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A Descriptive Model of Decision Analysis for Mitigating Earthquake Risks

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Abstract:This paper aims at developing a methodology of decision analysis for mitigating big earthquake risks arising with low probability for which the expected utility theory of von Neumann and Morgenstern is inadequate. We propose an alternative approach of decision analysis for such problems by using a value function under risk. First, we describe the statement of the decision problem of a person who will try to choose an alternative from six alternatives of remodeling his/her building to minimize earthquake risk. We set up hypothetical alternatives to improve buildings and to take or not to take earthquake insurance, some scenarios of earthquake, cost for improving buildings and cost for taking earthquake insurance, probability of death and injury, probability of fire and cost of restoring building's damage for each scenario. Then, it is shown that the value function under risk is a useful model for evaluating public risks of extreme events like big earthquakes that influence numerous people enormously or environment badly with low probability.

Keywords: Decision analysis, modeling, risk analysis, utility theory, low probability high consequence events, planning earthquake safety

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1. Introduction

Recently, we came across big earthquakes in various places in the world. Now, many people are doing reseach on predicting earthquakes, but there are many difficulties. However, if people would prepare for an earthquake and strengthen their buildings, it is possible to decrease the costs of reparing buildings' damage and the number of injured and dead people when we come across a big earthquake. If people would take an earthquake insurance, it is possible to decrease the damage costs.

In this paper we deal with a systems methodology of decision analysis for mitigating natural disaster risks of high consequence events like big earthquake arising with low probability. The expected utility theory of von Neumann and Morgenstern [10] has widely been used for normative decision analysis, however this is not an adequate approach of decision analysis for low probability high consequence events [4],[7]. Besides the expected utility theory and its extensions there exist Bayes model [1], maximum entropy model [5], extreme value theory [3] and so on. But it is said that they are not appropriate models for decision analysis [2]. We propose an alternative approach of modeling decision analysis for such problems by using a value function under risk [8],[9].

The value function under risk is a suitable approach of modeling behavioral legitimacy of decision making. Furthermore, this model includes von Neumann and Morgenstern expected utility model as a special case. In this case probability is regarded as an attribute of evaluation. In this paper, we show that value function under risk is an appropriate approach to model and analyze decision making problem with low probability high cosequence events like earthquakes.

In Section 2, we assume alternatives of remodeling building, costs of remodeling and taking an earthquake insurance, scenarios of earthquakes, damage costs, percentage of death and injury and percentage of fire. We then describe a decision making problem. In Section 3, we describe two mathematical models. One is an expected utility model, and the other is a model described by value function under risk. In Section 4, we show numerical results for these two models. In Section 5, we summarize the results and describe further problems.

2. Statement of a decision making problem

As a realistic hypothetical setting we show a model of decision analysis of a decision maker who will try to choose an alternative from six alternatives of remodeling his building to minimize natural disaster risk arising from big earthquake with low probability[6] and taking an earthquake insurance or not.

In this decision making problem, we assume that a decision maker live in Osaka at a wooden house which is valued at 20 million yen today, and that the house is insured against fire for 80% of the current price. Then, which alternatives the decision maker would choose? If he would insure the earthquake insurance, it costs 22400 yen per year. Since we consider this problem for 10 years span, the total premium of the earthquake insurance for 10 years is 224000 yen. We do not dare to take into account fluctuations in the current price of the house.

We assume that if the decision maker would rebuild the building with new earthquake-proof design, it costs 20 million yen. If he would repair the building in two or three-dimensinal viewpoint and so on, it costs 8 million yen (= 20 million yen × 40%). Even if he would not remodel the building, it costs 2 million yen for maintenance (= 20 million yen × 10%). It is assumed that in the cost of 20 million yen and 8 million yen the cost for maintenance is included.

2.1 Alternatives and remodeling costs

Six altrnatives are considered as follows:

Alternative 1. Rebuild the building with new earthquake-proof design, and take the earthquake insurance.

Alternative 2. Repair the building in two or three-dimensinal viewpoint, perform diagonal reinforcement, exchange heavy roofing tiles by light ones, and so on, and take the earthquake insurance.

Alternative 3. Leave as it is, but take the earthquake insurance.

Alternative 4. Rebuild the building with new earthquake-proof design, but do not take the earthquake insurance.

Alternative 5. Repair the building in two or three-dimensinal viewpoint, perform diagonal reinforcement, exchange heavy roofing tiles by light ones, and so on, but do not take the earthquake insurance.

Alternative 6. Leave as it is, and do not take the earthquake insurance.

