a particular individual state s occurs by $\pi_h(s|t)$, where there holds $\sum_{s=0}^{S} \pi_h(s|t) = 1$.

Let us describe catastrophe risk by a 'two-stage compound lottery', which is composed by individual state s, and collective state t. Suppose that first, nature selects a collective state; then individual states are determined by nature in order that statistical aggregation of individual states across the economy exactly map into the predetermined collective state. Denote the joint probability that collective state t and individual state s occurs for household of type h, by

$\pi_h(s,t) > 0$. There holds $\sum_{s,t} \pi_h(s,t) = 1$. Then, the probability that household h stands at individual state s the economy being in collective state t, denoted by $\pi_h(s|t)$, is defined by

$$\pi_h(s|t) = \frac{\pi_h(s,t)}{\sum_{s=0}^{S} \pi_h(s,t)}. \tag{2}$$

It is worth while noting here that there exactly holds a statistical relations $n_{ht}^s = \pi_h(s|t)N_h$[15]. All households in the respective type are endowed with the same state-contingent wealth; endowment depends solely on households' individual state s. State-contingent endowment of the representative household of type h in individual state s is given by $e_h(s) = W_h - \xi_h(s)$, where W_h is state-independent endowment of type h household and $\xi_h(s)$ is the loss of wealth in individual state s. For a moment, money transfer among victims is supposed to be taken place by some financial means. Let us denote state-contingent wealth after redistribution of h type household in joint state (s,t) by $x_h(s,t)$. Assume that the households in the same type and individual state are guaranteed to possess the same wealth irrespective of collective states. In other words, state-contingent wealth of each type household may differ only due to differences in individual states. The representative household is assumed to have von Neumann-Morgenstern utilities:

$$u_h(\boldsymbol{x}_h) = \sum_{s,t} \pi_h(s,t) v_h(x_h(s,t)), \tag{3}$$

where $\boldsymbol{x}_h = \{x_h(0,0), \cdots, x_h(s,t), \cdots, x_h(S,T)\}$ is the vector of the state-contingent wealth after redistribution. All households are risk aversive, and indirect utility function $v_h : R \to R$ satisfies $dv_h(x_h(s,t))/dx_h(s,t) > 0$, $d^2v_h(x_h(s,t))/dx_h(s,t)^2 \le 0$.

2.2 The Social Optimization

Consider the optimal risk allocation problem in the society. The central planner is supposed to maximize the weighted sum of individual expected utility functions. Thus, the Social Optimization problem (SO) is formulated as follows:

$$\max_{\boldsymbol{x}_h} \left\{ \sum_h \nu_h N_h u_h(\boldsymbol{x}_h) \right\} \tag{4}$$

subject to

$$\sum_h N_h \sum_s \pi_h(s|t)(x_h(s,t) - e_h(s)) = 0 \tag{5}$$

$$x_h(s,t) \ge 0 \quad \text{for all } s,t \tag{6}$$

where ν_h represents the weight attached to type h households. Note that constraints (5) should hold for the respective collective states. It means that the state-contingent redistribution of wealth is regulated by the total amount of state-contingent wealth in the society. Hence, there holds that $\sum_h N_h \sum_s \pi_h(s|t)x_h(s,t) = \sum_h N_h \sum_s \pi_h(s|t)e_h(s)$. The weights attached to the respective household types will be discussed further in 3.6. Denote Lagrange multiplier associated with constraint (5) by $p(t)$. Assuming interior solutions, the first-order condition is given by

$$\pi_h(s,t)\frac{dv_h(x_h(s,t))}{dx_h(s,t)} = \lambda_h \pi_h(s|t)p(t) \quad \text{for all } h,s,t \tag{7}$$

where $\lambda_h = 1/\nu_h$ is the converse of weight ν_h. Since $\pi_h(s,t) = \pi_h(s|t)\pi(t)$, eq.(7) can be transformed into

$$\pi(t)\frac{dv_h(x_h(s,t))}{dx_h(s,t)} = \lambda_h p(t) \quad \text{for all } h,s,t. \tag{8}$$

The Lagrange multiplier $p(t)$ evaluates the shadow price of the collective state t. The optimal condition (8) means that, for any t, h, the marginal utility with respect to state-contingent wealth of the households, $dv_h(x_h(s,t))/dx_h(s,t)$, should be equilibrated across all individual states, and congruent with $\lambda_h p(t)/\pi_h(t)$. Hence we have

$$x_h(0,t) = \cdots = x_h(S,t) = \hat{x}_h(t), \quad \text{for all } h, t, \tag{9}$$

where $\hat{x}_h(t)$ represents the optimal state-contingent wealth of h type households, conditional upon the economy being in collective state t. The efficient allocation of the catastrophe risk is given by the set of state-contingent wealth, $\boldsymbol{x}_h$ $(h = 1, \cdots, H)$, which satisfy eqs. (5) and (8). In other words, socially efficient risk allocation is attained by redistributing state-contingent wealth among individuals in such a way that the households' marginal utility with respect to state-contingent wealth is equalized among all households in the the same type, and satisfies the optimal condition (8).

3 Market Allocation of Catastrophe Risk

3.1 Mutual Insurance and Pure Arrow Security

As discussed in the previous section, the efficient allocations of catastrophe risk can be realized by redistributing state-contingent wealth across the population. In this section, we try to design a disaster insurance system to realize an efficient allocation of catastrophe risk in a decentralized way. Recently, Cass *et al.* show the catastrophe risk could be efficiently allocated by means of mutual insurance contracts and pure Arrow securities[10]. The pooling of individual risk is accomplished by means of mutual insurance contracts, each of which is designed to cover a particular joint state for a particular type of household in the respective segmented insurance markets. The spreading of collective risk is accomplished by means of pure Arrow securities, each covering a particular collective state for all types of households. Though Cass *et al.* (1996) only consider an exchange economy with catastrophe risk, this paper tries to expand their model by introducing Arrow=Malinvaud insurance systems to show that an efficient disaster insurance system can be designed as a combination of mutual insurance contracts and pure Arrow securities.

