# Public-goods games under time pressure

Christian Hilbe

*Department of Mathematics,*
*University of Vienna, Vienna, Austria*

## Goal

To investigate the role of time and time pressure in the contributions to a general public good.

## Background and motivation

A group's public goods require costly investments by individual group members while benefiting all group members irrespective of their investments. This leads to a so-called social dilemma: as non-contributors cannot be excluded from the benefits of the public good, there is a strong incentive for free riding (Hardin 1968). Without mechanisms ensuring the cooperation of players, this usually results in the failure of the public good (Ledyard 1995).

Much research in the last decade has focused on public-goods games and on mechanisms that maintain cooperation. These approaches include the possibility of punishing free riders (e.g., Fehr & Gächter 2000; Sigmund et al. 2001), voluntary participation (e.g., Semmann et al. 2003; Hauert et al. 2002), and the combination of the public-goods game with pairwise interactions (e.g., Milinski et al. 2002; Panchanathan & Boyd 2004).

However, important examples of public goods contain the strategic element of time, which has largely been neglected by experimental studies and theoretical analyses to date. Examples are given by effort levels in joint projects or by the "probably greatest public-goods game" (Milinski et al. 2006), the earth's climate. In both games, the strategy "wait and see" obviously plays a key role. Additionally, both examples are characterized by time pressure: joint projects usually have a deadline, and actions against climate change are more effective the earlier they are implemented. If the effectiveness of contributions to the public good decreases over time, the social dilemma is intensified, as the "wait and see" strategy may lead to late (and hence expensive) contributions to the public good. Without explicitly incorporating a temporal dimension, these important features are not adequately captured.

There is another interesting feature of the public good given by earth's climate: Unlike in typical public-goods games, the benefits of investments in stabilizing the earth's climate are not equally shared, since the consequences of global warming on single countries differ. From the point of view of a country for which consequences are moderate, such a setting might provide an additional incentive to adopt a "wait and see" strategy.

Surprisingly, despite the importance of these and other examples of public-goods games, no theoretical model and only a handful of experimental studies (Milinski et al. 2006, Milinski et al. 2008) as yet explicitly address the dimension of time in these games. The experiments of Milinski et al. (2008) try to imitate investments into the prevention of climate change. They find that, if investments have to exceed a certain threshold to be effective, the social dilemma can only be relieved if a failure to invest enough results in grave losses for individuals.

Some theoretical studies have dealt with the strategic element of time in a different setting: for example, Eskola (2009) discusses the optimal timing of reproduction, and Uehara et al. (2007) deal with the optimal display duration in animal contests. Eriksson et al. (2004) examine the war of attrition with an implicit time cost. We plan to transfer ideas developed in those other contexts to social dilemmas over public goods.

## Research questions

The aim of this research project is to obtain a better understanding of optimal schedules for timing investments into a public good under time pressure. More specifically, we will try to answer the following questions:

- What are the conditions that prevent a public good under time pressure from failing? Will there be a stable mixture of different strategies (e.g., a coexistence of high contributors with low contributors), or will the adaptation of investment schedules lead to uniform contributions?
- What is the eventual shape of the investment schedule? Will there be some last-minute effects, i.e., will there be a jump in the level of contributions when players perceive the impending failure of a public good?
- What is the impact of group size on the cooperation level of individuals, especially if the benefit of a public good does not depend linearly on contributions?
- As an extension, one may consider the effects of heterogeneity in the benefits different players derive from a public good. Does this lead to an increase of contributions by those players who gain a relatively high benefit from the public good? At equilibrium, will all players obtain the same net benefit from the public good?

## Methods and work plan

As these questions have so far not been treated explicitly, we will firstly use agent-based simulations to get an overview of possible settings for which a public good can be maintained under time pressure. As a starting point, we will use the following model. We consider a population of $N$ players who are engaged in a public-goods game. Each player $i = 1,\ldots,N$ decides independently of the others about the time $t$ and level $c$ of its contributions, based on a function-valued strategy $c_i(t)$, with $c_i(t) \geq 0$ for all $t$. Additionally, we assume that the capacities of players to contribute to the common resource at a given time are bounded, $c_i(t) \leq \overline{c}$ for all $t$.