We assume the cost for each alternative as follows:

Alternative 1. 20 million and 224 thousand yen

Alternative 2. 8 million and 224 thousand yen

Alternative 3. 2 million and 224 thousand yen

Alternative 4. 20 million yen

Alternative 5. 8 million yen

Alternative 6. 2 million yen

2.2 Scenarios of earthquakes

We assume three scenarios of earthquakes as follows:

Scenario 1. A tremor with an intensity of 7 on the Japanese seven-stage scale (An earthquake of magnitude 7 or 8 class) [1%]

Scenario 2. A tremor with an intensity of 7 on the Japanese seven-stage scale (An earthquake of magnitude 6 or 7 class) [10%]

Scenario 3. No damage cost even if an earthquake occurs [89%]

We consider that the damage of fire is serious when a big earthquake happens, so we also assume the next three scenarios taking into account probability of fire. If Scenario 1' happens, the probability of fire is 25%, and if Scenario 2' happens, it is 5%.

Scenario 1'. A tremor with an intensity of 7 on the Japanese seven-stage scale (An earthquake of magnitude 7 or 8 class) [1%] Probability of fire is 25%

Scenario 2'. A tremor with an intensity of 7 on the Japanese seven-stage scale (An earthquake of magnitude 6 or 7 class) [10%] Probability of fire is 5%

Scenario 3'. No damage cost even if an earthquake occurs [89%]

Each percentage denotes probability of coming across each scenario during the next ten years. It is said that we come across an earthquake of magnitude 7 or 8 class once in thousand years. This is the reason why we set up 1% probability for Scenario 1 to come across such a big earthquake during the next ten years.

2.3 Percentage of death, injury and collapse and damage cost

We assume percentage of death and injury, percentage of collapse of buildings, degree of collapse and damage cost as shown in Table 1.

Table 1: Percentage and damage cost in each scenario

Alternative	Scenario	Percentage of death and injury (%)	Percentage of collapse (%)	Degree of collapse	Damage cost (million yen)
1, 4	1, 1'	2	2	Partially	2
	2, 2'	1	1	Partially	1
	3, 3'	0	0	Nothing	0
2, 5	1, 1'	10	40	Half	12
	2, 2'	5	20	Half	8
	3, 3'	0	0	Nothing	0
3, 6	1, 1'	40	80	Wholly	20
	2, 2'	20	60	Half	10
	3, 3'	0	0	Nothing	0

Obviously, spending more money for building will result less damage. Problem is how much money do people want to spend to decrease the probability of getting disasters when they come across a big earthquake with low probability.

If the decision maker chooses altenative 1, 2 or 3, then he takes the earthquake insurance. So when a big earthquake happens, he gets benefit. If he suffers partially damage of the building, he gets 4 hundred thousand yen, and if he suffers half damage, he gets 4 million yen, and if he suffers wholly damage, he gets 8 million yen.

Of course, if he chooses alternative 4, 5 or 6 and when a big earthquake happens, he cannot get benefit.

2.4 Percentage and degree of fire and damage cost

We assume percentage of getting fire, degree of being burnt and the damage cost as shown in Table 2.

Table 2: Getting fire and the damage cost in each scenario

Alternative	Scenario	Percentage of getting fire (%)	Degree of being burnt	Damage cost (million yen)
1, 4	1'	25	Wholly	20
	2'	5	Half	10
	3'	0	Nothing	0
2, 5	1'	25	Wholly	20
	2'	5	Half	10
	3'	0	Nothing	0
3, 6	1'	25	Wholly	20
	2'	5	Half	10
	3'	0	Nothing	0

If the decision maker chooses altenative 1, 2 or 3, then he takes the earthquake insurance. So when a fire breake out and the building is burnt, he gets benefit. If he suffers half burnt, he gets 4 million yen, and if he suffers wholly burnt, he gets 8 million yen.

Of course, if he chooses alternative 4, 5 or 6 and when a fire break out, he cannot get benefit.

2.5 Attribute

We consider following three attributes.

