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## Learning as combining imprecise evidence

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# Alternative models of beliefs suit alternative questions

1. Classical, precise probabilities: Detecting climate change trend
2. Imprecise probabilities: Inductive inference for rare events
3. Information fusion (Dempster-Shafer): social construction of belief from experts' opinions

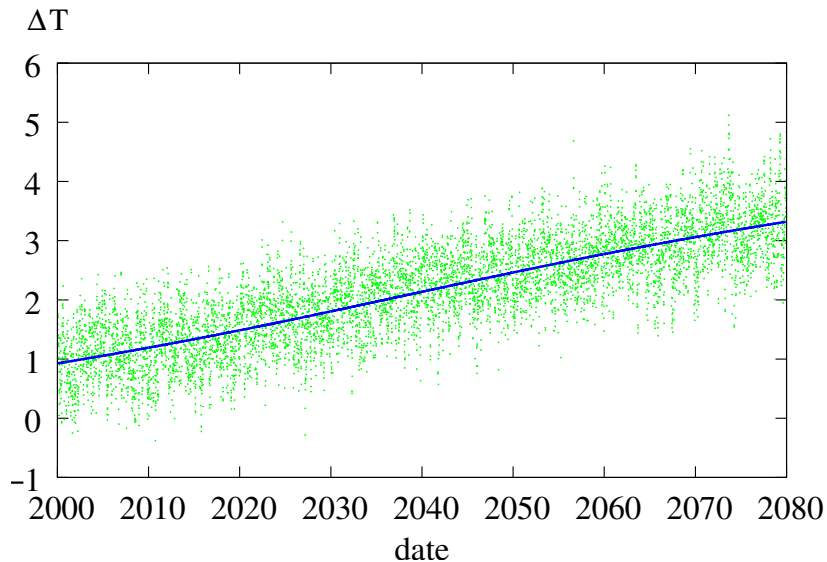
# 1. Classical, precise probabilities: Detecting climate change trend

## Is anticipation of climatic change important ?

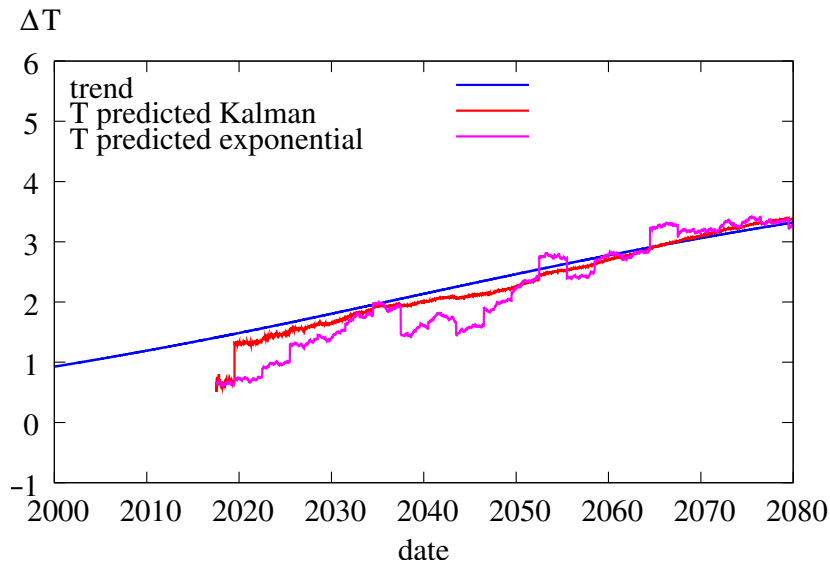
Regarding weather-related infrastructures, assumed to last  $\sim 55$  years, compare three investment rules:

1. Reactive adaptation. Investment designed for current temperature.
2. Simple proactive adaptation. Investment designed for predicted temperature at capital mid-life. No model, exponential forgetting for temperature and its trend.
3. Sophisticated proactive adaptation: Linear model with a Kalman filter to detect climate sensitivity.

