Discounting and sustainability in applied IAMs

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ABSTRACT

The debate on the proper level of discount rates in view of sustainability is based on an assumed relation between a 'pure time preference', economic growth and the interest rate. The relation is common in most dynastic Integrated Assessment Models (IAMs), and has led to much debate, since quantitative results of the IAMs highly depend on future discounting. One group of researchers advocates an 'descriptive' time preference based on historic data to ensure efficient resource use. Another group of researches advocates the use of a 'prescriptive' low time preference to ensure sustainable resource use.

In this paper, it is argued that, though the assumption underlying the discussion is convenient for the analysis, it is fictitious and misleading. In particular, the assumed relation cannot be used to predict future interest rates. Consequently, dynastic IAMs cannot be used to provide policy makers with quantitative figures about the desirable emission reduction levels and the supporting prices. This is illustrated by use of an OLG-IAM model, ALICE 2.0, that shows how various assumptions on demographic change and public policy affect the interest rate and optimal emission reductions. We suggest that economists should inform policy makers as to which instruments can be used to bring forward both efficient and sustainable resource use.

1 INTRODUCTION

Since the potential climate change caused by antropogenic greenhouse gas (GHG) emissions has been recognized as a major environmental problem, a number of applied Integrated Assessment over generations. There has been much analysis of the dynastic model, and the analysis has led to much debate on the use of different welfare functions. In this paper, we will argue that the dynastic mechanism of welfare distribution is unrealistic and misleading. Its prevalent use in IAMs has led to unnecessary controversies between researchers that advocate efficient and those that advocate sustainable resource use.

The dynastic IAMs commonly use a constant Rate of Pure Time Preference (RPTP) to discount future welfare levels before aggregation in a net present welfare measure. The discount rate substantially affects the calculated optimal emissions and their supporting prices. This finding is confirmed by the theory, in which it is shown that ‘discounting advances the doomsday’ [Koopmans 1974 {refid #527 difer}], or, in other words, where it is shown that the choice for the RPTP determines whether a sustainable allocation is optimal or not [Pezzey 1992 {refid #105 difer}]. The importance of discounting is intuitively clear. The environmental resource can produce an indefinite stream of valuable services, the aggregate net present value of which directly depends on the discount rate. This potential future value has to be balanced with the immediate returns of present exhaustive resource use that decreases future output. The discount rate determines which value is maximal, and thus, which use is most profitable.

Given the importance of the RPTP in the sustainability debate, many papers have been devoted to argue about the proper choice. Some economists suggest that historic data on interest rates can be used to calibrate the level of the RPTP in IAMs, while these in turn determine the optimal environmental resource use in applied models [Nordhaus 1994{refid %34 difer}, Ch. 6]. In other words, they assume that historic data determine whether a sustainable environmental resource use is desirable. Others strongly oppose this so called ‘descriptive’ view, and argue for a ‘prescriptive’ approach in which one should use a low, or even zero, discount rate, since future generations should not be discriminated by lower welfare weights [Broome 1992 {refid #9 difer}]. Between these two views, no agreement seems to be possible. In this paper, we will argue that part of the discord is due to the use of a dynastic framework, which gives a convenient simplification of the dynamic competitive equilibrium, but which cannot be used to calculate whether sustainable environmental resource use is optimal or not. Stated in other words, dynastic models are not fit for calculating optimal emission reductions and supporting prices if these emissions affect welfare levels in the very long run.

The argument developed in this paper is straightforward. First, let us note that the RPTP as used in a dynastic framework represents an fictitious parameter that cannot be observed in a competitive dynamic equilibrium. The observed discount rate, say, the interest rate is a variable of goods over which property rights are established, also fictional, a political debate about it is more straightforward than one on the abstract concept of pure discounting.