Attribute 1. Necessary cost x_1

Attribute 2. Death and injury x_2 $(=\ 0$ or $1)$

Attribute 3. Loss cost x_3

Necessary cost is the sum of remodelig cost and premium of the earthquake insurance. Death and injury indicate whether people receive serious injury or not and that they are dead. Loss cost is the balance of damege cost and the insurance.

$$x_1 = \begin{cases} 20 & \text{million and 224 thousand yen, if alternative 1 is chosen} \\ 8 & \text{million and 224 thousand yen, if alternative 2 is chosen} \\ 2 & \text{million and 224 thousand yen, if alternative 3 is chosen} \\ 20 & \text{million yen, if alternative 4 is chosen} \\ 8 & \text{million yen, if alternative 5 is chosen} \\ 2 & \text{million yen, if alternative 6 is chosen} \end{cases}$$

$$x_2 = \begin{cases} 1, & \text{for death and injury} \\ 0, & \text{otherwise} \end{cases}$$

Loss cost x_3 takes various values. If we chose alternative 1 and spent much money to rebuild the building and took the earthquake insurance, we would have little damage cost even if we come across a big earthquake. If we chose alternative 2 or 3, and if we came across a big earthquake, we would loose much money for the damage of building. But, for alternative 1, 2 or 3 we can get benefit from insurance. If we chose altenative 4, 5 or 6, and if we came across a big earthquake, we would loose much money.

3. Mathematical modeling

3.1 Decision tree

Altenatives which are evaluated by the expected utility theory are described by lottery. The same is true for value function under risk. Figure 1 shows a decison tree which represents the evaluation of six alternatives where a square node represents decision node and a circular node represents chance node. In Fig. 1, A_i denotes a decision tree we come across when alternative a_i is chosen. It is different for scenarios 1 $\sim$ 3 and for scenarios 1' $\sim$ 3', so these decision trees of alternative 1 are shown separately. Figure 2 shows the decision tree of scenarios 1 $\sim$ 3, and Fig. 3 shows the decision tree of scenarios 1' $\sim$ 3', where p_j denotes the probability of scenario j, (x_2, x_3) means that x_2 and x_3 arise simultaneously with probability p_{ijk}. For example, this shows that we come across death and injury and damage of 1.6 million yen with probability p_{114}.

The value of p_{ijk} of scenarios 1 $\sim$ 3 can be calculated from Table 1 and is obtained as shown in Table 3. The value of p_{ijk} of scenarios 1' $\sim$ 3' can be calculated from Table 1 and 2 and is obtained as shown in Table 4.

