MODELING EQUITY IN NATURAL DISASTER RISKS UNDER MULTIPLE MISFORTUNE LEVELS

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Abstract: In this paper we develop a model of decision analysis to mitigate natural disaster risks such as floods, typhoon, big earthquakes and so forth, taking into account the notion of equity among individuals in the society. In evaluating such natural disaster risks we deal with three issues:

1. The undesirability of misfortunes (damages) caused by natural disaster, aside from equity considerations.
2. Ex-post equity, meaning the equity associated with the misfortunes that are actually met by the individuals.
3. Ex-ante equity, meaning the equity of the risks which eventually lead to the misfortunes.

Specific forms of utility functions for evaluating the amount of misfortunes, ex-post equity and ex-ante equity are proposed. A multiattribute utility function, which is assumed to be additive for these three attribute, is constructed. The implications of the concept proposed in this paper are clarified using a simple numerical example with 2-person society and 2 levels of misfortunes. A hypothetical numerical example is included to evaluate two alternative plans of strengthening the social infrastructure to mitigate natural disaster risks taking into account the equity notions among many residential areas along a river basin.

Keywords: Catastrophe risk, natural disaster risk, misfortune profile, ex-post equity, ex-ante equity, risk assessment, risk management.

1. Introduction

An equitable distribution of risk is an important criterion besides minimizing the total amount of misfortunes (damage) in catastrophe risk management. Keeney and Winkler (1985), Sarin (1985), and Fishburn and Straffin (1989) discussed equitable distribution of risk, besides decreasing fatalities, in evaluating public risks. They have developed von Neunmann-Morgenstern utility functions for evaluating decision strategies. They described fatality profiles in which each individual would be either living or dying at the end of the time period of concern. They represented risk as risk profiles that give the probability of dying each individual due to the cause of concern.

This paper deals with the use of a model to improve catastrophe risk management. In modeling risk profile, we propose misfortune profile instead of fatality profile that describes not only living or dying but also multiple levels of misfortunes. For modeling natural disaster risks this generalization is more appropriate, since natural disaster risks induce various levels of misfortunes (damages) to each individual. Under this generalized problem setting we will develop a utility theoretic model to evaluate the amount of misfortunes, ex-post equity which means the equity associated with the misfortunes that actually be met, and ex-ante equity which means the equity of the possibility of risks eventually leading to the misfortunes.

2. A Model for Evaluating Natural Disaster Risks

Suppose that we evaluate the natural disaster risks in an N-person society. Let M be the number levels of misfortune. **Figure 1** shows a lottery model for evaluating risks where event i (i=1,2,…,L) will occur with probability p_i, X_i denotes a misfortune matrix, and Q denotes an ex-ante risk matrix.

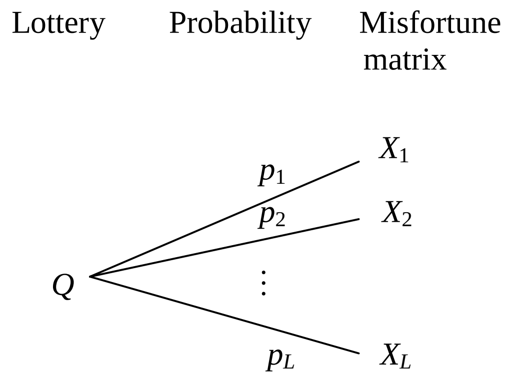

Figure 1. A lottery model for evaluating risks.

Misfortune matrix X_i is defined as

$$X_i = \begin{pmatrix} x_{i11} & x_{i12} \cdots & x_{i1N} \\ x_{i21} & x_{i22} \cdots & x_{i2N} \\ \vdots & \vdots & \vdots \\ x_{iM1} & x_{iM2} \cdots & x_{iMN} \end{pmatrix}, \qquad i = 1,2,\ldots,L \tag{1}$$

where x_{ijk}=1 or 0 (j=1,2,…,M, k=1,2,…,N) depending on whether the individual k will meet the misfortune of level j or not as a consequence of event i. In other words, X_i denotes a misfortune profile which shows the distribution of misfortunes when the event i occurs.