2 DISCOUNTING IN DYNASTIC MODELS

For the purpose of generality, let us consider the case of a multiple dynasty economy. Every dynasty is characterized by its dynastic planner's welfare function that aggregates utilities of different generations within the same dynasty into one welfare measure. A commonly used welfare function aggregates utilities by weights that are proportional to the size of the generation, and decline geometrically in time. Such a welfare function is referred to as having constant pure time preference. The form can be found in all text books on growth models, and is used in all IAMs mentioned above. It can be traced back as far as Ramsey [1928 {refid #38 difer}]. In the discrete time model, welfare is written:

$$w_0^i = \sum_{t=0,\dots,\infty} \beta^t u_t^i \,, \tag{1}$$

where β is the constant utility discount factor, and where u_t^i is the utility of generation *t* in dynasty *i*, or alternatively, the utility of dynasty *i* in period *t*. Another commonly used form is the so called maximin, or Rawlsian welfare function. It is particularly popular among advocates of intergenerational equity. A discussion of its merits in relation to Hartwick's rule for optimal capital planning can be found in [Solow 1974, 1986 {refid #522,347 difer}]. In this form, welfare is taken to be equal to the lowest per capita utility level of all generations:

$$w_0^i = \min_{t=0,..,\infty} \mathrm{u}_t^i \,. \tag{2}$$

Both welfare functions can be represented in recurrent form, as $w_t = n_t u_{\mathrm{t}} + \beta w_{t+1}$, and $w_t = \min\{u_{\mathrm{t}}, w_{t+1}\}$, respectively, where we omit the superscript *i* for convenience and where u_t is the utility per capita. Koopmans [1960 {refid #324 difer}] shows that there is a broad class of welfare functions that can be written in such a recursive way, using the general recursive welfare function:

utilities. Then, one allocation is overtaken by another if from period T on, the summed utilities for t=1,…,T of the latter allocation exceeds the summed utilities of the former:

$$\forall t \geq T: \sum_{i=0,\ldots,t} \tilde{u}_i \leq \sum_{i=0,\ldots,t} \hat{u}_i \tag{4}$$

where i denotes the generations, the hat denotes the overtaking optimal allocation, and the tilde denotes the former allocation. However, orderings on sequences with unbounded values are typically incomplete[2]. To avoid this undesirable feature, in applied IAMs, it is typically assumed that there is a $\beta \in (0,1)$ such that

$$|\mathrm{H}(u, \tilde{w}) - \mathrm{H}(u,w)| \leq \beta |\tilde{w} - w| \tag{5}$$

for all $u, \tilde{w}, w \geq 0$. The contractor $\beta<1$ enables us to set an upper bound on aggregated welfare, given an upper bound on utility. Let $\bar{\mathrm{u}}$ be the upper bound for extended utility. Then, for any feasible utility u:

$$\mathrm{H}(u, w) \leq \mathrm{H}(\bar{\mathrm{u}}, w) \leq \mathrm{H}(\bar{\mathrm{u}}, 0) + \beta w, \tag{6}$$

where the first inequality follows from non-decreasingness and the second inequality follows from the contraction property. If we take an upper bound $\overline{\mathrm{w}}$ for w_{t+1} that satisfies $\overline{\mathrm{w}} > \mathrm{H}(\bar{\mathrm{u}},0)/(1-\beta)$, it follows that:

$$u_t \leq \bar{\mathrm{u}} \ \wedge w_{t+1} \leq \overline{\mathrm{w}} \Rightarrow \mathrm{H}(u_t, w_{t+1}) < \overline{\mathrm{w}}. \tag{7}$$

which implies that $\overline{\mathrm{w}}$ is an upper bound for w_t as well. Hence, we can consistently assume that $\overline{\mathrm{w}}$ is an upper bound for welfare in all periods.