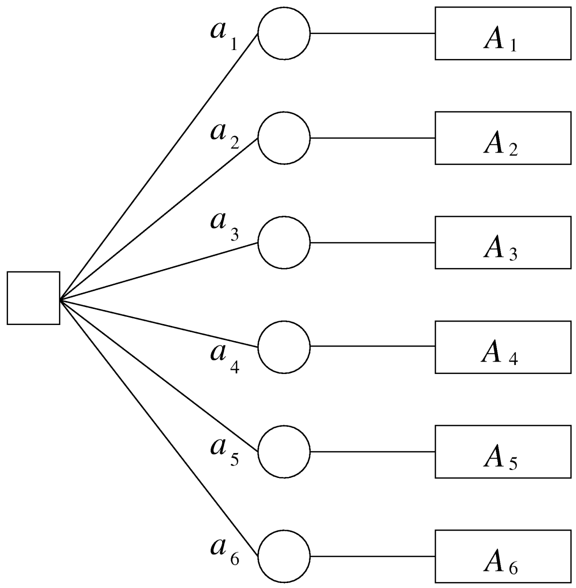

Fig. 1: Decision Tree

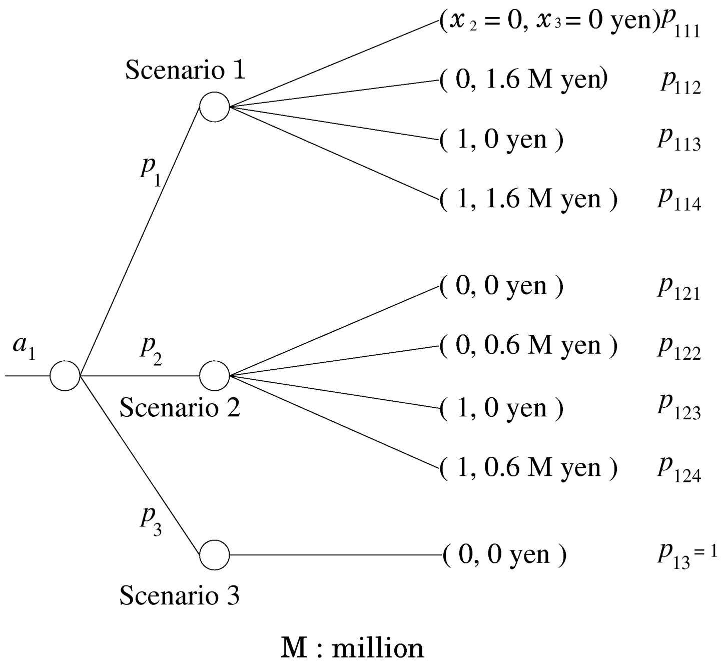

Fig. 2: Decision Tree of Alternative 1 for Scenarios 1 ∼ 3

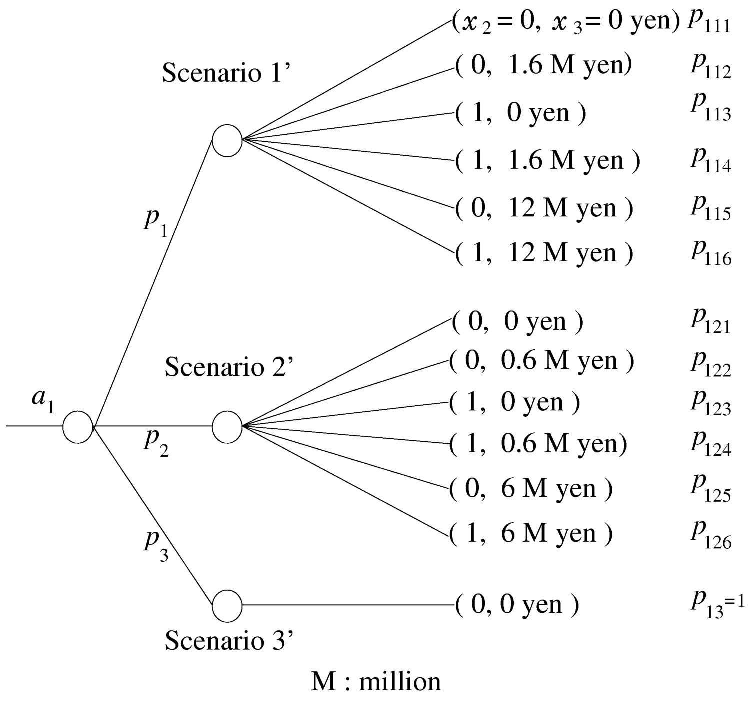

Fig. 3: Decision Tree of Alternative 1 for Scenarios 1' ∼ 3'

Table 3: The value of p_{ijk} for scenarios $1 \sim 3$

Alternative	Scenario	p_{ijk}	Probability(%)
1, 4	1	p_{111}	96.04
		p_{112}	1.96
		p_{113}	1.96
		p_{114}	0.04
	2	p_{121}	98.01
		p_{122}	0.99
		p_{123}	0.99
		p_{124}	0.01
	3	p_{13}	0.0
2, 5	1	p_{211}	54.0
		p_{212}	36.0
		p_{213}	6.0
		p_{214}	4.0
	2	p_{221}	76.0
		p_{222}	19.0
		p_{223}	4.0
		p_{224}	1.0
	3	p_{23}	0.0
3, 6	1	p_{311}	12.0
		p_{312}	48.0
		p_{313}	8.0
		p_{314}	32.0
	2	p_{321}	32.0
		p_{322}	48.0
		p_{323}	8.0
		p_{324}	12.0
	3	p_{33}	0.0

Table 4: The value of p_{ijk} for scenarios 1' $\sim$ 3'

Alternative	Scenario	p_{ijk}	Probability(%)
1 , 4	1	p_{111}	72.03
		p_{112}	1.47
		p_{113}	1.47
		p_{114}	0.03
		p_{115}	24.5
		p_{116}	0.5
	2	p_{121}	93.1095
		p_{122}	0.9025
		p_{123}	0.9025
		p_{124}	0.0095
		p_{125}	4.95
		p_{126}	0.05
	3	p_{13}	100.0
2 , 5	1	p_{211}	40.5
		p_{212}	27.0
		p_{213}	4.5
		p_{214}	43.9
		p_{215}	22.5
		p_{216}	2.5
	2	p_{221}	72.2
		p_{222}	18.05
		p_{223}	3.8
		p_{224}	0.95
		p_{225}	4.75
		p_{226}	0.25
	3	p_{23}	100.0
3 , 6	1	p_{311}	9.0
		p_{312}	36.0
		p_{313}	6.0
		p_{314}	24.0
		p_{315}	15.0
		p_{316}	10.0
	2	p_{321}	30.4
		p_{322}	45.6
		p_{323}	7.6
		p_{324}	11.4
		p_{325}	4.0
		p_{326}	1.0
	3	p_{33}	100.0

3.2 Mathematical model based on the expected utility theory

When we chose an alternative, the value of attribute x_1, that is necessary cost would be determined. Therefore, we deal with two attributes x_2 and x_3. Utility function $u_2(x_2)$ of death and injury is given by eqn.(1), where attribute x_2 takes 0 (no death or no injury) or 1 (getting death or heavy injury).