An ex-ante risk matrix Q which denotes an ex-ante profile, is defined as

$$Q = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1N} \\ q_{21} & q_{22} & \cdots & q_{21} \\ \vdots & \vdots & & \vdots \\ q_{M1} & q_{M2} & \cdots & q_{MN} \end{pmatrix} \tag{2}$$

where

$$q_{jk} = \sum_{i=1}^{L} p_i x_{ijk} \tag{3}$$

and q_{jk} denotes the marginal probability that the individual k will meet the misfortune of level j.

Let $\mathbf{y}_i$ be a misfortune vector which describes the distribution of number of individuals who will meet the various level of misfortunes as a consequence of event *i* as

$$\mathbf{y}_i = (y_{i1}, y_{i2}, \cdots, y_{iM}) \tag{4}$$

where y_{ij} denotes the number of individuals who will meet the misfortune of level j as a consequence of event *i*, and is obtained as

$$y_{ij} = \sum_{k=1}^{N} x_{ijk} \tag{5}$$

For clarifying the underlying idea of assessing natural disaster risks a simple example with 2 individuals and 2 misfortune levels (to meet or not to meet misfortune) is shown below. In this case the amount of misfortunes is described as the number of individuals who met the misfortune. From the point of view of ex-post equity, if both of the two individuals met the misfortune or if both of them did not meet the misfortune, the complete ex-post equity is obtained. From the point of view of ex-ante equity, if the marginal probability of meeting misfortune takes the same value for both individuals, the ex-ante equity is obtained.

As an example **Figure 2** shows three lotteries L1, L2 and L3. In L1 just the individual 1 will meet the misfortune with probability 0.5 and just the individual 2 will meet the misfortune with probability 0.5. There exists no chance that both individuals will meet the misfortune at the same time. In lottery L2 only the individual 1 will meet the misfortune for sure. For both lotteries, the misfortune vector is obtained as (1, 1), therefore, from the point of view of the amount of misfortune and the ex-post equity, L1 and L2 are indifferent. If we compare the ex-ante risk matrix Q_1 and Q_2, L1 is more equitable than L2 from the point of view of ex-ante equity.

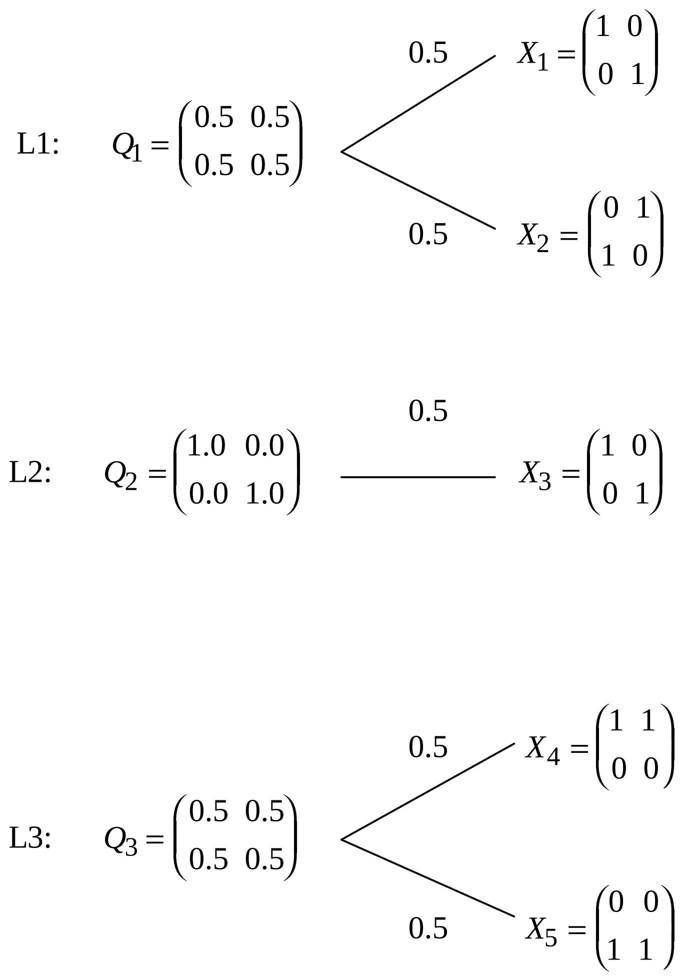

Figure 2. Three ex-ante risk matrices and three lotteries described in terms of misfortune matrices.