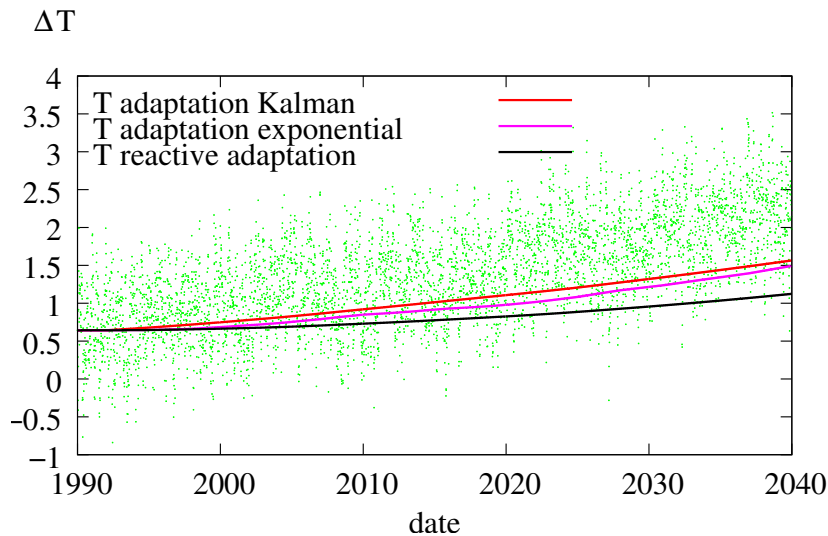
## (Unknown) Trend and variability



# Temperature prediction



# Evolution of the capital stock average design temperature



# Discussion of 1. Climatic change anticipation

- ▶ Without learning, we are out of the natural variability range by mid-century
- ▶ Proactive learning makes a difference, even with the simple rule
- ▶ What about model uncertainty ?

## 2. Inference for rare events with imprecise probabilities

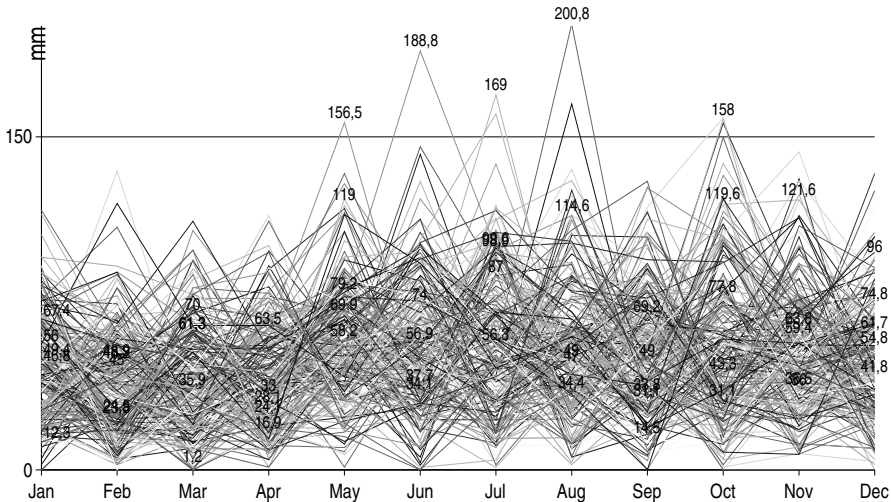
### Probability of a wet month in Paris next year?

An imprecise answer is a probability range : 2.5 to 5 percent. It may be better justified than a precise number:

- ▶ Trade off between precision and confidence (back of the envelope calculation)
- ▶ When subjective priors inadequate, information imprecise, data missing
- ▶ Extreme case: possibility of an event that has never been observed.

In 219 years, 9 months over 150mm precipitation

## Precipitations at PARIS LE BOURGET



## Inductive learning under uncertainty (ambiguity)

The frequency after the next (unknown) observation will be:

$$\text{More than } \frac{9}{219+1} \text{ but less than } \frac{9+1}{219+1}$$

For  $m$  positive outcomes in  $n$  trials, imprecise beta model infers:

$$\left[ \frac{m}{n+s}, \frac{m+s}{n+s} \right] \quad (1)$$

Parameter  $s$  determines the degree of imprecision in posterior inferences. It can be interpreted as a number of additional unknown observations.

## Mathematical break: the imprecise beta model as robust bayesian inference

Let  $\theta$  denote the chance of success in a Bernoulli trials experiment. Assume the prior on  $\theta$  is the family of PDFs

$$M = \{\beta(s, t), 0 < t < 1\} \quad (2)$$

where the beta laws  $\beta(s, t)(\theta) \propto \theta^{st-1}(1-\theta)^{s(1-t)-1}$  are parametrized by their mean  $t$ . Bayesian updating for  $m$  successes in  $n$  trials lead to posterior PDFs

$$M' = \left\{ \beta\left(s + n, \frac{st + m}{s + n}\right), 0 < t < 1 \right\} \quad (3)$$

The lower probability bound is

$$\inf_{p \in M'} E\theta = \frac{m}{s + n}$$

Google "Imprecise Dirichlet Model" for the multinomial case.