Through the aggregator, the welfare of the whole sequence of utilities is incorporated in the variable w, evaluated at t=0. Abstracting from distortions such as taxes, the dynastic welfare program can then be written as the weighted maximization of all w_0^i,

$$w_0^i = \sum_{i=1,\ldots,I} \alpha^i w_0^i, \tag{8}$$

The second derivative of the aggregator function, $H_2(.)$, is known as the pure time preference factor. In terms that are more common in the IAM literature, the rate pure of time preference (RPTP), which we denote by ρ_t, is given by

$$\rho_t = H_2(u_t,w_{t+1})^{-1}-1 \;. \tag{9}$$

As one sees, in the general model, the RPTP is a non-constant variable. There is good reason for this. As Lucas and Stokey [1984 {refid #269 difer}] and Epstein [1987 {refid #323 difer}] point out, if one presumes a constant PRTP as in (1), that is $H_2(.)\equiv\beta$, the steady states of the economy form a continuum that represents no economic feature, but is a straightforward implication of the assumed time-additivity. This artifact of the model is considered undesirable as there is no *economic* explanation, and thus, Lucas and Stokey and Epstein strongly prefer the use of a non-trivial aggregator function H(.). Despite these arguments in the literature against it, the applied model makers continue to use the model with constant RPTP, because, it is the most convenient way of welfare aggregation.

Whereas β reflects the discount factor for welfare – the pure time preference – discounting also refers to the price depreciation for marketed goods. In a general model with various goods, every good has its own price depreciation and thus there is no unambiguous interest rate. Many integrated assessment models however specify a single consumer good, and take the price depreciation for this good as the interest rate. If utility has a constant elasticity of marginal consumption, say γ, then the discount rate for the single consumer good, the interest rate r_t, satisfies

$$r_t \approx \rho + \gamma g_t. \tag{10}$$

where g_t is the consumption growth rate for the single consumer good. The equation gives the linear approximation of the equality $(1+r)=(1+\rho)(1+\gamma g_t)$. This equation has been given a central role in the debate on discounting [e.g. Cline 1992 {refid #147 difer}]. Its clear implication is that the future interest rate decreases gradually as the consumption growth decreases,

$$r_{t+1} - r_t \approx \gamma(g_{t+1} - g_t), \tag{11}$$

a common assumption in IAMs, converging to a level $r^{LT} = \delta + \gamma g^{LT}$ where the superscript LT denotes the long term. The model maker only needs to estimate the parameters δ and γ by mechanisms against unsustainability. As Ginsburgh and Keyzer [1997 {refid #154 difer}, pp. 281,282] point out, the dynastic model easily leads to allocations in which, from a certain point in time towards the indefinite time horizon, future generations may have to pay a debt that was incurred before their time of birth (‘slavery’).

Because the use of a ‘descriptive’ pure time preference (that describes historic data) can lead to an unsustainable optimal solution, some researchers advocate a ‘prescriptive’ approach, using a low, or zero, pure time preference. In General, the prescriptive low discount rate will ensure a sustainable environmental resource use. However, the advocates of the descriptive approach argue that if one uses a low discount rate for environmental assessment, which is different from the actual discount rate observed in the market, this implies an inefficient resource use. If using the dynastic model, it seems that efficiency and sustainability are at contrast.

3 DISCOUNTING IN OLG MODELS

In OLG models, there is no central planner whose preferences determine the interest rate. Instead, the interest rate is one element of the price vector that adjusts in order to match demand and supply of all goods, in all periods. For the general OLG economy with multiple goods and producers, it is impossible to predict changes in the interest rate as other exogenous parameters unfold over time. Nevertheless, under some simplifying assumptions, it is possible to relate the interest rate to certain parameters.

Consider a two-generations OLG economy. Supply consists of the endowments of the young, ω_t^y, and the old, ω_t^o, plus net production, $y(p_t,\psi_t,\psi_{t+1})$, which is a function of prices for the goods, p_t, and stock prices at the beginning and end of the period ψ_t, and ψ_{t+1}, respectively.[4] Since we did not demand that there is a single good or stock, prices are vectors. Both the old and the young generation demand goods for consumption, based on prices over their life-cycles, $d^y(p_t, p_{t+1})$ and $d^o(p_{t-1}, p_t)$, respectively. Production and demand functions are homogeneous of degree zero in prices. In a steady state, prices satisfy $1+r = p_t / p_{t+1} = \psi_t / \psi_{t+1}$, where r is the interest rate. In equilibrium, in every period, supply matches demand:

$$d^o(p_{t-1}, p_t) + d^y(p_t, p_{t+1}) = \omega_t^y + \omega_t^o + y(p_t,\psi_t,\psi_{t+1}). \tag{12}$$

This equation defines a recursive relation between prices in period $t-1$, period t, and period $t+1$. It number of welfare weights that respect the budgets for the dynasties. This difference in complexity also explains why in an OLG model, the interest rate does not in a rather trivial way depend on some exogenous parameters.