In L3, both individuals 1 and 2 will meet the misfortune with probability 0.5 and both of them will not meet the misfortune with probability 0.5. If we compare L1 and L3, ex-ante matrix Q_1 is equal to Q_3, therefore, from the point of view of ex-ante equity, L1 and L3 are indifferent. But since the misfortune vector is (1, 1) for L1 and (2, 0) or (0, 2) for L3, L3 is more equitable than L1 from the ex-post equity point of view.

As seen from this simple example, we need the concept of both ex-ante and ex-post equity besides the amount of misfortunes for evaluating natural disaster risks.

3. Disutility Functions for Evaluating Natural Disaster Risks

In this section we describe a specific form of disutility functions for evaluating the amount of misfortunes, ex-post equity and ex-ante equity where there exist multiple levels of misfortune.

3.1 Amount of Misfortunes

One specific form of a disutility function for evaluating the amount of misfortunes can be described as

$$U_y = \sum_{i=1}^{L} p_i u_{yi} \tag{6}$$

where

$$u_{yi} = -\sum_{j=1}^{M} w_{yj} \left(y_{ij} / N \right)^2 \tag{7}$$

and w_{yj} denotes the weighting coefficient for the undesirability of misfortunes of level j with respect to the amount of misfortune. These weighting coefficients satisfy

$$0 = w_{yM} < \ldots < w_{y2} < w_{y1} = 1.$$

Equation (7) shows a risk averse disutility function over the amount of misfortunes, and eqn.(6) shows its expected disutility.

For example, in the lottery L1 shown in Fig. 2, we know that

$$M = N = 2, \qquad w_{y1} = 1, \qquad w_{y2} = 0$$

$$y_1 = (1,1), \qquad y_2 = (1,1), \qquad p_1 = p_2 = 0.5$$

Therefore, we easily get the result of

$$u_{y1} = u_{y2} = -0.25$$

and, hence

$$U_y(\mathrm{L1}) = -0.25$$

Similarly, we obtain

$$U_y(\mathrm{L2}) = -0.25 \quad \text{and} \quad U_y(\mathrm{L3}) = -0.50.$$

This implies that for the amount of misfortune

$$L1 \sim L2 \succ L3$$

where ~ denotes "is indifferent to" and $\succ$ denotes "is preferred to."

3.2 Ex-post equity

A specific form for a utility function for evaluating the ex-post equity can be described as

$$U_p = \sum_{i=1}^{L} p_i u_{pi} \tag{8}$$

where

$$u_{pi} = -4 \sum_{j=1}^{M} n_{ij} y_{ij} \tag{9}$$

$$n_{ij} = \sum_{k>j} \left(w_{pj} - w_{pk} \right) y_{ik} / N^2 \tag{10}$$

and w_{pj} denotes the weighting coefficient for the undesirability of misfortune of level j with respect to the ex-post equity. These weighting coefficients satisfy

$$0 = w_{pM} < \ldots < w_{p2} < w_{p1} = 1.$$

An individual, who got misfortune of level j, feels inequity n_{ij} in eqn.(10) when the event i occurred, since the inequity felt by this individual depends on the number of individuals y_{ik} who met the better consequence and also on the difference of the undesirability of the misfortunes. Equation (9) shows a disutility function for evaluating the ex-post equity of the misfortune matrix X_i , and eqn.(8) shows its expected disutility. As discussed in Keeney (1980) the disutility function (9) gives a risk prone type function with respect to the number of individuals who will meet the misfortune of various levels.