$$u_2(x_2) = \begin{cases} 0.0 & (x_2 = 0) \\ 1.0 & (x_2 = 1) \end{cases} \tag{1}$$

Utility function $u_3(x_3)$ of damage cost is represented by a cost function. When utility function $u(x_1)$ for remodeling cost x_1 is decided, evaluation of alternatives a_i is given by

$$U(a_i) = k_1 u(x_1) + k_2 \sum_{j=1}^{3} p_j u(a_i, e_j,), \quad i = 1, 2, 3 \tag{2}$$

where p_j denotes the probability of occurring of each scenario, that is occurrence probability of each earthquake, a_i denotes each alternative and e_j denotes each scenario, and

$$u(a_i, e_j) = \sum_{k=1}^{l} p_{ijk} U_{2_{ijk}}, \qquad i = 1, 2, 3, \quad j = 1, 2, 3 \tag{3}$$

$$l = \begin{cases} 4 & (\text{Scenario } 1 \sim 3) \\ 6 & (\text{Scenario } 1' \sim 3') \end{cases}$$

We denote U_2 at each end of the decision tree by $U_{2_{ijk}}$. Equation (3) shows expected utility of the chance node with respect to attributes 2 and 3. For simplicity, we assume that two-attribute utility function U_2 is given by

$$U_2 = k_3 u_2(x_2) + k_4 u_3(x_3), \tag{4}$$

where k_3 and k_4 denote scaling constants for death and injury and damage cost, respectively.

3.3 Mathemathical model based on the value function under risk

The value function to evaluate alternative a_i is written as

$$V(a_i) = \alpha_1 u(x_1) + \alpha_2 \sum_{j=1}^{3} p_j f_{ij}(x_2, x_3), \qquad i = 1, 2, 3 \tag{5}$$

where p_j denotes occurrence probability of each earthquake, and

$$f_{ij}(x_2, x_3) = \sum_{k=1}^{l} f(x_2, x_3, p_{ijk}), \qquad i = 1, 2, 3, \quad j = 1, 2, 3 \tag{6}$$

$$l = \begin{cases} 4 & \text{(Scenarios 1} \sim 3\text{)} \\ 6 & \text{(Scenarios 1'} \sim 3'\text{)} \end{cases}$$

Now we assume mutually weak risk difference independence between x_2 and x_3 [8]. This means that normalized conditional value function $v_2(x_2|x_3, p_{ijk})$ does not depend on x_3 and $v_3(x_3|x_2, p_{ijk})$ does not depend on x_2, where $v_2(x_2|x_3, p_{ijk})$ denotes a value function for death and injury x_2 under given damage cost x_3 and occurrence probability p_{ijk}, and $v_3(x_3|x_2, p_{ijk})$ denotes a value function for damage cost x_3 under given death and injury x_2 and occurrence probability p_{ijk}. Then, the value function under risk $f(x_2, x_3, p_{ijk})$ is given by eqn.(7) where p_{ijk} denotes probability of getting (x_2, x_3) at each end of the decision tree shown in Fig. 2 or 3.

$$\begin{aligned} f(x_2, x_3, p_{ijk}) &= f(x_2, x_3^R, p_{ijk}) + f(x_2^R, x_3, p_{ijk}) \\ &\quad + K(p_{ijk}) f(x_2, x_3^R, p_{ijk}) f(x_2^R, x_3, p_{ijk}), \end{aligned} \tag{7}$$

where $K(p_{ijk})$ is defined as a function of p_{ijk}. Function f which denotes disutility, is defined based on some reference points (RP) x_2^R and x_3^R on X_2 and X_3, respectively, where we regard $x_2^R = 0$, $x_3^R = 0$. Then we get

$$f \geqq 0, \quad x_2 \geqq 0, \quad x_3 \geqq 0$$

In this example, x_2 takes 0 or 1 indicating whether people suffer from death and injury or not. So, (x_2, p_{ijk}) is considered only when $x_2 = 1$ and $p_{ijk} \leq 1$. When $x_2 = 0$, we assume $f(x_2, x_3^R, p_{ijk}) = 0$.