## Results: probability of occurrence next year (per cent)

### Wet month in Paris

Observation period	$n$	$m$	$s$	Result
1870–1989	219	9	0	4.1 sharp
	219	9	1	4.1 – 4.5
	219	9	2	4.1 – 5.0
1900–	89	3	1	3.3 – 4.4
1950–	39	1	1	2.5 – 5.0

### Major nuclear accident

Observation period	$n$	$m$	$s$	Result
1950–2006	56	2	1	3.5 – 5.3
1986–2006	20	0	1	0 – 4.8

## Discussion of 2: inference with imprecise probabilities

- ▶ A robust bayesian approach, imprecision meaningful when  $s/n$  is not negligible.
- ▶ Far-reaching consequences: decision making with imprecise expected utility, logic
- ▶ Some empirical evidence for expected value as an intervall
- ▶ Events never or rarely observed: maximum probability = degree of possibility

### [3.] Learning in the Transferable Belief Model: Fusion of experts opinion

Possibility distribution of climate sensitivity  $\Delta T(2 \times \text{CO}_2)$  ?

'Evidence' to learn from:

Morgan and Keith (1995) experts elicitation survey.

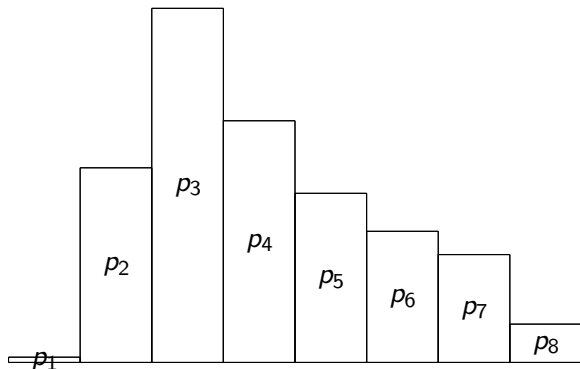
Problems:

- ▶ Information given as probability distributions functions
- ▶ Experts are not independent and not equally trusted
- ▶ Conflicting opinions

## Probability distribution from expert 1

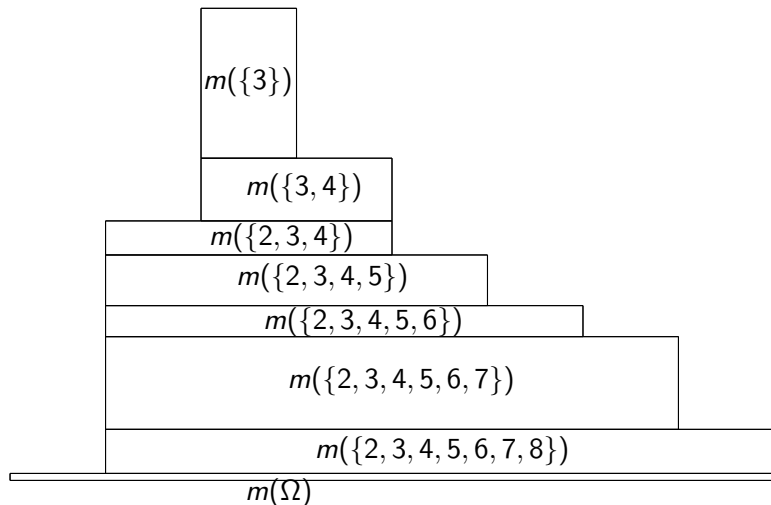
$$p_1 = P(-6 \leq \Delta T_{2 \times \text{CO}_2} \leq 0)$$

$$p_2 = P(0 \leq \Delta T_{2 \times \text{CO}_2} \leq 1.7) \dots$$

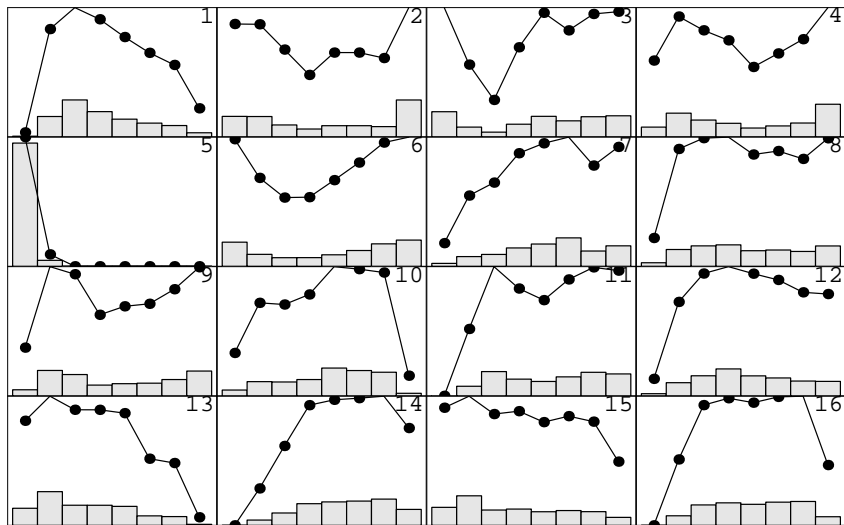


## Belief function from expert 1

Define the possibility function :  $\pi(\omega) = \sum_{\omega \in A} m(A)$



## Results from 16 experts: find the outlier!



Dotted lines: possibility, grey histograms: probability

## Fusion in two stages

1. Ignoring the outlier 5, and pool other expert's belief functions using a conjunction operator that do not assume independence (idempotent).