In the lottery L1 shown in Figure 2, we know that

$$w_{p1} = 1, \qquad w_{p2} = 0$$

Therefore, we obtain

$$n_{11} = 0.25, \quad n_{12} = 0.00, \quad n_{21} = 0.25, \quad n_{22} = 0.00$$
$$u_{p1} = u_{p2} = -1.00$$

and hence

$$U_p(\text{L1}) = -1.00.$$

Similarly, we obtain

$$U_p(\text{L2}) = -1.00 \quad \text{and} \quad U_p(\text{L3}) = 0.00$$

This implies that for the ex-post equity

$$\text{L3} \succ \text{L1} \sim \text{L2}.$$

3.3 Ex-ante Equity

A specific form of a disutility function for evaluating the ex-ante equity can be described as

$$U_a = -4\sum_{k=1}^{N}\sum_{j=1}^{M}\left(w_{aj}q_{jk} - m\right)^2 / N \tag{11}$$

where

$$m = \sum_{k=1}^{N}\sum_{j=1}^{M} w_{aj}q_{jk} / N \tag{12}$$

and w_{aj} denotes the weighting coefficient for the undesirability of misfortune of level j with respect to the ex-ante equity. The weighting coefficients satisfy

$$0 = w_{aM} < \ldots < w_{a2} < w_{a1} = 1.$$

Equation (12) shows a weighted mean of the marginal probability distribution. The disutility for the ex-ante equity is measured as the variance of the marginal probability distribution as shown by eqn.(11).

In the lottery L1 shown in Figure 2, we know that

$$w_{a1} = 1, \quad w_{a2} = 0, \quad q_{ij} = 0.5 \quad (\text{i}, \text{j} = 1, 2)$$

therefore, we obtain

$$m = 0.50$$

and hence

$$U_a(\text{L1}) = 0.00$$

Similarly, we obtain

$$U_a(\text{L2}) = -1.00 \quad \text{and} \quad U_a(\text{L3}) = 0.00.$$

This implies that for the ex-ante equity

$$\text{L1} \sim \text{L3} \succ \text{L2}.$$

3.4 Multiattribute Disutility Function

Table 1 summarizes the value of three disutility functions U_y, U_p and U_a for three lotteries L1, L2 and L3. If we compare these three lotteries under the criterion of expected number of individuals who will meet the misfortune, all the three lotteries are indifferent. If we compare these three lotteries under the value of three disutility functions U_y, U_p and U_a , we obtain

$$\text{L1} \succ \text{L2}.$$

But we cannot compare L1 with L3 unless we get an aggregated disutility function for U_y, U_p and U_a .

Table 1. Value of utility functions.

	U_y	U_p	U_a
L1	−0.25	−1.00	0.00
L2	−0.25	−1.00	−1.00
L3	−0.50	0.00	0.00

Three disutility functions U_y, U_p and U_a can be aggregated as a three-attribute disutility function

$$U = k_y U_y + k_p U_p + k_a U_a, \tag{13}$$

if we postulate additive independence (Keeney and Raiffa, 1993) among three attribute, where k_y, k_p and k_a denote the scaling coefficients. If the additive independence or utility independence property fails among three attributes, we need to take into account more complex dependence properties (Tamura and Nakamura, 1983) when we construct a multiattribute utility function.

4. A Realistic Hypothetical Problem to Mitigate Natural Disaster Risks

In this section we set up a hypothetical numerical example to evaluate two alternative plans A1 and A2 of strengthening the social infrastructure to mitigate natural disaster (flood) risks taking into account the equity notions among multiple residential areas in a river basin shown in **Figure 3**. As shown in Figure 3 there are 20 residential areas along with this river basin.

