

The Pure Economics of Tradable Pollution Permits

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Abstract

The paper addresses the issue of international trade of pollution permits, in the context of domestic economies with non-neutral fiscal systems, i.e. distortionary taxation. It also generalizes the existing literature to the consideration of pollution emitted by both firms and households, and in the case of a non-linear production function of the economy.

A main issue arises from the possible divergence, in second-best economies, between the pollution tax and the marginal abatement cost. Several applied studies and models concerning the Greenhouse Gases mitigation policy, which are reviewed, confirm this divergence.

A close examination shows that the results depend crucially on the existing tax-systems, and in particular on the available fiscal tools. If there is a sufficiently large array of fiscal tools, then, as in the first best framework, the marginal abatement cost is equal to the pollution tax and markets of tradable permits are PARETO efficient.

When there are constraints on taxation, none of the previous results verify in all generality. However, if some separability condition on production functions is satisfied, tradable permits markets are efficient.

Beside theoretical analysis, the paper presents numerical simulations in order to give an idea of the behavior of the two types of models.

Keywords: Tradable Pollution Permit, Terms of Trade (gains and losses from change in); Marginal Abatement Cost; Double-Dividend, Competitive Exchange Equilibrium.

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1 Introduction: Pollution Internalizing in First-Best and Second-Best Contexts

In a first-best context, internalizing an externality such as a pollution can be efficiently performed either by imposing a Pigovian tax or by setting a market of pollution permits, the level of the tax or the price of permit being equal to the marginal desutility of the externality. If the externality is a public bad -i. e. its abatement a public good, benefiting to several economic agents- the optimal level of abatement can be obtained through a Lindhal-type market and personalized prices where “demand” for abatement by each economic agent (firm or household) is determined according to his willingness to pay¹.

Tax on pollution is then equal to the marginal damage -which is the essence of the Pigovian rule-, and also to the marginal abatement cost, i. e. the welfare loss for each economic agent -and by aggregation for the whole economy- of a unit abatement of pollution. The latter is the *social* marginal abatement cost, for instance the decrease of the social welfare function when redistribution between households is implemented consistently to such a criterion (and through lump-sum transfers, in order to reach first-best PARETO efficiency).

How can this Pigovian framework be transposed to a second best context, in economies where there exist “distortions”, either in markets (e. g. labor market) or through a non-neutral fiscal system? This has been the topic of numerous papers in the near past, mainly about the Greenhouse Effect. The inquiry has focused on the eventual existence of a “double dividend”, with the idea that a policy of pollution abatement may correct pre-existing distortions and then increase welfare (independently of the welfare gain directly accruing from pollution abatement)².

However, existing literature on the subject is limited in scope for two main reasons: i) it considers exclusively pollution generated by final consumption of households, moreover assuming a *linear* production function of the economy (production costs of all goods being constant); and ii) pollution internalizing is analyzed exclusively at the domestic level, without consideration of possible bargaining between countries on emissions levels (when pollution is an international public bad) and also without consideration of the effects of international trade on domestic policies of pollution abatement (some countries being net exporters of polluting goods, other being net importers).

Theoretical literature has focused exclusively or near-exclusively on the problem of the effect of pollution internalizing on commodity taxation and in particular on the “additivity property”: how are optimal tax formulas modified, and in particular are taxes on polluting goods higher or lower than the Pigovian “marginal environmental damage”? Diverging results, arising obviously from different theoretical representations of the economy, are exhibited in the literature.

Beyond their theoretical aspect, the two problems which are raised are of critical importance in the present political context of world environmental policies, and in particular with respect to the United Nations Framework Convention on Climate Change. On the one hand, it results from statistical data on CO₂ emissions that the largest part (from 70% to 80% according to the countries and the levels of

¹I am indebted to Roger GUESNERIE for comments on a first version of the paper, and to Marc VIELLE for a long collaboration on the model GEMINI-E3, which was at the origin of the present reflections.

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²Prominent papers are from Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Goulder (1995), Bovenberg and Goulder (1996), Fullerton and Metcalf (1997a and 1997b), Ekins (1997), Goulder, Parry and Burtraw (1996), Goulder & Alii (1997 and 1998), Barker (1997), Bohm (1997 and 1998), Hoel (1998) and Schneider (1998).

economic development) originates from production and, in the case of a non-linear technology, cannot be traced directly to final consumption.

On the second hand, due to the obviously worldwide "public bad" character of CO₂ concentration in the atmosphere, coordination of domestic policies is required if the problem is to be addressed efficiently, and this can take the form of an allocation of pollution rights among countries and the implementation of flexible mechanisms such as tradable pollution permits. Equity considerations also can only be tackled within the global framework of general equilibrium between countries³.

In such a global framework, and due to second-best constraints on domestic economies, the resulting picture can be sensibly different from the traditional Pigovian -and first-best- one.

Firstly, at the domestic level, there are two different issues which are pollution taxation for the production sector and pollution taxation ("commodity taxation") for households. What are the relations between these two taxes, and can they be termed "Pigovian"? As it will appear, taking into account taxation of pollution in the production sector rather clarifies the debate on commodity taxation, as we then have a yardstick which is precisely the tax applying to firms.

Secondly, in a second best context, it is not warranted that the social marginal abatement cost be equal to the individual marginal abatement cost, i. e. the pollution tax set on private agents (and in particular on firms) so as to obtain the desired level of emissions or to fully internalize the pollution cost (when the damage of pollution is precisely estimated, which is not presently the case for GHG). Then, the question is on which bases are pollution rights to be traded between countries: pollution taxes (with the result that the same tax applies all over the world) or social marginal abatement costs (so that social marginal abatement costs are equalized all over the world but not, then, domestic pollution taxes)? Obviously, the sensible answer is the second: effectively, why would a country be willing to buy a permit for more than it costs to him (or sell for less)?