2. To combine these prior beliefs with expert 5 beliefs. To learn from new evidence, one need to answer three questions:

- ▶ Discount outlier's opinion and how much ?



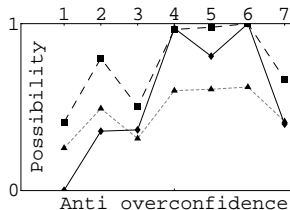
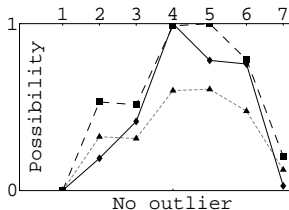
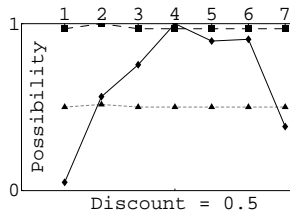
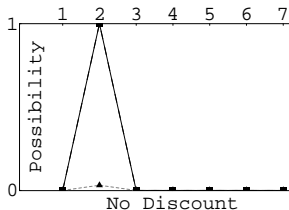
pool AND 5 are true

**Disjunction** pool OR 5 is true

**Exclusive disjunction** Either pool XOR 5 is true, not both

- ▶ Independence ?

# Conjunction: pool AND discounted(5) are right



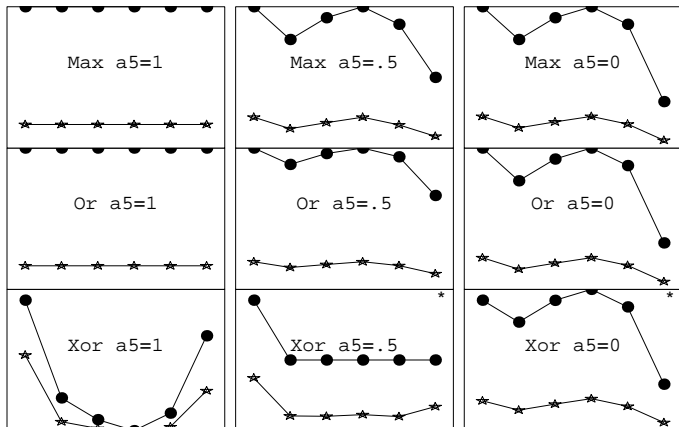
Combination rule	
-----▲-----	Minimum
---■---	Min Normal
—◆—	Dempster

Climate sensitivity	
Bin	
1	-6.0 to 0.9 °C
2	0.9 to 1.7 °C
3	1.7 to 2.2 °C
4	2.2 to 2.7 °C
5	2.7 to 3.3 °C
6	3.3 to 4.2 °C
7	4.2 to 12.0 °C

# Disjunction: OR (first two rows), XOR bottom row

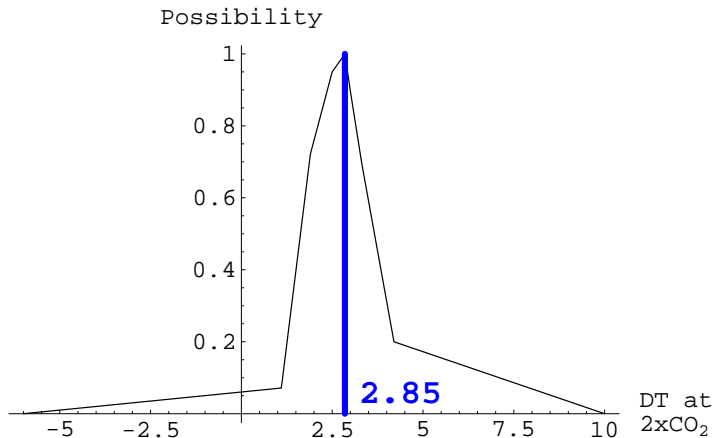
Left col:  $a_5 = 1$  means consider outlier's beliefs void (full discount)

Right col:  $a_5 = 0$  means consider them fully.



# Subjective assessment of $\Delta T(2\times\text{CO}_2)$

Conjunction, discounting outlier 90 percent.



## Discussion of 3. Fusion of experts opinion

- ▶ Learning from conflicting informations requires explicit modeling of evidence reliability and sources interactivity
- ▶ Defeasible reasoning:  $X \text{ XOR TRUE} = \sim X$
- ▶ Formalization of social construction of belief

## Conclusion: approaches to formalize learning

- ▶ Detecting climate change matters
- ▶ Inference for rare events: imprecise models are robust
- ▶ Construction of belief: learning as combining evidence, sometimes conflicting