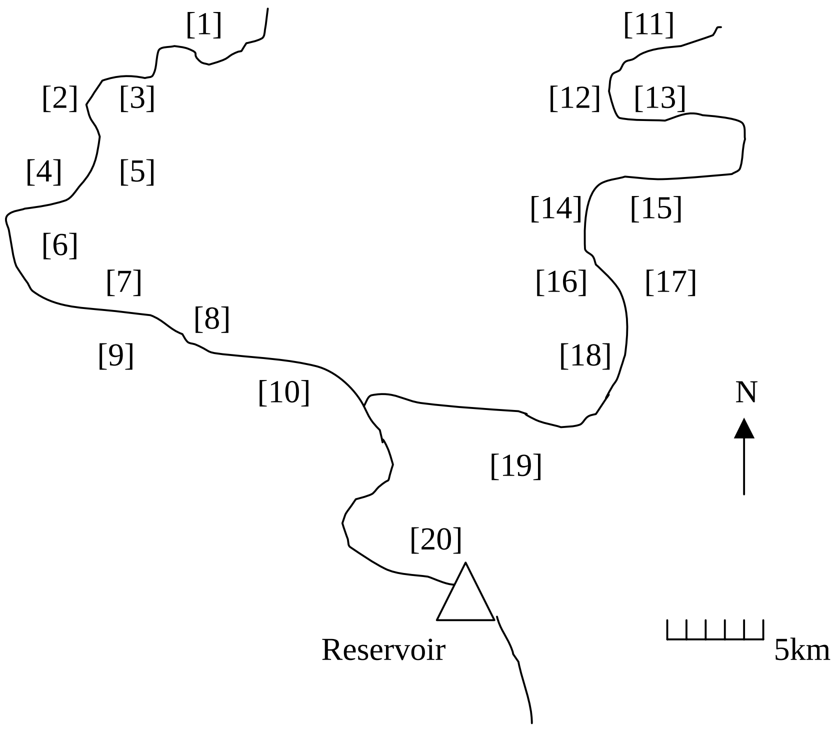

Figure 3. Hypothetical river basin and residential area.

Table 2. Distribution of population and misfortune level

Area No.	Misfortune level X_1	X_2	Population ($\times 1000$)
1	Medium	Low	207
2	Medium	Medium	29
3	High	Medium	8
4	Medium	Medium	281
5	High	High	1
6	Medium	High	1
7	High	Medium	3
8	High	High	1
9	Medium	High	2
10	Low	Medium	27
11	Low	High	1
12	Medium	Low	711
13	High	Low	200
14	High	Medium	81
15	High	Medium	5
16	High	Low	2
17	Medium	Low	14
18	Medium	Medium	6
19	Low	Medium	20
20	Medium	Medium	1

Table 2 shows the distribution of population in each residential area and two kinds of misfortune level that each residential area comes across. In this Table 2, "High", "Medium" and "Low" show the level of misfortune, that is, "High" implies that the

concerning residential area will get serious damage by the flood, and so forth. There are two alternative plans A1 and A2 of strengthening the social infrastructure to mitigate the natural disaster risk. **Table 3** shows the probability of getting two misfortune matrices X_1 and X_2 when the alternative plans A1 and A2 are adopted where, from Table 2, misfortune matrices are obtained as follows:

$$X_1 = \begin{pmatrix} 001010110000011110000 \\ 11010100100100001101 \\ 000000000110000000010 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} 000011011010000000000 \\ 01110010010001100111 \\ 10000000000110011000 \end{pmatrix}$$

Table 3. Probability of getting misfortune matrices X_1 and X_2.

	Alternatives	
	A1	A2
$p(X_1)$	0.664	0.277
$p(X_2)$	0.336	0.723

Table 4 shows the expected utility value for various weighting coefficients, where

$$w_{y1} = w_{p1} = w_{a1} = 1$$
$$w_{y2} = w_{p2} = w_{a2} = w_2$$
$$w_{y3} = w_{p3} = w_{a3} = 0$$

and w_2 is varied from 0 to 1. For larger w_2 the misfortune level "Medium" is seriously considered that this misfortune level is closer to the misfortune level "High". In Table 4 the better value of each disutility is underlined. When w_2=0.00 or 0.25, the alternative A2 is better than A1 with respect to all the attributes. But when w_2=0.50 or 0.75, there exists trade-off between the amount of misfortune and the equity.

Table 4. Utility values of three attributes for various weights.