Then, $f(x_2, x_3^R, p_{ijk})$ can be written as

$$f(x_2, x_3^R, p_{ijk}) = \alpha_3 v_2(x_2|x_3^R, 1) w(p_{ijk}|1, x_3^R), \tag{8}$$

where

$$\alpha_3 = f(1, x_3^R, 1), \tag{9}$$

$$v_2(x_2|x_3^R, 1) = \begin{cases} 0.0 & (x_2 = 0) \\ 1.0 & (x_2 = 1) \end{cases} \tag{10}$$

$$w(p_{ijk}|1, x_3^R) = f(1, x_3^R, p_{ijk})/f(1, x_3^R, 1), \tag{11}$$

and α_3 denotes the greatest impact of $f(x_2, x_3^R, p_{ijk})$. When the impact for the damage cost is 0 ($x_3^R = 0$), we obtain $w(p_{ijk}|1, x_3^R) = w_2(p_{ijk})$, and $w_2(p_{ijk})$ gives normalized value with respect to the situation of suffuring from death and injury.

We need to decide $f(x_2^R, x_3, p_{ijk})$ in the range of $0 \leq x_3 \leq 20$ million yen. Then, $f(x_2^R, x_3, p_{ijk})$ is obtained as

$$f(x_2^R, x_3, p_{ijk}) = \alpha_4 w_3(p_{ijk}) v_3(x_3|p_{ijk}) \tag{12}$$

where

$$\alpha_4 = f(x_2^R, x_3^*, 1.0), \tag{13}$$

$$w_3(p_{ijk}) = f(x_2^R, x_3^*, p_{ijk})/f(x_2^R, x_3^*, 1.0), \tag{14}$$

$$v_3(x_3|p_{ijk}) = f(x_2^R, x_3, p_{ijk})/f(x_2^R, x_3^*, p_{ijk}), \tag{15}$$

and x_3^* denotes the highest damage cost.

Assume that $v_3(x_3|p_{ijk})$ does not depend on p_{ijk}, which is denoted as $v_3(x_3|p_{ijk}) = v_3(x_3)$, and α_4 denotes the greatest value of $f(x_2^R, x_3, p_{ijk})$. When there is no death and injury ($x_2 = 0$) $w(p_{ijk}|0, x_3) = w_3(p_{ijk})$. Equation (12) is then written as

$$f(x_2^R, x_3, p_{ijk}) = \alpha_4 w_3(p_{ijk}) v_3(x_3) \tag{16}$$

This shows the same form as in the expected utility theory.

Equation (7) is then rewritten as

$$f(x_2, x_3, p_{ijk}) = \alpha_3 w_2(p_{ijk}) + \alpha_4 w_3(p_{ijk}) v_3(x_3)$$
$$+K(p_{ijk})\alpha_3\alpha_4 w_2(p_{ijk}) w_3(p_{ijk}) v_3(x_3) \quad (17)$$
$$(x_2 = 1)$$

$$f(x_2, x_3, p_{ijk}) = \alpha_4 w_3(p_{ijk}) v_3(x_3) \quad (x_2 = 0) \quad (18)$$

Therefore, evaluation of death and injury and damage costs is obtained as

$$\begin{aligned} f(x_2, x_3, p_{ijk}) &= x_2\{\alpha_3 w_2(p_{ijk}) + \alpha_4 w_3(p_{ijk}) v_3(x_3) \\ &+ K(p_{ijk})\alpha_3\alpha_4 w_2(p_{ijk}) w_3(p_{ijk}) v_3(x_3)\} \\ &+ (1 - x_2)\alpha_4 w_3(p_{ijk}) v_3(x_3) \end{aligned} \quad (19)$$

We evaluate eqn. (19) for branches of the end nodes of the decision tree in Fig. 1, and use it to compute eqn. (6).

We consider $K(p_{ijk}) = p_{ijk}$ and disregard the 3rd term in eqn.(19) because the 3rd term is very small compared with the 1st and 2nd term.

Table 5 shows three equations assumed as utility function (disutility) of necessary cost and as weighting function on probability of getting each disaster when he gets death and injury (x_2=1) and when he gets damage cost. We use the same utility function of necessary cost for damage cost. Figures 4, 5 and 6 show the shape of these three functions.

Table 5: Utility function and weighting function

$u(x)$	$\sqrt{-x^2+2x}$
	$x = x_1/20,224,000 \quad (0 \le x_1 \le 20,224,000)$
	$x = x_3/20,000,000 \quad (0 \le x_3 \le 20,000,000)$
$w_2(p_{ijk})$	$\log_{10}(999999p_{ijk}+1)/6$
	$(0 \le p_{ijk} \le 1)$
$w_3(p_{ijk})$	$\log_{10}(9999p_{ijk}+1)/4$
	$(0 \le p_{ijk} \le 1)$