Consider the market opportunities for insuring individual risk faced by households in particular type h. Suppose that these households can buy or sell units of each of mutual insurance policies which pay a net benefit of $1 - \pi_h(s|t)$ if they find themselves in a joint state (s,t), and carry a premium of $\pi_h(s|t)$ in a joint state (s',t) with $s' \neq s$. The mutual insurance system Ω_h for type h households can be represented by a bundle of contingent insurance contracts of insurance payment $m_h(s,t)$ and premium $\mu_h(t)$:

$$\Omega_h = (\boldsymbol{m}_h, \boldsymbol{\mu}_h), \tag{10}$$

where $\boldsymbol{m}_h = \{m_h(0,0), \cdots, m_h(s,t), \cdots, m_h(S,T)\}$, $\boldsymbol{\mu}_h = \{\mu_h(0), \cdots, \mu_h(t), \cdots, \mu_h(T)\}$. In this scheme, while all households in type h pay out $\mu_h(t)$, in collective state t, insurance money that individuals in state s receive is represented by $m_h(s,t)$. With the mutual insurance system, the state-contingent wealth of the household in type h being in state (s,t) is defined by

$$\hat{e}_h(s,t) = e(s) + m_h(s,t) - \mu_h(t). \tag{11}$$

The total amounts of insurance claims when collective risk t occurs are given by $\sum_h \sum_s \pi_h(s|t) N_h m_h(s,t)$, while the total premium income of the damaged amounts to $\sum_h N_h \mu_h(t)$. In perfectly competitive mutual insurance markets, the total premiums will necessarily equal

$$\mu_h(t) = \sum_{s=0}^{S} \pi_h(s|t) m_h(s,t). \tag{12}$$

This is idealization of mutual insurance contracts, and all transaction costs associated with administrative expenses, payment to the risk-bearer and moral hazard are neglected.

Now, consider the other market opportunities for insuring the collective risk faced by the households of all types. Suppose that, in addition to mutual insurance policies, the households can buy and sell pure Arrow securities[8][16]. Also, suppose that $T+1$ types of Arrow securities are available in the society, each of which exactly corresponds to the respective collective states. Arrow securities cost one price unit ex ante, and return one price unit if they find themselves in collective state t, but nothing otherwise. An ex ante price of unit security, $p(t)$, is endogeneously determined in the market. Denote an individual position of the respective securities by a vector $\boldsymbol{a}_h = (a_h(0), \cdots, a_h(t), \cdots, a_h(T))$. Thus, the total cost of holding a bundle of securities $\boldsymbol{a}_h$ is expressed by

$$y_h = \sum_{t=0}^{T} p(t) a_h(t) \tag{13}$$

Given these opportunities, state-contingent wealth of households of type h in joint state (s,t) is given by

$$x_h(s,t) = e_h(s) + m_h(s,t) - \sum_{s'=0}^{S} \pi_h(s'|t) m_h(s',t) + a_h(t) - y_h \quad \text{for all } s,t. \tag{14}$$

where the second term of the R.H.S is insurance claim corresponding to joint state (s,t); the third term mutual insurance premium for collective state t and the term $a_h(t) - y_h$ are net profits (losses) of Arrow securities when positive (negative). Given a collective state, from eq.(12), the expected wealth over individual states can be expressed by

$$\sum_{s=0}^{S} \pi_h(s|t) x_h(s,t) = \sum_{s=0}^{S} \pi_h(s|t) e_h(s) + a_h(t) - y_h \quad \text{for all } s,t, \tag{15}$$

which can be decomposed into expected wealth without redistribution denoted by $\sum_{s=0}^{S} \pi_h(s|t) e_h(s)$ and net profits (losses) driven by holding Arrow securities $a_h(t) - y_h$. Hence, rearranging eq. (15). the demand for Arrow securities against collective risk t is given by

$$a_h(t) = \sum_{s=0}^{S} \pi_h(s|t) \{x_h(s,t) - e_h(s)\} + y_h. \tag{16}$$

3.2 Market Equilibrium

Given market price of Arrow securities $p(t)$, the individual optimization problem IO is formulated as follows:

$$\max_{\boldsymbol{m}_h, \boldsymbol{a}_h, \boldsymbol{x}_h, y_h} \left\{ \sum_{s=0}^{S} \sum_{t=0}^{T} \pi_h(s,t) v_h(x_h(s,t)) \right\} \tag{17}$$

subject to

$$\sum_{t=0}^{T} p(t)a_h(t) = y_h \tag{18}$$

$$x_h(s,t) = e_h(s) + m_h(s,t) - \sum_{s'=0}^{S} \pi_h(s'|t)m_h(s',t) + a_h(t) - y_h \quad \text{for all } s,t \tag{19}$$

$$x_h(s,t) \geq 0,\ a_h(t) \geq 0,\ m_h(s,t) \geq 0,\ y_h \geq 0. \tag{20}$$

Let us denote Lagrange multipliers associated with constraints (18) and (19) by $\lambda_h, \lambda_h(s,t)$ respectively. Assuming an interior solution, the first-order optimal conditions are given by

$$\pi_h(s,t)\frac{dv_h(x_h(s,t))}{dx_h(s,t)} = \lambda_h(s,t) \tag{21}$$

$$\pi_h(s|t)\sum_{s'=0}^{S} \lambda_h(s',t) = \lambda_h(s,t) \tag{22}$$

$$\lambda_h p(t) = \sum_{s=0}^{S} \lambda_h(s,t) \tag{23}$$

$$\lambda_h = \sum_{s=0}^{S}\sum_{t=0}^{T} \lambda_h(s,t) \tag{24}$$

From eq.(22),(23), we have

$$\lambda_h(s,t) = \pi_h(s|t)p(t)\lambda_h \tag{25}$$

Putting this result into eq.(21), we get

$$\pi_h(t)\frac{dv_h(x_h(s,t))}{dx_h(s,t)} = \lambda_h p(t) \tag{26}$$

From eqs.(21) and (24), we have

$$\lambda_h = \sum_{s=0}^{S}\sum_{t=0}^{T} \pi_h(s,t)\frac{dv_h(x_h(s,t))}{dx_h(s,t)}. \tag{27}$$

Since the R.H.S. of eq.(26) is independent from s, $dv_h(x_h(s,t))/dx_h(s,t)$ should be constant for arbitrary s. Thus, we find that the relations (9) also hold for arbitrary t, h in a decentralized economy. From eq.(12), the optimal demand for mutual insurance is given by

$$m_h(s,t) = e_h(0) - e_h(s) = L(s). \tag{28}$$

Thus, individual risks are fully covered by mutual insurance. From eqs. (23) and (24), the relation between market price of Arrow security, $p(t)$, and shadow price of state-contingent wealth, $\lambda_h(s,t)$, is given by

$$p(t) = \frac{\sum_{s=0}^{S} \lambda_h(s,t)}{\sum_{s=0}^{S}\sum_{t=0}^{T} \lambda_h(s,t)}. \tag{29}$$

From eq. (27), we see that shadow price of holding a bundle of Arrow securities by type h households should be equalized with expected marginal utility of state-contingent wealth with redistribution.