In an OLG model, the equilibrium on the goods markets implies an equilibrium on the savings-capital market as well: life-cycle savings match the value of the capital stock. Different from a dynastic model, capital is not transferred to future generations as a bequest, but it is held by old generations who sell their capital stock to the new young generation to pay for their pension. Both the level of private life-cycle savings and the level of investments depend on the interest rate, and thus, if some exogenous change affects one of these variables, it will affect the interest rate as well. For example, if, as part of a social security policy, a pay-as-you-go pension system is introduced where the currently young pay for the pensions of the currently old, this will decrease the need for private life-cycle savings, and thus affect the interest rate. In the general case, the effect is ambiguous (whether there is an increase or a decrease of the interest rate), but in a simple OLG model where endowments are not too unevenly spread over the life-cycle, a decreasing need for life-cycle savings leads to an increase in the interest rate [Blanchard and Fisher 1989 {refid #7 difer}, Section 3.5 and Problem 4]. This finding implies that the social security policy directly affects the interest rate.

Another potential major cause for changing future discounting is the expected demographic change in the next century. It is expected that life-expectancy will increase by about 20 years [WB 1994 {refid #497 difer}], whereas the productive life-time does not extend with the same amount. The question is now whether this demographic change will increase or decrease the interest rate. Again, the answer is ambiguous. In case of a pay-as-you-go system, the ‘aging’ will increase transfers from the present to the previous generation, and this can lead to an increased interest rate [Auerbach *et al*. 1989 {refid #339 difer}]. On the other hand, if there is a fully-funded system, as opposed to a pay-as-you-go system, where every generation pays for its own pension, and if the endowments are not too unevenly distributed over the life-cycle (as above), ‘aging’ will increase the need for life-cycle savings to pay for the increased retirement period and this induces an increase in the interest rate. The effects are substantial. Gerlagh [1998, Section 3.3.1] shows by a simple numerical example that a process of aging that reflects the expected demographic change in the next century is capable of shifting a dynamically efficient equilibrium path with an interest rate of 5 per cent per year, to a dynamically inefficient equilibrium where the interest rate has dropped to 0.5 per cent per year, substantially below the economic growth rate.

Another major source of changing interest rates in the OLG model finds its cause in public is set aside for future generations, a policy that can be interpreted as a transfer of income to future generations, this will reduce the interest rate[5]{refid #427 difer}. Even more importantly, Gerlagh shows that the decrease in the interest rate ensures that the welfare of future generations exceeds a reference welfare level, set by a ‘strong sustainability’ policy that protects the environment by strict environmental measures. Thereby, the intergenerational distribution of the property rights over the environmental resource is effectively used to let the discount rate endogenously decrease to a sustainable level.

4 NUMERICAL ILLUSTRATION

To illustrate the arguments given above, we use an OLG-IAM model, ALICE 2.0 [Gerlagh 1998, Section 5.3], that is applied to the issue of climate change, a comprehensive description of which can be found in [Gerlagh 1998, Section 5.2]. A brief specification is given in the appendix. ALICE 2 distinguishes periods of 20 years each, the first of which corresponds to the interval 2000-2020. In every period, a new generation is born. The model includes a rise in life-expectancy that is supposed to take place during the 21st century (the first five periods), represented by a transition from generations that live two adult periods to generations that live three adult periods. In the figures that follow, state variables, such as CO_2 concentrations, are defined for the years between two periods: 2000, 2020, 2040, etc., while flow variables, such as consumption and emissions, describe a flow during a period, e.g. 2000-2020, and these periods are identified with the central year, e.g. 2010. ALICE 2 includes a simple production sector for man-made commodities, using labor, man-made capital and emission units as production factors.