The public-goods game is played in a randomly formed group of $n$ players. We denote the total instantaneous contributions at time $t$ by $C(t) = \sum_{i=1}^{n} c_i(t)$. Then the-benefit derived from the public good at time $T$ has the following general form, $B(T) = F\left(\int_0^T e(t)C(t)dt\right)$. In this expression, the function $F$ describes the benefit derived from the public good in dependence on the total effective contribution made toward the public good until time $T$. The function $e(t) \geq 0$ describes the efficiency of contributions made at time $t$. In the simplest case $e(t) = 1$, it does not matter at which time the contributions are made. By specifying functions $e(t)$ with $de/dt \leq 0$, one can incorporate the possible assumption that the efficiency of contributions toward the public good time decreases with time: the later contributions are made, the more input is needed to reach a certain level of total effective input. This assumption can be illustrated by considering the example of investing into the stabilization of the earth's climate. In this case, $e(t)$ can be regarded as a discounting factor for contributions made at time $t$. Generally, $e(t)$ might take any functional form, but for this project I will use two standard forms: $e_1(t)=1$ (no discounting of future contributions) and $e_2(t)=\exp(-\alpha t)$ (discounting with a constant rate $\alpha$). Also the function $F(x)$, which relates total effective inputs to benefits derived from the public good, can take any functional form that respects that benefits are non-negative ($F(x)\geq 0$ for all $x$) and that benefits do not decrease with the total effective input ($dF/dx \geq 0$ for all $x$). In addition, realistic examples often require respecting the following two conditions:

- There is a maximum benefit $B_{\max}$, so that even arbitrarily large total effective contributions toward the public good result in a limited benefit: $F(x)\leq B_{\max}$ for all $x$ with $\lim_{x\to\infty} F(x)=B_{\max}$, describing a diminishing return.
- A minimum level of total effective contribution is required for the public good to yield a benefit larger than $B_{\max}/2$.

For example, a function that fulfills both of these assumptions is $F(x)=B_{\max}e^{\alpha(x-\beta)}/(\gamma+e^{\alpha(x-\beta)})$ with positive parameters $\alpha$ and $\gamma$. Then $F(x)>B_{\max}/2$ requires $x>\beta+\alpha^{-1}\ln(2\gamma)$.

We furthermore assume that the duration $t^*$ of each public-goods game is a random variable drawn from the unit interval $[0,1]$. We consider several options for the distribution of $t^*$:

- The game duration is fixed to $t^*=1$ (i.e., $t^*$ follows a Dirac delta distribution with a peak at 1). This corresponds, for example, to joint projects with equally long deadlines.
- The game duration is drawn from a uniform, exponential, or Weibull distribution. This can be illustrated, for example, by investments into the defense of a community continually threatened by external attacks.

If the benefit of the public good is shared equally among all players, then the total payoff of each player is given by

$$P_i(t^*)=\frac{B(t^*)}{n}-\int_0^{t^*} c_i(t)dt\,.$$

In order to include strategic interactions among players, we additionally assume that the contributions of a player at time $t$ may depend on one or more of the following three quantities:

- The recent effective total contributions of all players, $\int_0^t e(\tau)C(\tau)d\tau$. This is a measure of the input that may still be needed to reach a certain level of benefit,
- The player's own effective total contributions, $\int_0^t c_i(\tau)d\tau$. This is relevant since each player's resources are limited and because players want to prevent being exploited and thus need to monitor their own investments.
- The total instantaneous investments of all players made at any given time $t$, $C(t)$.

In carrying out this research, we will first focus on separately exploring each of these three listed dependences. In order to maintain cooperation in the public-goods game, some mechanism is needed that favor cooperators over free-riders. The easiest mechanism we will consider is a snowdrift game with parameters such that a single cooperator excels in a population of free-riders. The type of game can vary with group size $n$, so that changing $n$ can turn a snowdrift game into a prisoner's dilemma. In such cases, an additional mechanism is needed for the evolution of cooperation to prevent the public good from failing.

A different approach would follow the assumptions of Eriksson et al. (2004). They consider a public-goods game that is played until a certain benefit $B_{\min}$ is reached. This implies that cooperators on average play longer games than free-riders, and hence may receive higher total payoffs. One way to include such an idea into our setting is to assume that, with the game's maximal duration being fixed to 1, players first may contribute toward the public good until the time $t^*$ when $B_{\min}$ is reached, after which each player obtains a benefit of $\sigma(1-t^*)$.