The prices of Arrow securities are determined so that the respective contingent markets are clearing. Given collective state t, total amount of the claims that all type h households pay is $\sum_h N_h a_h(t)$, while the total revenue from selling securities is $\sum_h \sum_t N_h p(t) a_h(t)$. Thus the following arbitrage conditions are satisfied.

$$\sum_{h=1}^{H} N_h a_h(t) = \sum_{h=1}^{H} \sum_{t'=0}^{T} N_h p(t') a_h(t') \quad \text{for all} \quad t \tag{30}$$

Eq.(24) and (25), the market prices automatically satisfy the normalizing condition $\sum_{t=0}^{T} p(t) = 1$.

It should be noted here that we have the equivalent conditions: eq. (8) for the centralized problem and eq. (26) for the market solution. In other words, by means of combinations of mutual insurance contracts and pure Arrow securities, the social optimum allocation of catastrophe risk can be achieved by individual utility maximizing behavior in a decentralized fashion. By comparing eqs. (8) and (7), the market risk allocation exactly corresponds to the solution of the centralized problem in which the associated weights ν_h is defined by the converse of the expected marginal utility of the state-contingent wealth represented by λ_h. Hence, we have the following proposition.

Proposition **By transaction of mutual insurance and pure Arrow securities ($\boldsymbol{m}_h$, $\boldsymbol{a}_h$) ($h = 1, \cdots, H$), the Pareto optimal allocation of catastrophe risk can be achieved in a decentralized way.**

3.3 Malinvaud=Arrow Insurance

So far, each household is supposed to transact securities in the markets. But, this assumption is not substantial. Instead, the equivalent risks allocation can be attained if the insurance industry provides the households with alternative disaster insurance systems. Let us call the disaster insurance that is defined by combinations of mutual insurance and securities as Malinvaud=Arrow insurance. Let us define the premium c_h for h type households of this insurance system by

$$c_h = \sum_{t=0}^{T} \{p^*(t) a_h^*(t) + \mu_h^*(t)\}, \tag{31}$$

where $p^*(t), a_h^*(t)$ and $\mu_h^*(t)$ denote the equilibrium price of Arrow securities and the equilibrium demand of securities by type h household, and the equilibrium premium of mutual insurance, respectively. The first term represents the payment for Arrow securities; the second term the premium for mutual insurance. On the other hand, the insurance money, denoted by $R(s,t)$, which h type household can expect to receive in joint state (s,t), is defined by

$$R_h(s,t) = m_h^*(s,t) + a_h^*(t) + \sum_{t' \neq t} \mu_h^*(t'). \tag{32}$$

Note that disaster insurance schemes $(c_h, R_h(s,t))$ are not unique. For example, by adding a constant A on the R.H.S. of both eqs. (31) and (32), we can design a new scheme $(c_h + A, R_h(s,t) + A)$ that may function equivalently as the original scheme.

The Malinvaud=Arrow insurance market is expected to broaden the capacity of risk financing to a great extent. It makes the insurance money by selling the contingent securities in international capital markets which are incomparably larger than insurance markets. For this purpose, some sorts of securities have already been designed for practical use. The disaster insurance of Malinvaud=Arrow-model is the highly abstracted model of risk spreading with securities, utilizing the mechanism of conditional market. There remain several unsolved issues that must be discussed in future.

First, in many case, households cannot fully percept disaster risks, and lack motivation to purchase insurance. This is especially the case, if they anticipate that relief programs for victims will be implemented by governments. Imperfect cognition, moral hazard, and transaction costs distort efficient market allocation of risks. One of the countermeasure against imperfect cognition or risk-neglecting behavior is that regional governments levy a tax on respective inhabitants as insurance premium, and buy the disaster insurance in the market to insure the losses of the inhabitants. This policy is valuable since regional governments usually hold more precise information than households and act better against risks with less moral hazard. However if we take possibility of migration of households into account, problems such as difference of subjective probabilities between regional governments and households, and moral hazard of households are still unavoidable. Accordingly this policy is also regarded as the second best policy.

As was referred in 3.4, the market equilibrium correspond to the Pareto optimal allocation of risks that is prescribed by the weights ν_h $(h = 1, \cdots, H)$, which is the converse of the expected equilibrium marginal utility of the contingent wealth, λ_h. It means that larger amounts of wealth are allocated to the households whose expected marginal utility is smaller (who own larger amounts of wealth). Namely, we gain more inverse allocation of wealth than the one which is given if we adopt the social welfare function where households' expected utility functions are simply added. Here arise the problems of fair risk allocation.