We specify four scenarios to show how the demographic change and the intergenerational distribution of property rights over the environmental resource affects the results. The first scenario, labeled *Grandf1*, assumes that the environmental resource is grandfathered to the first generation, and abstracts from the increase in life-expectancy. The second scenario, labeled *Grandf2*, includes the demographic change. The third scenario, labeled *Trust1*, assumes that the property rights over the environmental resource are equally distributed over all generations by use of a *trust fund* [Gerlagh 1998, Section 4.3.3], and abstracts from the demographic change. The fourth scenario, labeled *Trust2*, includes the demographic change.

TABLE 1. *Scenario specification*

Figure 1 shows that the interest rates for the four scenarios. There are two outstanding findings. First, the intergenerational distribution of the property rights over the environmental resource has a major impact on future interest rates. Sharing the property rights with future generations decreases the interest rate from 7.3 to 2.9 per cent per year in 2200, if we omit aging. Including the demographic change, we find that sharing the property rights decreases the interest rate from 2.5 to 0.5 per cent in 2200. Secondly, comparing the scenarios with and without aging, it is immediately clear that aging decreases the future interest rate substantially, from 7.3 to 2.5 per cent in 2200 under grandfathering, and from 2.9 to 0.5 per cent in 2200 if property rights are shared with future generations. The figure thus undoubtedly reveals the importance of both demographic changes and environmental policies for long term interest rates.

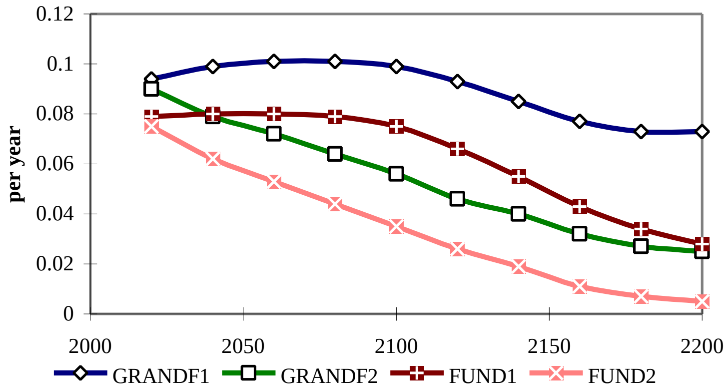


FIGURE 1. *Interest rates*

Now, we turn our attention to the consequences of the different interest rates for climate change itself. We have assumed that CO_2 emissions decrease by one per cent point for each 4 US$/tC increase in the emission price, and this implies that we abstract from endogenous technological improvement and transition costs associated with a shift towards a ‘backstop’ energy technology.

Omitting aging, grandfathering leads to a very low emission price that reaches 100 US$/tC in 2200. If we include aging, the discount rate decreases, and the carbon emission price slowly increases from nearly zero in 2000 to 100 US$/tC in 2100 and 400 US$/tC in 2200 (Figure 2). Consequently, in the first scenario, the reduction in emissions is mainly explained by an assumed autonomous increase in energy efficiency (Figure 3). In the second scenario, in the early periods