We embed the public-goods game described above into an evolutionary process, so that investment schedules evolve according to their relative success in the game. This can be modeled by a stochastic process, which may be similar to the one investigated by Nakamaru & Dieckmann (2009). Each individual may die according to the probabilistic rate $d_i = 1/N$. Its place is then filled by the offspring of player $i$ with probability $P_i / \sum_{j=1}^{N} P_j$. The offspring's strategy changes from that of its parent $i$ with probability $m$, from $c_i(t)$ to $c_i(t) + s_i(t)$, where the total change is small, $\int_0^1 |s_i(t)| \, dt < \varepsilon$. Finally, we assume that there is a small probability that player $i$ switches to full free-riding, $c_i(t) = 0$ for all $t$.

After the identification of interesting settings through agent-based simulations, we will try to obtain analytic results using the adaptive dynamics theory of function-valued traits (Dieckmann et al. 2006; Parvinen et al. 2006).

## Relevance and link to EEP's research plan

This project applies techniques that have largely been developed by researchers associated with EEP's research project on *Adaptive Dynamics Theory* (e.g., Dieckmann et al. 2006; Parvinen et al. 2006). Additionally, the theoretical analysis of public-goods games is a core theme of EEP's research project on the *Evolution of Cooperation*, which has made many interesting contributions to this field (e.g., Hauert et al. 2002; Nakamaru & Dieckmann 2009).

## Expected output and publications

The research is intended for publication as a coauthored article in an international scientific journal and will form an essential part of my PhD thesis.

## References

Fehr E & Gächter S (2000). Cooperation and punishment in public good experiments. *The American Economic Review* 90: 980-994

Hardin G (1968). The tragedy of the commons. *Science* 162: 1243-1248

Hauert C, De Monte S, Hofbauer J & Sigmund K (2002). Volunteering as Red Queen mechanism for cooperation in public good games. *Science* 296: 1129-1132

Dieckmann U, Heino M & Parvinen K (2006). The adaptive dynamics of function-valued traits. *Journal of Theoretical Biology* 241: 370-389

Eriksson A, Lindgren K & Lundh T (2004). War of attrition with implicit time cost. *Journal of Theoretical Biology* 230: 319-332

Eskola HTM (2009). On the evolution of the timing of reproduction. *Theoretical Population Biology,* in press

Ledyard JO (1995). Public goods: A survey of experimental research. In: *Handbook of Experimental Economics*, eds. Kagel JH & Roth AE, pp 111-194. Princeton University Press

Milinski M, Semmann D & Krambeck HJ (2002). Reputation helps to solve the 'tragedy of the commons'. *Nature* 415: 424-426

Milinski M, Semmann D, Krambeck HJ & Marotzke J (2006). Stabilizing the Earth's climate is not a losing game: Supporting evidence from public goods experiments. *Proceedings of the National Academy of Science of the USA* 103: 1994-3998

Milinski M, Semmann D, Krambeck HJ & Marotzke J (2008). The collective-risk social dilemma and the prevention of simulated dangerous climate change. *Proceedings of the National Academy of Science of the USA* 105: 2291-2294

Nakamaru M & Dieckmann U (2009). Runaway selection for cooperation and strict-and-severe punishment. *Journal of Theoretical Biology* 257: 1-8

Panchanathan K & Boyd R (2004). Indirect reciprocity can stabilize cooperation without the second-order free rider problem. *Nature* 432: 499-502

Parvinen K, Dieckmann U & Heino M (2006). Function-valued adaptive dynamics and the calculus of variations. *Journal of Mathematical Biology* 52: 1-26

Semmann D, Krambeck HJ & Milinski M (2003). Volunteering leads to rock-paper-scissors dynamics in a public goods game. *Nature* 425: 390-393

Sigmund K, Hauert C & Nowak MA (2001). Reward and punishment. *Proceedings of the National Academy of Science of the USA* 98: 10757-10762

Uehara T, Iwasa Y & Ohtsuki H (2007). ESS distribution of display duration in animal contests to assess an opponent before fighting or fleeing. *Evolutionary Ecology Research* 9: 395-408