4 Economic Valuation of Disaster Mitigation Investment

4.1 The Cost-Benefit Rule

Disaster mitigation investment is the basic means to reduce collective risks. Let $\pi(t : z)$ be the conditional probability that collective state t occurs when a society is equipped with mitigation capital stocks, represented by a single parameter z. Consider a single mitigation project, which can increase mitigation capital stocks from z_0 to z_1. Accordingly, the project will change the conditional probability of collective risk t from $\pi(t : z_0)$ to $\pi(t : z_1)$. Suppose further that the investment change probabilities vector, state-contingent wealth, and price of contingent securities from $\boldsymbol{\pi}_h^0$, $\boldsymbol{x}_h^{*0}$ and $\boldsymbol{p}^{*0}$ to $\boldsymbol{\pi}_h^1$, $\boldsymbol{x}_h^{*1}$ and $\boldsymbol{p}^{*1}$, respectively. The non-contingent compensating valuation (OP_h^C) can be defined by the amount of money such that

$$E[v(x_h^{*1}(s,t) - OP) : \pi_h^1(s,t)] = E[v(x_h^{*0}(s,t)) : \pi_h^0(s,t)], \tag{33}$$

where $E[\cdot : \pi_h^i(s,t)]$ is expectation operator with respect to $\pi_h^i(s,t)$ $(i = 0, 1)$. The project is supposed to be sufficiently small and perfectly divisible. Further, the probability of the occurrence of disaster, denoted by $\pi_h(s, t : z)$, is assumed to be continuously differentiable with respect to z. By totally differentiating $dE[v_h(\boldsymbol{x}_h^{*1} - OP_h^C) : \boldsymbol{\pi}_h^1] = 0$ in the neighborhood of $\boldsymbol{x}_h^1, \boldsymbol{\pi}_h^1$, we have

$$\begin{aligned}
&\sum_{s=0}^{S}\sum_{t=0}^{T}\{\frac{\partial \pi_h^1(s,t)}{\partial z} v_h(x_h^{*1}(s,t))(-dz) \\
&+\pi_h^1(s,t)\frac{\partial v_h(x_h^{*1}(s,t))}{\partial x_h(s,t)}[(1-p^*(t))da_h^*(t) - a_h^*(t)dp^*(t) - dOP_h^C]\} = 0,
\end{aligned} \tag{34}$$

where $x_h^*(s,t), p^*(t)$ and $a_h^*(t)$ are the equilibrium state-contingent wealth, the equilibrium price of contingent securities and the equilibrium demand for contingent securities in the market. Totally differentiating both side of securities market clearing condition (30), we have

$$da_h^*(t) = p^*(t)da_h^*(t) + dp^*(t)a_h^*(t). \tag{35}$$

Substituting eq. (35) into (34), and taking the optimal conditions (21),(24) into account, the noncontingent compensating valuation (the compensating option price), can be given by

$$\begin{aligned} dOP_h^C &= \frac{1}{\lambda_h^1}\sum_{s,t}\frac{\partial \pi_h^1(s,t)}{\partial z}v_h(x_h^{*1}(s,t))\cdot(-dz) \qquad (36)\\ \lambda_h^1 &= \sum_{s=0}^{S}\sum_{t=0}^{T}\pi_h^1(s,t)\frac{\partial v_h(x_h^{*1}(s,t))}{\partial x_h^1(s,t)}, \end{aligned}$$

where λ_h^1 is the expected marginal utility, which given $\boldsymbol{\pi}_h^1$, is equal to the shadow price of the bundle of contingent securities at the equilibrium. If the Malinvaud=Arrow insurance is available in the society, the individual risk could be covered by mutual insurance so that there hold eq.(9) for each collective state t. Accordingly, we finally have

$$dOP_h^C = \frac{1}{\lambda_h^1}\sum_{t=0}^{T}\frac{\partial \pi^1(t)}{\partial z}v_h(\hat{x}_h^{*1}(t))\cdot(-dz). \tag{37}$$

In the same way, the equivalent option price is defined by

$$\begin{aligned} dOP_h^E &= -\frac{1}{\lambda_h^0}\sum_{t=0}^{T}\frac{\partial \pi^0(t)}{\partial z}v_h(\hat{x}_h^{*0}(t))dz \qquad (38)\\ \lambda_h^0 &= \sum_{t=0}^{T}\pi_h^0(t)\frac{\partial v_h(\hat{x}_h^{*0}(t))}{\partial \hat{x}_h^0(t)}. \end{aligned}$$

By definition of option price indices, we see that the compensating and equivalent option price indices are sign-preserving, and there holds $dOP_h^C \geq dOP_h^E$ (see **Appendix** for proof.). The the willingness-to-pay, if the project is large, can be straightforwardly derived from eqs. (37) and (38). The compensating option price with large project, denoted by ΔOP_h^C, and the equivalent option price, ΔOP_h^E, are respectively given by

$$\Delta OP_h^C = \int_{z^0}^{z^1} dOP_h^C dz \tag{39}$$

$$\Delta OP_h^E = \int_{z^0}^{z^1} dOP_h^E dz. \tag{40}$$

4.2 Option Price and Disaster Insurance

From eqs. (14), (31) and (32), the state-contingent wealth is given by

$$x_h(s,t) = e_h(s) + R_h(s,t) - c_h. \tag{41}$$

Substituting this to eq. (33), the compensating option price is defined by

$$E[v_h(e_h(s) + R_h^{1*}(s,t) - c_h^{1*} - OP_h^C) : \boldsymbol{\pi}_h^1] = E[v_h(\boldsymbol{x}_h^{*0}) : \boldsymbol{\pi}_h^0] \tag{42}$$

where $R_h^{1*}(s,t)$ and c_h^{1*} are ex post insurance money and premium after mitigation is made. Considering that premium c_h^{1*} becomes independent from states, from eq. (42), we see that option price is exactly measured by to the reduction in insurance premium which a household pays for.

Conventionally, economic benefits of disaster mitigation have been evaluated by use of the reduction in expected losses. The conventional method, however, is relevant if and only if individual losses are fully covered by mutual insurance (see **Appendix II**). This is not the case for catastrophe risk. Aggregating state-contingent individual wealth, represented by eq.(14), across all households, we have

$$\sum_h \sum_s \pi_h(s|t) N_h x_h(s,t) = \sum_h \sum_s \pi_h(s|t) N_h \{e_h(s) + m_h(s,t) \\ - \sum_{s'} \pi_h(s'|t) m_h(s',t) + a_h(t) - y_h\} \quad \text{for all} \quad t. \tag{43}$$

From conditions (12) and (30), we obtain

$$\sum_h N_h \hat{x}_h(t) = \sum_h N_h \sum_s \pi_h(s|t) e_h(s) \quad \text{for all} \quad t. \tag{44}$$

This explains that the total amount of direct monetary losses caused by disaster cannot be reduced by any means of risk financial initiative, though the psychological cost of the victims could be largely reduced by reallocating ex post wealth after disaster. On the other hand, the mitigation investment could reduce the expected direct losses by disaster. The benefits of mitigation investment in its broader term, including the psychological as well as direct monetary costs, are dependent upon the risk financial initiative that are available at the time.