Alternative	w_2	Utility value		
		U_y	U_p	U_a
A1	0.00	-0.024	-0.417	-0.281
	0.25	-0.134	-0.400	-0.171
	0.50	-0.244	-0.382	-0.104
	0.75	-0.354	-0.365	-0.082
	1.00	-0.464	-0.347	-0.103
A2	0.00	-0.010	-0.171	-0.051
	0.25	-0.065	-0.288	-0.060
	0.50	-0.120	-0.406	-0.125
	0.75	-0.175	-0.523	-0.247
	1.00	-0.231	-0.640	-0.425

Table 5 shows the value of multiattribute utility function (13) for various values of w_2 and various values of scaling coefficients k_y, k_p and k_a. When w_2=0.00, 0.25 or 0.50, the alternative A2 is better than A1, but when w_2=0.75 or 1.00, that is, the misfortune level "Medium" is seriously considered, the alternative A1 is better than A2 unless the scaling coefficient k_y for the misfortune level is very large.

If the value of w_2 and and k_y, k_p and k_a are given, we could choose the appropriate alternative A1 or A2. Roughly speaking, the alternative A1 is to be chosen, if the misfortune level "Medium" is considered to be as serious as the misfortune level "High", otherwise the alternative A2 is to be chosen in this hypothetical example.

Table 5. Utility values of two alternatives for various weighting coefficients and scaling constants.

w_2	k_y	k_p	k_a	A1	A2
	0.2	0.2	0.6	-0.257	-0.067
	0.2	0.4	0.4	-0.284	-0.091
	0.2	0.6	0.2	-0.311	-0.115
0.00	1/3	1/3	1/3	-0.240	-0.077
	0.4	0.2	0.4	-0.205	-0.058
	0.4	0.4	0.2	-0.233	-0.082
	0.6	0.2	0.2	-0.154	-0.050
	0.2	0.2	0.6	-0.209	-0.107
	0.2	0.4	0.4	-0.255	-0.152
	0.2	0.6	0.2	-0.301	-0.198
0.25	1/3	1/3	1/3	-0.235	-0.138
	0.4	0.2	0.4	-0.202	-0.107
	0.4	0.4	0.2	-0.248	-0.153
	0.6	0.2	0.2	-0.194	-0.108
	0.2	0.2	0.6	-0.188	-0.180
	0.2	0.4	0.4	-0.243	-0.236
	0.2	0.6	0.2	-0.299	-0.292
0.50	1/3	1/3	1/3	-0.243	-0.217
	0.4	0.2	0.4	-0.216	-0.179
	0.4	0.4	0.2	-0.271	-0.235
	0.6	0.2	0.2	-0.244	-0.178
	0.2	0.2	0.6	-0.193	-0.288
	0.2	0.4	0.4	-0.249	-0.343
	0.2	0.6	0.2	-0.306	-0.398
0.75	1/3	1/3	1/3	-0.267	-0.315
	0.4	0.2	0.4	-0.247	-0.274
	0.4	0.4	0.2	-0.304	-0.329
	0.6	0.2	0.2	-0.302	-0.259
	0.2	0.2	0.6	-0.224	-0.429
	0.2	0.4	0.4	-0.273	-0.472
	0.2	0.6	0.2	-0.322	-0.515
1.00	1/3	1/3	1/3	-0.305	-0.432
	0.4	0.2	0.4	-0.296	-0.390
	0.4	0.4	0.2	-0.345	-0.433
	0.6	0.2	0.2	-0.368	-0.352

5. Concluding Remarks

We have proposed a misfortune profile having various levels of misfortunes, and shown a utility theoretic model of evaluating the amount misfortune and the equity of natural disaster risks. The misfortune profile is described as a misfortune matrix in which the distribution of misfortunes of various levels among the individuals is described.

Ex-post equity is evaluated by the misfortunes that are actually met by the individuals. Ex-ante equity is evaluated by the ex-ante risk profile. The ex-ante risk profile is described as an ex-ante risk matrix in which each component of the matrix denotes the marginal probability that each individual will meet the misfortune of a specified level.

Using a simple numerical example the implications of the concepts proposed in this paper is clarified. Furthermore, how to apply the idea described in this paper to more realistic problems is shown by using a hypothetical numerical example of mitigating natural disaster risks for flood in a residential area along with a river basin. In this example two alternative plans of strengthening the social infrastructure is compared to mitigate natural disaster risks taking into account the equity notions among many residential areas for various values of a weighting coefficient and scaling constants. For further research we need to find a systematic way of finding these values that are suited to the situation undergoing.

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