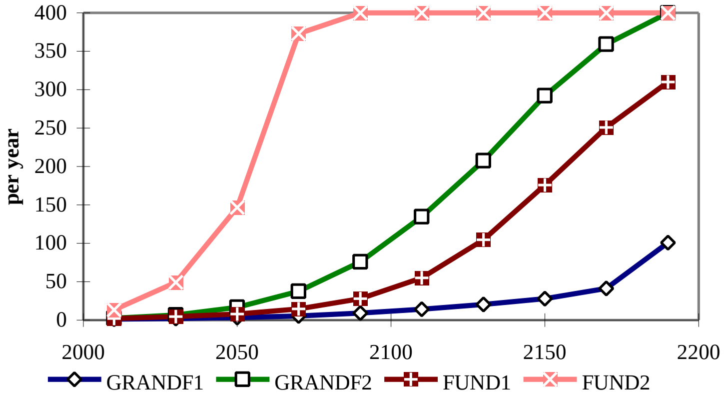
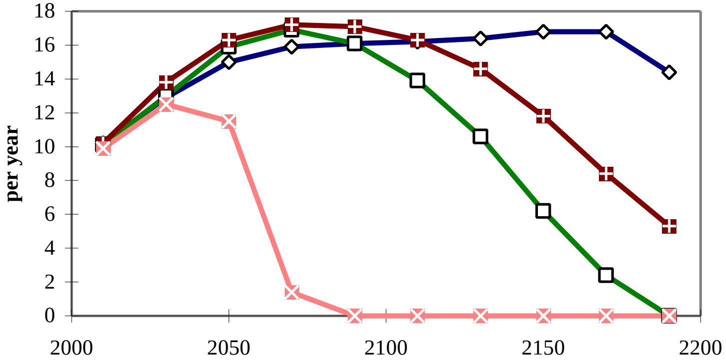


FIGURE 2. CO_2 *emission prices*

The trust fund substantially increases the emission price already in the first period, but as we have already seen for the discount rate, its total effect depends on the inclusion of aging. If aging omitted, the emission price remains between the two grandfathering scenarios, and this more or less also applies to the emission levels. If aging is included in the calculations, the price increases rapidly towards 400 US$/tC in 2080, which by assumption implies a 100 per cent reduction of net CO_2 emissions. The policy does not limit the expansion of net emissions in the medium term, leaving present generations the possibility to adapt and to develop alternative energy sources. After 2050, net emissions decrease rapidly, and by 2100 a complete substitution of fossil fuel energy carriers by fossil free energy carriers has taken place.



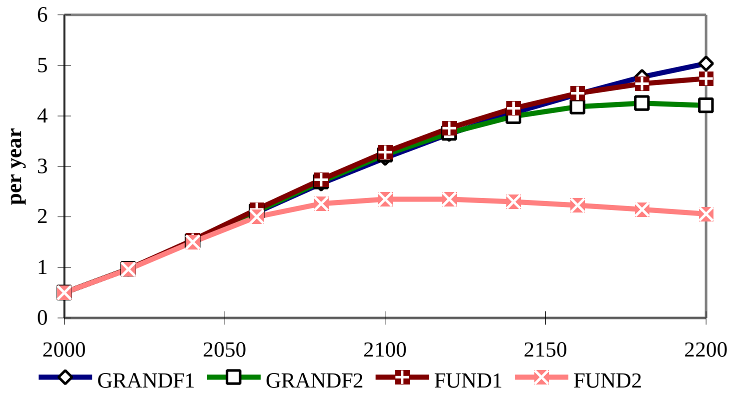


FIGURE 4. *Global mean temperature*

Let us reiterate that the figures presented are meant to illustrate the importance of taking into account various changes that may affect future discounting, but that are easily neglected in dynastic models. It is not our purpose to present a new set of numbers that can be used for applied greenhouse gas policies.

5 CONCLUSION

Summarizing the findings. Discounting is of major importance to the sustainability issue, since too high a discount rate makes it unprofitable to sustain a high quality environment, whereas a low discount rate makes sustainable use the most profitable venture. In the standard dynastic model with additive discounted welfare functions (at constant RPTP), the future discount rates depend on a limited number of parameters only. Since these parameters are calibrated on historic data, it seems possible to determine whether sustainability is optimal or not. It is undeniable that the dynastic model simplifies the analysis. However, it makes no sense to use the dynastic model for predicting future interest rates, since there is no reason to assume that welfare is distributed as if there was a central planner.

In OLG models, on the other hand, the discounting is related to many other variables in the economy that may change in the future. Demographic changes, changes in social security, and the specification of property rights over environmental resources all affect the discounting. These forward an efficient and sustainable environmental resource use [Gerlagh 1998]. If a policy maker prefers to combine efficiency and sustainability, he should not use a dynastic model that is not capable of doing so.