5 Numerical Examples

5.1 Specification

Consider a simple society consisting two types ($h = 1, 2$) of households, whose population are N_1 and N_2, respectively. The type 1 households are faced by disaster risk, while the type 2 households no disaster risk. Consider two different collective states: no disaster occurs when $t = 0$, and a disaster occurs and causes losses when $t = 1$. For the respective states, the number of victims are represented by $(0, 0), (n, 0)$. The probabilities of collective risk, denoted by $\pi(0)$, are defined by $\pi(0) = 1 - q$ and $\pi(1) = q$. The individual risk of type 1 households are represented by conditional probabilities:

$$\pi_1(0|t) = \begin{cases} 1 & (t = 0) \\ 1 - \frac{n}{N_1} & (t = 1) \end{cases} \tag{45}$$

$$\pi_1(1|t) = \begin{cases} 0 & (t = 0) \\ \frac{n}{N_1} & (t = 1) \end{cases} \tag{46}$$

The utility function is assumed to be CRRA (constant relative risk aversive):

$$v_h(x_h(s,t)) = \frac{x_h(s,t)^{1-\gamma} - 1}{1 - \gamma} \quad (\gamma \neq 1) \\ = \log x_h(s,t) \qquad (\gamma = 1) \tag{47}$$

$$R_r(x) = -x \frac{v_h''(x)}{v_h'(x)} = \gamma = const. \tag{48}$$

for $h = 1, 2$, where $R_r(x)$ is the degree of relative risk aversion. The social optimization problem (SO) gives us the optimal solution such that

$$\hat{x}_h(0) = \left(\frac{1-q}{\lambda_h p(0)}\right)^{1/\gamma}, \quad \hat{x}_h(1) = \left(\frac{q}{\lambda_h p(1)}\right)^{1/\gamma} \tag{49}$$

$$p(0) = (1-q) \cdot \left(\frac{\frac{N_1}{\lambda_1^{1/\gamma}} + \frac{N_2}{\lambda_2^{1/\gamma}}}{w(0)}\right)^{\gamma}, \quad p(1) = q \cdot \left(\frac{\frac{N_1}{\lambda_1^{1/\gamma}} + \frac{N_2}{\lambda_2^{1/\gamma}}}{w(1)}\right)^{\gamma}, \tag{50}$$

where $w(0) = N_1 e_1(0) + N_2 e_2(0)$ and $w(1) = (N_1 - n)e_1(0) + n e_1(1) + N_2 e_2(0)$. Clearly, we see $w(0) \geq w(1)$. The solution of the individual optimization problem (IO) is given by

$$\hat{x}_h(0) = \left(\frac{1-q}{\lambda_h p(0)}\right)^{1/\gamma}, \ \hat{x}_h(1) = \left(\frac{q}{\lambda_h p(1)}\right)^{1/\gamma} \tag{51}$$

$$\lambda_h = \left[\frac{\left(\frac{1-q}{p(0)^{1-\gamma}}\right)^{1/\gamma} + \left(\frac{q}{p(1)^{1-\gamma}}\right)^{1/\gamma}}{\Phi_h}\right]^{\gamma}, \tag{52}$$

where $\Phi_h = \{p(0) + p(1)\pi_h(0|1)\}e_h(0) + p(1)\ \pi_h(1|1)\ e_h(1)$. From eqs. (51), (52) and (30), we obtain $p(0), p(1)$ explicitly as

$$p(0) = \frac{(1-q)w(1)^{\gamma}}{\Gamma}, \quad p(1) = \frac{q w(0)^{\gamma}}{\Gamma} \tag{53}$$

$$\lambda_1 = \frac{\Gamma \Xi^{\gamma}}{w(0)^{\gamma} w(1)^{\gamma} \Theta_1^{\gamma}}, \quad \lambda_2 = \frac{\Gamma \Xi^{\gamma}}{w(0)^{\gamma} w(1)^{\gamma} \Theta_2^{\gamma}}, \tag{54}$$

where $\Gamma = q w(0)^{\gamma} + (1-q) w(1)^{\gamma}$, $\Xi = q w(0)^{\gamma} w(1) + (1-q) w(0) w(1)^{\gamma}$, $\Theta_1 = \{(1 - \frac{n}{N_1}) q w(0)^{\gamma} + (1-q) w(1)^{\gamma}\} e_1(0) + \frac{n}{N_1} q w(0)^{\gamma} e_1(1)$ and $\Theta_2 = \Gamma e_2(0)$. Consequently, the equilibrium states of state-dependent individual wealth are defined by

$$\hat{x}_1(0) = \frac{\Theta_1}{\Xi} w(0), \quad \hat{x}_1(1) = \frac{\Theta_1}{\Xi} w(1) \tag{55}$$

$$\hat{x}_2(0) = \frac{\Theta_2}{\Xi} w(0), \quad \hat{x}_2(1) = \frac{\Theta_2}{\Xi} w(1). \tag{56}$$

At the equilibrium, there hold the following relationships among state-contingent wealth, demands for mutual insurance and those for Arrow securities:

$$\begin{aligned} \hat{x}_1(0) - \hat{x}_1(1) &= \pi_1(1|1)\{e_1(0) - e_1(1)\} + \{a_1(0) - a_1(1)\} \\ &= \pi_1(1|1) m_1(1,1) + \{a_1(0) - a_1(1)\} \end{aligned} \tag{57}$$

$$\hat{x}_2(0) - \hat{x}_2(1) = a_2(0) - a_2(1) \tag{58}$$

The first term of eq. (57) represents expected losses of household of type h (identical to premium for mutual insurance $\pi_1(1|1)m_1(1,1)$). If $a_h(0)\ - a_h(1)$ is negative, the second term expresses net monetary losses driven by holding Arrow securities. The difference in the states of individual wealth between collective states $t = 0$ and $t = 1$ is exactly identical to the aggregated sum of insurance premium to cover both of individual and collective risks. Considering $w(0) \geq w(1)$, and eqs. (57) and (58), we see that $a_1(0) \leq a_1(1)$, $a_2(0) \geq a_2(1)$. Further, from $y_h = p(0)a_h(0) + p(1)a_h(1)$ and $p(0) + p(1) = 1$, we see that y_h is an interior point between $a_h(0)$ and $a_h(1)$. Hence, we have