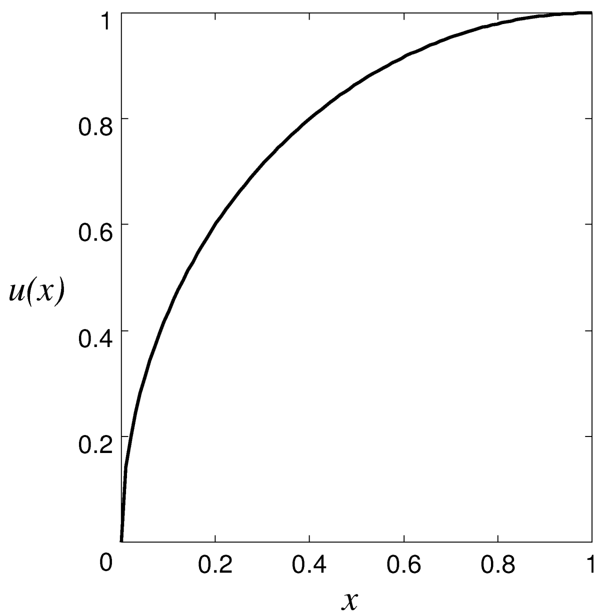

Fig. 4: Utility function on the cost

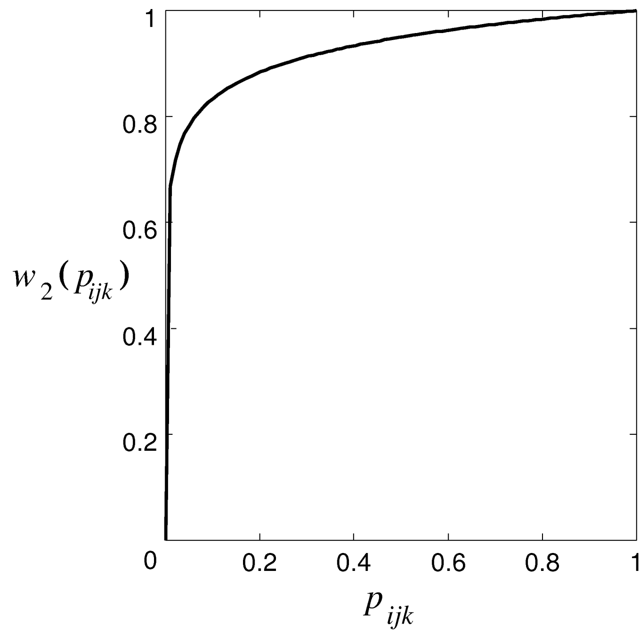

Fig. 5: Weighting function on p_{ijk} to get death and injury, that is, when $x_2 = 1$

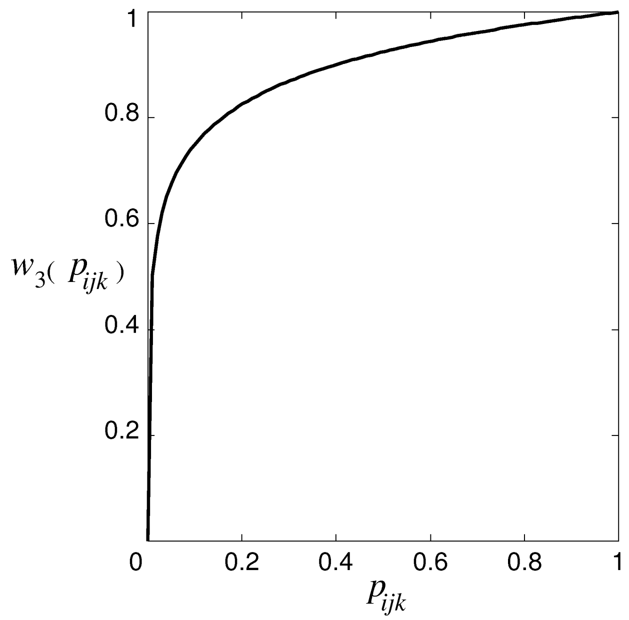

Fig. 6: Weighting function on p_{ijk} to get damage when $x_2 = 0$

4. Numerical results

In Tables 6 and 7, we show numerical results obtained from mathematical models of the expected utility theory and the value function under risk. The evaluated value of each altenative represents disutility.

We assumed the value of each scaling constant taking into account the people's life and remodeling cost. When k_1 or α_1 is small, that is, the scaling constant of remodeling cost is small, it is legitimate that people want to spend much money to get more safety for avoiding catastrophic risk arising from earthquake. This means that for small k_1 or α_1, alternative 1 should be preferred to alternatives 2 and 3.