APPENDIX. CONDENSED MODEL DESCRIPTION FOR ALICE 2.0

The ALICE 2.0 model distinguishes discrete time steps, $t \in \mathbf{T}=\{1,\ldots,\infty\}$, each representing periods of 20 years duration, the first of which corresponds to 2000-2020. In every period, a new generation is born, denoted by the date of birth, using the label t or i if convenient. The model includes a rise in life-expectancy that is supposed to take place during the 21st century (the first five periods), represented by a transition from generations that live two (adult) periods to generations that live three (adult) periods. The first and third scenario abstract from the demographic change, assuming that all generations live only two (adult) periods. See [Gerlagh 1998, Table 3.5] for a comparison of modeled life-expectancy with data from the literature.

Generations maximize their lifetime utility $\mathrm{U}(C_i,B_i)$ derived from rival consumption of the consumer good during the life-cycle, $C_i=(C_i,C_{i,i+1},C_{i,i+2})$, and non-rival consumption of the resource amenity, for convenience referred to as ‘environmental services’, $B_i=(B_t,B_{t+1},B_{t+2})$ where we use the subscript $t=i$ to stress that the amenity level is the same for all consumers. Their utility function U(.) is a nested CES function with a constant expenditure share of 90 per cent for the consumer good and 10 per cent for the resource amenity, and an intertemporal elasticity of substitution of 0.67. The generations maximize utility subject to the budget constraint:

$$max\ \{\ \mathrm{U}(C_i\ ,B_i\ )\ \mid \sum_{t=i,\ldots,i+2} p_t\, C_{i,t} + \varphi_{i,t} B_t\ \leq \sum_{t=i,\ldots,i+2} w_t\, L_{i,t} + H_{i,t}\ \}, \tag{13}$$

where p_t, w_t denote the prices of the (rival) consumer good and labor, respectively, $\varphi_{i,t}$ are the Lindahl prices for non-rival consumption of the resource amenity of generation i in period t, $L_{i,t}$ denotes the labor endowment and $H_{i,t}$ is the income generation i receives in period t as its share in the value of the environmental resource. The first generations not living three periods have adjusted utility functions and budget constraints in the obvious way.

There is one private firm that uses ‘emission units’, E_t, and ‘labor’, $L_t=L_{t-2,t}+L_{t-1,t}+L_{t,t}$, for the production of the consumer good, and also a capital stock, K_t, which is itself produced by the same sector. For convenience, we assume that the capital stock is made up of the consumer good.

USD/tC price increase for the emission units, and such that the maximum emission levels follow the IS92a scenario [IPCC 1992].

The climate change issue is represented through an ‘environmental firm’. Peck and Teisberg [1992] and Nordhaus [1994] have much contributed to the development of stylized economic IAMs by providing highly simplified representations of geophysical interactions to make these applicable in macro-economic models. The typical simplified aggregate representation that has thereafter evolved links emissions to concentrations, concentrations to temperatures, and temperatures to damages. We follow this literature.

As the emissions of CO_2 account for the main antropogenic contribution to the greenhouse effect, we focus on the carbon-cycle and its relation to climate change. Let us assume that the atmospheric GHG accumulation can be represented by a ‘linear box’ model in which each of the boxes i=1,…,5 is well mixed as described by Maier-Reimer and Hasselman [1987]. If CO_2 is emitted, it is distributed over the boxes for given shares a_i. Within a box, CO_2 concentrations exponentially adjust to their ‘natural’ levels at annual adjustment rate $1/\tau_t$, where τ_i is the so called ‘e-folding time’ or turnover time, which is the expected period that a gas particle will remain in the box. Let M_t^i denote the accumulated antropogenic emissions in box i at the beginning of period t and E_t the emissions during period t, we then have:

$$M_{t+1}^i = e^{-N\tau_i} M_t^i + a_i E_t \tag{15}$$

for $\Sigma_i a_i = 1$, where N is the period length of the discrete time model. Maier-Reimer and Hasselman [1987] have estimated the following parameters a and τ for a 5-box model:

TABLE 1. *Parameter values for the linear Maier-Reimer and Hasselman 5-box model*

e-folding time (τ_i) (yr)	∞	313.8	79.8	18.8	1.7
share (a_i)	0.14	0.24	0.32	0.21	0.09

Remarkably, 14 per cent of emissions remains in the atmosphere for the infinite horizon, which implies that the absorption capacity of the biogeochemical system is exhaustible.