$$a_1(0) \leq y_1 \leq a_1(1) \tag{59}$$

$$a_2(0) \geq y_2 \geq a_2(1). \tag{60}$$

The type 1 household receives claim $a_1(1)$ when disaster occurs and $a_1(0)$ when no disaster occurs. The Type 2 household gains $a_2(0) - y_2$ when no disaster occurs, while loses $y_2 - a_2(1)$ when disaster occurs. As it were, the disaster insurance serves type 2 households as high-risk-high-return asset. Given the above solution, let us observe that Malinvaud=Arrow insurance does not guarantee the fair premium. The Principle of Lexis should be violated. Compare premium c_h with expected insurance money $\bar{R}_h$. As mentioned before, the scheme of Malinvaud=Arrow insurance system is note unique; only the differences in holding levels of securities between two collective states, $a_h(0)-a_h(1)$, have substantial meaning. Hence, without loss of generality, let us standardize as $a_1(0) = 0$ and $a_2(0) = 0$. Then, holding positions of Arrow securities in collective state 1 are given by

$$a_1(1) = \frac{N_2}{N_1}\frac{\Theta_2}{\Xi}n(e_1(0) - e_1(1)) \tag{61}$$

$$a_2(1) = -\frac{\Theta_2}{\Xi}n(e_1(0) - e_1(1)) \tag{62}$$

for the respective household types. Hence, we have the following results:

$$c_1 - \bar{R}_1 = \frac{N_2}{N_1}\frac{n}{\Xi}q(1-q)(w(0)^\gamma - w(1)^\gamma)(e_1(0) - e_1(1))e_2(0) \quad > 0 \tag{63}$$

$$c_2 - \bar{R}_2 = -\frac{n}{\Xi}q(1-q)(w(0)^\gamma - w(1)^\gamma)(e_1(0) - e_1(1))e_2(0) \quad < 0. \tag{64}$$

For type 1 households, the premium becomes larger than the expected insurance money, while for type 2 households, the expected insurance money exceed their insurance premium.

5.2 Economic Benefit of Mitigation Investment

The disaster mitigation investment may contribute to changes of the probabilities that an large earthquake arrives, as well as changes of the distribution of disaster loss magnitude. Though the cost-benefit rule presented in this paper can be applied to both changes. In what follows, for simplicity of discussions, let us focus upon the case where mitigation investment reduces the probabilities that disasters hit the society, but does not change the distribution of the magnitude of disaster losses.

Let us first consider the cases where mitigation investment only affects probabilities that an earthquake hits the society. Assume that probabilities of collective state 1 is regulated by mitigation capital stock z, specified as $q(z) = q_0 \exp(-z)$. From eqs. (37), (55a), and (56b), the compensating option price (referring to 'the option price' hereafter) is given by

$$dOP_h^C = -\frac{\Theta_h}{(1-\gamma)\Gamma\Xi}\{w(0)^\gamma w(1) - w(0)w(1)^\gamma\}\frac{dq(z^1)}{dz}dz \ (h = 1, 2). \tag{65}$$

For numerical calculations, we set $N_1 = 1$, $N_2 = 2$, $n = 0.5$, $e_1(0) = e_2(0) = 10$, $e_1(1) = 5$, $q_0 = 0.1$ and $dz = 1$. In this benchmark case, half of type 1 households are supposed to be deprived of half of their wealth. Define mark-up index by $\rho = \overline{OP}^C/\bar{L}$, where $\overline{OP}^C = N_1 OP_1^C + N_2 OP_2^C$ is the aggregated value of individual option prices and $\bar{L} = \{q(z^0) - q(z^1)\}\{w(0) - w(1)\}$ is the total expected losses reduction driven by mitigation investment.

Figure.1 and **Figure.2** respectively illustrate the relations between individual states and mark-up indices, and those between collective states and mark-up indices, where the degree of expected losses reduction is assumed to remain constant at $\bar{L}^0 = 0.25$. The degree of individual risk is measured by $d = \frac{e_1(0)-e_1(1)}{e_1(0)}$, while the magnitude of collective risk is expressed by $c = \frac{n}{N_1}$.