In Table 6 by using the expected utility theory, when k_1 is small, namely the scaling constant of remodeling cost is small, alternative 1 is not preferred to alternatives 2 and 3. On the other hand, in Table 7 by the value function under risk, when α_1 is small, alternative 1 is preferred to other two alternatives. This means that the value function under risk is an adequate model to evaluate catastrophic risk of natural disaster such as earthquake.

Table 6: Evaluated value of alternatives by the expected utility theory

Scaling constant				$U(a_1)$	$U(a_2)$	$U(a_3)$
k_1	k_2	k_3	k_4			
0.1	0.9	0.05	0.95	0.105	0.098	**0.091**
		0.1	0.9	0.105	0.097	**0.090**
		0.2	0.8	0.105	0.096	**0.087**
0.2	0.8	0.05	0.95	0.205	0.176	**0.132**
		0.1	0.9	0.205	0.176	**0.131**
		0.2	0.8	0.204	0.175	**0.128**
0.5	0.5	0.05	0.95	0.503	0.412	**0.253**
		0.1	0.9	0.503	0.412	**0.253**
		0.2	0.8	0.503	0.411	**0.251**

Table 7: Evaluated value of alternatives by the value function under risk

Scaling constant				$V(a_1)$	$V(a_2)$	$V(a_3)$
α_1	α_2	α_3	α_4			
0.1	0.9	0.05	0.95	**0.181**	0.241	0.256
		0.1	0.9	**0.185**	0.243	0.257
		0.2	0.8	**0.191**	0.247	0.259
0.2	0.8	0.05	0.95	**0.272**	0.304	0.278
		0.1	0.9	**0.275**	0.306	0.279
		0.2	0.8	**0.281**	0.309	**0.281**
0.5	0.5	0.05	0.95	0.545	0.492	**0.345**
		0.1	0.9	0.547	0.493	**0.345**
		0.2	0.8	0.551	0.495	**0.347**

In Tables 8 and 9, we show numerical results obtained from mathematical models of either taking earthquake insurance or not . Table 9 obtained by the value function under risk shows altenative 1 is preferred to alternative 4 at each scaling constant. It is because people consider the damage by fire and the reduction of loss cost by taking the earthquake insurance. But Table 8 obtained by the expected utility function does not shows such preference difference.

Table 8: Comparison of taking earthquake insurance or not by the expected utility theory

Scaling constant				$U(a_1)$	$U(a_4)$
k_1	k_2	k_3	k_4		
0.1	0.9	0.05	0.95	0.105	0.105
		0.1	0.9	0.105	0.105
		0.2	0.8	0.105	0.105
0.2	0.8	0.05	0.95	0.205	0.205
		0.1	0.9	0.205	0.205
		0.2	0.8	0.204	0.204
0.5	0.5	0.05	0.95	0.503	0.503
		0.1	0.9	0.503	0.503
		0.2	0.8	0.503	0.503

Table 9: Comparison of taking earthquake insurance or not by the value function under risk

Scaling constant				$V(a_1)$	$V(a_4)$
α_1	α_2	α_3	α_4		
0.1	0.9	0.05	0.95	0.181	0.196
		0.1	0.9	0.185	0.199
		0.2	0.8	0.191	0.204
0.2	0.8	0.05	0.95	0.272	0.286
		0.1	0.9	0.275	0.288
		0.2	0.8	0.281	0.292
0.5	0.5	0.05	0.95	0.545	0.553
		0.1	0.9	0.547	0.555
		0.2	0.8	0.550	0.558

5. Conclusion

In this paper, we proposed a mathematical model of evaluating alternatives in decision making problem for low probability high consequence events like earthquake. We compared two models of the expected utility theory and the value function under risk. It is clarified that the expected utility theory which has widely been used for multiattribute decision analysis under risk is not adequate for risk analysis with small probability of 10^{-n} order, where $n > 2$.

By using the value function under risk, in which probability is regarded as an attribute of evaluation, we tried to evaluate risk analysis problem with low probability high consequence events.

We found that the value function under risk is useful for evaluating public risks which influence unspecific numerous people or environment badly with low probability as follws:

1. It could represent impact of risk which the expected utility model cannot represent.

2. In the value function under risk, probability p is regarded as an attribute, so it could represent preference relation of people which may change variously depending upon the attributes or level of attributes.

3. It is adequate to evaluate the multiobjective decision making problem because the value function under risk could deal with multiattribute problems.

In this paper, utility function and weighting function are not based on real data. We need further study on the basis of actual survey like questionnaires. We considered death and injury as one attribute. But, since utilities of the two may be different, we need to build decision making model more precisely. Since the damage of buildings is closely related with the ground or the distance from the earthquake center, we need a further study taking these factors into consideration.

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