The accumulation of GHGs causes an increase of the equilibrium global mean temperature (see [IPCC 1992, Table 2.2] for a list of GHGs and their potential contribution). For CO_2, the temperature increase is expected to be of approximate logarithmic nature.

$$T_{t+1} = e^{-Ne} T_t + (1 - e^{-Ne}) T_{t+1}^{eq} . \tag{17}$$

where *e* is the annual adjustment rate, which we set to 2 per cent per year following Peck and Teisberg [1992].

So far, the above equations relate to complex geophysical relationships, and though present knowledge of these relationships is limited, the approximations are based on the firm foundation of a quantitative understanding of physical processes. However, when calculating impacts of climate change, the scientific understanding is grossly insufficient to warrant even something like a ‘best guess’. In general, it is assumed that damages caused by climate change will outweigh the benefits, but the lack of knowledge is unmistakably revealed by many sensitivity analyses that are carried out with a variety of so called damage functions. These damage functions are supposed to provide a reduced form for the many complex damages associated with climate change, such as loss of coastal zones due to sea level rise, loss of biodiversity, vector borne diseases, and extreme events. Some damage functions take the global temperature as arguments, others take the rate of increase of global temperature as an argument, some damage functions are quadratic, other are of higher or lower order, cf. [Tol 1995]. The lack of understanding is also recognized by the IPCC [1996a, Section 6.2.13]. Yet, in the same report, several damage estimates are listed [IPCC 1996a, Table 6.4]. The use of these figures for our analysis does not mean that we consider them reliable, but reflects our wish to maintain compatibility with the prevalent IAMs. Typically, the IAMs include a reduced damage function h(.):

$$D_t = \mathrm{h}(T_t) , \tag{18}$$

where D_t is the damage, usually a function of actual temperature increase, expressed in monetary units or as a percentage of GDP.

The IAMs have different ways of incorporating the damage functions, either by subtracting damages from production or consumption, or directly from utility. We find this practice misleading, because environmental damage is better understood as a decrease in the quality or quantity of environmental functions, than as a reduction in a flow of man made goods. Environmental degeneration can lead to a reduction both in biomass and in biodiversity, with the former measuring the quantity and the latter the quality of the environmental functions [IPCC 1996b, Section 1.3.2]. A decrease in biodiversity constitutes a change in environmental quality, as opposed to the quantity, and in a reduced model, this can best be described via a non-rival

Now, let us return to the resource income shares which, by definition, should sum to the total value of the resource, that is:

$$\sum_{i=1,\ldots,\infty} H_i = \sum_{t=1,\ldots,\infty} (p_t^e E_t + p_t^b B_t), \tag{20}$$

where p_t^e and $p_t^b = \varphi_{t-2,t} + \varphi_{t-1,t} + \varphi_{t,t}$ represent the price of the emission units and the environmental services level respectively, and $H_i = H_{i,i} + H_{i,i+1} + H_{i,i+2}$ is the aggregate resource share received by generation *i* over the life-cycle.

In the grandfathering scenario, the environmental resource is given to the first generation, t=0, who receives all present and future revenues as income:

$$H_0 = \sum_{t=1,\ldots,\infty} (p_t^e E_t + p_t^b B_t), \tag{21}$$

$$H_t = 0 \quad \text{for } t=1,2,\ldots, \tag{22}$$

where p^e and p^b represent the price of the emission units and the environmental services respectively.

In the trust fund scenario, all generations receive a claim for the maximum environmental services level:

$$H_{i,t} = \varphi_{i,t} \tag{23}$$

and the trust fund implements the required redistribution of revenues over the generations; see [Gerlagh 1998, Section 5.2.4] for a detailed description.

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