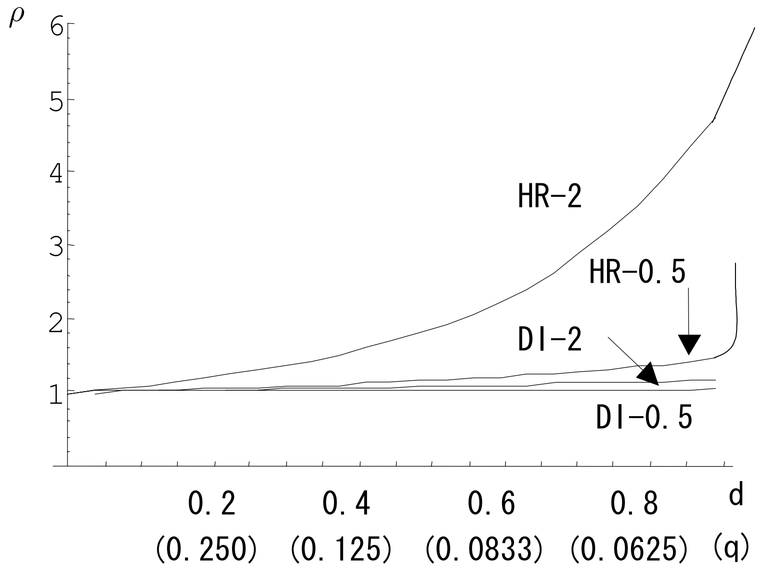

Figure.1 The relationships between individual losses and mark-up indices

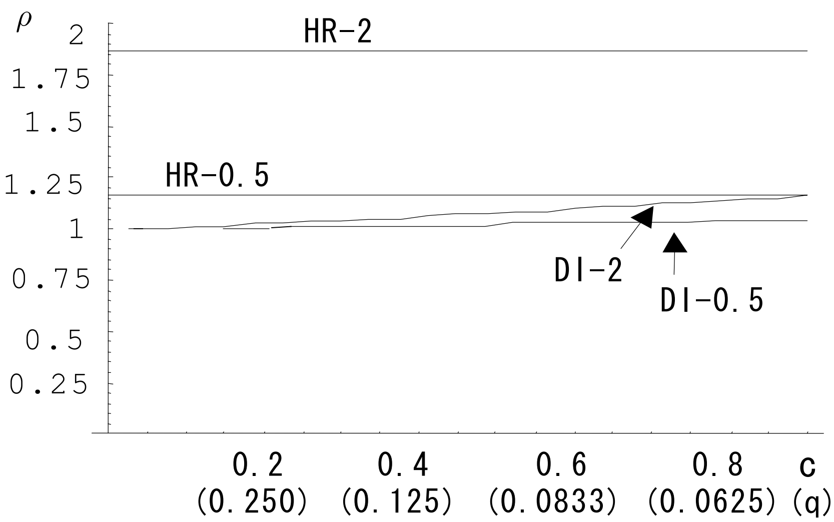

Figure.2 The relationships between collective states and mark-up indices

Given the predetermined levels of the expected losses, $\bar{L}^0$, the relationships among q, d, and c satisfy

$$\frac{\bar{L}^0}{N_1 e_1(0)} = qcd. \tag{66}$$

Figure.1 shows the relationships between mark-up indices and $d = \frac{e_1(0)-e_1(1)}{e_1(0)}$, given $c = 0.5$. The horizontal axis represents the levels of d, which are inversely related to q. As the values of d increase, the size of disaster loss increase, while frequencies become smaller. In this figure, four different lines are drawn. The line corresponding to HR-2(Holding Risk; $\gamma = 2$) explains the relations in the case where no disaster insurance is available, and the relative risk aversion of the households is fixed to 2, while that in the case DI-2(Disaster Insurance,$\gamma = 2$), the disaster insurance is supposed to become available. In addition, the lines in the cases HR-0.5 and DI-0.5 illustrate the similar relationships, where the degree of relative risk aversion γ goes down to 0.5. For all cases, the mark-up indices are always larger than unity. The larger the degrees of individual risk are, the larger the mark-up indices are. When the disaster insurance is not available in the economy, the values of mark-up indices go to infinity as d approaches to 1. On the other hand, once the disaster insurance become available, the mark-up indices take finite values even if d approaches to 1. The mark-up indices increase as the degrees of risk aversion increase.

Figure.2 shows the relationships between the mark-up indices and the magnitude of collective risks $c = \frac{n}{N_1}$, given the fixed degree of individual risk at $d = 0.5$ and $\bar{L}^0 = 0.25$. In both cases of HR-2 and HR-0.5, where no disaster insurance is available, the mark-up indices remain constant, being independent from the magnitude of collective risks. As far as risk allocation among individuals is prohibited, the welfare of the respective households is not influenced by the welfare of others. On the other hand, once the disaster insurance becomes available, the mark-up indices inevitably increase as the magnitude of collective risk increase.

Figure.3 illustrates the relationships between the aggregated option values of marginal mitigation investment and the existing stocks of mitigation capitals. Three lines of four correspond to the cases where 1) no initiative are available (indicated by HR), 2) only mutual insurance contracts are available (MI, Mutual Insurance), and 3) the disaster insurance is fully available (DI). For a vehicle of comparison, the line, indicated by (EL, Expected Loss) illustrate these relationships when the economic benefits are measured based upon the terms of expected losses reduction. This figure clearly explains that economic benefits of mitigation investment measured by option prices largely exceed the ones estimated by conventional methods based upon expected loss reduction. In other words, the conventional methods may lead to underestimation of the economic benefits of the projects to mitigate catastrophe risk. This is particularly true if the society is not equipped with disaster insurance systems.

6 Concluding Remarks

Summing up the main findings of the paper, catastrophe risks by natural disaster can be characterized by composition of both individual risks and collective risks. Catastrophe risks can be optimally, but not fully, insured by the new types of disaster insurance, which are state-contingently composed of mutual insurance to spread individual losses among the households of the same type and of contingent securities to transfer wealth among the pools of different types. The paper has further proposed the method to evaluate economic benefits of disaster prevention investment, which is made in conjunction with the optimal allocation of disaster risks by the disaster insurance.

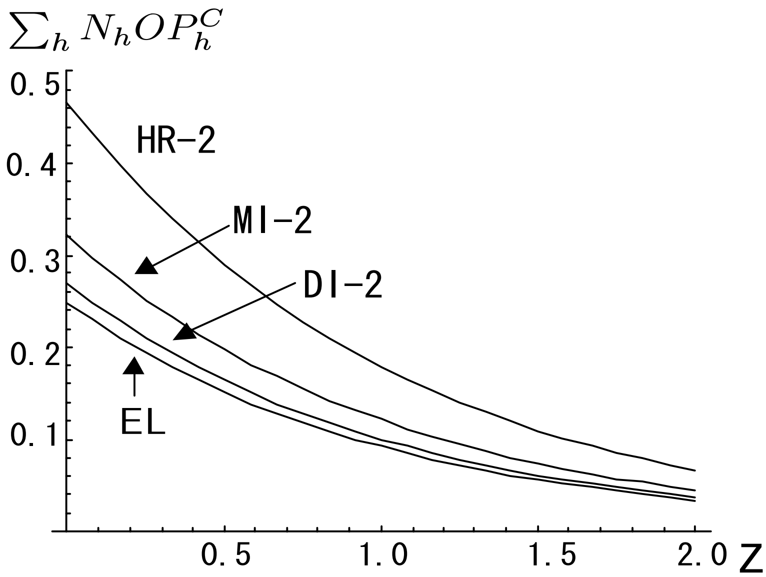

Figure.3 The relationships between mitigation stocks and economic benefits of mitigation investment

The future extension of the models presented in the paper may include: (1) The problems caused by asymmetric information conditions in insurance markets, most typically exemplified by moral hazard and incomplete cognition of risks, must be investigated. (2) Fair allocation of disaster risks must be considered. The disaster insurance systems presented in this paper result in regressive allocation of contingent wealth among households. (3) The possibility of migration among regions (types) by households will be one of the most substantial elements composing the problem. It is nothing other than the right of choosing the 'type' given to each household in our model, and it is probable to distort Pareto efficient allocation of disaster risks. The mechanism allowing migration must be analyzed. (4) In the real world, disaster insurance competes with another derivatives in financial market. The assumption of complete contingent markets is far from the reality. More practical insurance scheme should be discussed. The general equilibrium modeling with incomplete security markets is an interesting subject to challenge[18].

Appendix I Proof of Property

The expected utility is represented with certainty equivalent $v_h(\bar{x}_h) = E[v_h(\boldsymbol{x}_h^*) : \boldsymbol{\pi}_h]$. Regarding that OP^C is state-independently defined, the difference between the expected utility after the project and the one before the project is given by $\Delta v_h = v_h(\bar{x}_h^1) - v_h(\bar{x}_h^0) = v_h(\bar{x}_h^1) - v_h(\bar{x}_h^1 - OP^C) \geq 0$. Both compensating option price and equivalent option price are equipped with sign-preserving property. $\Delta v_h = v_h(\bar{x}_h^1) - v_h(\bar{x}_h^1 - OP_h^C) = v_h(\bar{x}_h^0 + OP_h^E) - v_h(\bar{x}_h^0)$. It follows $v_h(\bar{x}_h^0 + OP_h^E) + v_h(\bar{x}_h^1 - OP_h^C) = v_h(\bar{x}_h^1) + v_h(\bar{x}_h^0)$. Now, the risk premium ι is introduced to satisfy $(v_h(\bar{x}_h^1) + v_h(\bar{x}_h^0))/2 = v_h(\bar{\bar{x}}_h + \iota)$ where $\bar{\bar{x}}_h = (\bar{x}_h^1 + \bar{x}_h^0)/2$. The assumption of risk averse preference implies $\iota \leq 0$. Now, $(\bar{x}_h^0 + OP_h^E + \bar{x}_h^1 - OP_h^C)/2 - \bar{\bar{x}} = (OP_h^E - OP_h^C)/2 = \iota \leq 0$. It follows that $OP_h^E \leq OP_h^C$.

Appendix II Insurability and Expected Losses Reduction

Suppose that households ($h = 1, \cdots, H$) are likely to be damaged with the identical probability π. Every household is endowed with wealth e, and is deprived of L by disaster. Households pay the insurance premium μ in advance and is paid the insurance claims m if it is damaged. If perfect competition is assumed in the market, resulting in $\mu = \pi m$ where risk averse households will prefer fully-covered contracts. Hence the wealth position of undamaged state is determined as $x(0) = e - \pi L$, and that of damaged is $x(1) = (e - L) + L - \pi L = e - \pi L$. It implies that individual risk is completely covered by insurance. The maximized expected utility is given by $(1 - \pi)v_h(x(0)) + \pi v_h(x(1)) = v_h(x(0))$. Now the probability of disaster is mitigated from π to $\bar{\pi}$. Compensating valuation is represented by expected losses reduction such as $C = (\pi - \bar{\pi})L$. The identical proof is applied to the case of equivalent valuation.

References

[1] Johansson, P.-O.: 'Cost-Benefit Analysis of Environmental Change', *Cambridge University Press*, 1993.

[2] Ueda, T.:'On the Benefit of Natural Disaster Prevention Project from the Points of Uncertainty and Economic Disequlibrium', *Proceedings of Infrastructure Planning*,No.14, pp.17-34, 1997 (in Japanese).

[3] Tatano, H.: 'Project Evaluation under Uncertainty, Issues and Perspective', *Proceedings of Infrastructure Planning*, No.20(2), pp.19-30, 1997 (in Japanese).

[4] Takagi, Y., Morisugi, H. Ueda, T., et al. : 'The Benefit Evaluation of The Flood Control Works with Location Equilibrium Model', *Proceedings of Infrastructure Planning*, No.13, pp.339-348, 1996 (in Japanese).

[5] Lundberg, F.: 'Über die Theorie der Rück- versicherung', *Trans VI International Congress Actuaries*, Vol. I, pp. 877-955, 1909.

[6] Crameér, H.: 'Collective risk theory', *Skandia Jubilee Volume, Stockholm.*

[7] Borch, K.: 'Economics of Insurance', *North-Holland*, 1990.

[8] Arrow, K.J.: 'Le role des valeurs boursieres pour la repartition la meilleure des risques', *Econometrie*, 41-47, English translation as 'The role of securities in the optimal allocation of risk-bearing', *Review of Economic Studies*, Vol. 31, pp. 91-96, 1964.

[9] Borch, K.: 'Equilibrium in a reinsurance market', *Econometrica*, Vol. 30, pp. 424-444, 1962.

[10] Cass, D., Chichilinsky, G., and Wu, H.-M.: 'Individual risk and mutual insurance', *Econometrica*, Vol.64, pp. 333-341, 1996.

[11] Johansson P.-O. and Löfgren , K.-G.: 'Wealth from optimal health', *Journal of Health Economics*, Vol. 14, pp.65-79, 1995.

[12] Froot, K. A. (ed.): 'The Financing of Catastrophe Risk', *The University of Chicago Press*, 1999.

[13] Malinvaud, E.: 'The allocation of individual risks in large markets', *Journal of Economic Theory*, Vol. 4, pp. 312-328, 1972.

[14] Malinvaud, E.: 'Markets for an exchange economy with individual risks', *Econometrica*, Vol. 41, pp.383-410, 1973.

[15] de Finettti, B.: ‘Forsight: its logical laws, its subjective sources’, in: Kyburg, Jr. H. E. and Smokler, H. E. (eds.), ‘Studies in Subjective probability’, *Wiley*, 1964.

[16] Arrow, K.J. and Lind, R. C.: ‘Uncertainty and the evaluations of public investments’, *American Economic Review*, Vol. 53, pp. 941-973, 1970.

[17] Starret, D. A.: ‘Foundations of Public Economics’, *Cambridge University Press*, pp. 233-245, 1988.

[18] Geanakopolos, J.: ‘An introduction to general equilibrium with incomplete asset markets’, *Journal of Mathematical Economics*, Vol. 19, pp. 1-38, 1990.