An immediate consequence is that, in this context, international trade of permits is to be managed directly by governments, which can in parallel implement their domestic environmental policy either through taxation or through a domestic market of permits (the domestic tax or the equilibrium price of the permit being different from the international price). An alternative way of implementing a world market of tradable permits, where private agents as direct traders, is to set differential taxes (or subsidies) at the domestic level so that the discrepancy between the domestic pollution tax and the international price of permits is corrected.

The paper addresses the two above-mentioned problems and the related issues just evoked. The international dimension requires a new approach which is the analysis of *general equilibrium between second-best economies*. As far as the author knows, this approach has never been followed previously. We will term this type of equilibrium a "competitive exchange equilibrium", understanding that the concerned entities are second-best economies i.e. optimizing domestic governments. A directly related issue is the (second-best) PARETO efficiency of such an equilibrium, the question being also relevant without pollution and pollution internalizing.

As a critical aspect is the possible (and probable) divergence between the pollution tax and the (social) marginal abatement cost, section 2 will review evidence arising from some applied studies and applied models, at the domestic or world levels, in which relations between these two quantities are closely examined. When independent measures are performed, they show a difference which may be

³International markets of tradable permits have been investigated in a first best context by Chichilnisky and Heal (1995), Heal (1997) and Chichilnisky, Heal and Starrett (1998)

significant⁴.

The results which can be expected at the theoretical level depend obviously on the second-best models considered as representations of domestic economies, and in particular on the available fiscal tools and on the constraints put on them.

If there is a sufficiently large array of fiscal tools (excluding, of course, individualized lump-sum transfers which would lead to a first-best optimum), then it can be expected that the management of environmental policies, at the domestic and international levels, closely parallels the first-best Pigovian case (commodity taxation remaining an issue). As section 3 will show, this is in particular the case within the Diamond & Mirrlees paradigm, and this is due to a fundamental property of this model which is "production efficiency" and, correlatively, the complete disconnection between production and consumption prices⁵.

Outside the scope of the Diamond & Mirrlees paradigm and associated assumptions, a very great number of models can be considered, differing by the available fiscal tools and the constraints put on them. Section 4 will deal with a model differing from the D&M in one crucial assumption, which is taxation of profits. In the D&M model, possibility (and in fact effectiveness) of total taxation of profits is assumed and this assumption is decisive for the property of "production efficiency".

Two reasons justify the consideration of such a model. First real fiscal systems all over the world have limited or even zero taxation on profits, and this is probably to be linked to problems of incentive in economies where pure profits are confiscated. The second, and maybe more important, reason is that existing models -either applied models like general equilibrium models, or theoretical-applied models such as for instance the one developed by Bovenberg and Goulder (1996)- assume, explicitly or implicitly, that profits are untaxed or taxed up to a limited rate. It is then important to understand more fully how these models behave and how to interpret their results.

This type of model is in the pure Boiteux tradition⁶, with an explicit representation of production prices (which are endogenous variables and not dual prices, contrary to the D&M case). The "cost" of such a generalization is that only a unique household (either a representative household, or an "aggregate" household represented by a social welfare function) can be taken into account if we want to obtain readable results. Pollution internalizing can then tackle efficiency aspects, not equity considerations as in the case of the D&M model.

In order to distinguish the two cases, notably in the formulation of results and the expression of theorems, the first will be referred as "*when there is a sufficiently large array of fiscal tools*" and the second as "*general case*", though it is not general but represents an example where assumptions of the D&M paradigm are not all fulfilled.

In both sections 3 and 4, numerical simulations are presented in order to give an idea of the behavior of the two models. Finally, section 5 draws the main teachings for policy implementation and suggests desirable further analysis and generalizations.

⁴Thereafter, as there is not ambiguity, marginal abatement cost (MAC in short) will design the social marginal abatement cost, to distinguish from the pollution tax, equal to the individual marginal abatement cost.

⁵For a complete presentation of optimal taxation based on the D&M model, see Guesnerie (1995). Other important contributions are by Atkinson and Stiglitz (1972) and Sandmo (1974). Sandmo (1975) is the first to address the problem of optimal taxation with externalities.

⁶Boiteux (1956). Boiteux deals with shadow taxes or shadow tolls, not effective taxes and tolls. For an application of the Boiteux model to taxation and pricing, see Guesnerie (1975) and Bernard (1977) and (1990).

2 From Empirical Assessment to Theory

Numerous appraisals of environment policies, and particularly the policy of mitigation of GHG emissions, have been implemented either at the domestic level, for a given country, or at a regional or world scale. Most appraisals, and notably regional and world appraisals, have been implemented with theoretical aggregate models or with computable general equilibrium models, usually detailed in some 8 to 15 sectors (including the 5 energy sectors).

2.1 Assessment at the domestic level

Most notable among appraisals performed at the domestic level and not implementing a general equilibrium model is the study conducted by the Energy Information Administration of the American Department of Energy (EIA-DOE, 1998). It is based on a whole array of models, on the one hand the detailed energy models serving for long term forecasts (NEMS), on the other hand a macro-economic model, the Data Resources, Inc. (DRI) model of the U.S. economy.

The study aims at determining, for different levels of CO₂ emissions abatement in energy consumption, and for the years 2010 and 2020, the policy to be implemented and in particular the carbon tax to be instituted in order to reach these targets. It also determines -through the macro-economic model - the global cost of the policy.

In the study, the cost is represented by the change in GDP at constant prices, with two measures of GDP loss: *actual*, meaning that adjustment (transitory) costs are included, and *potential*, i. e. excluding these costs. Results are represented in the two graphs of figure 1, related to respectively year 2010 and year 2020.

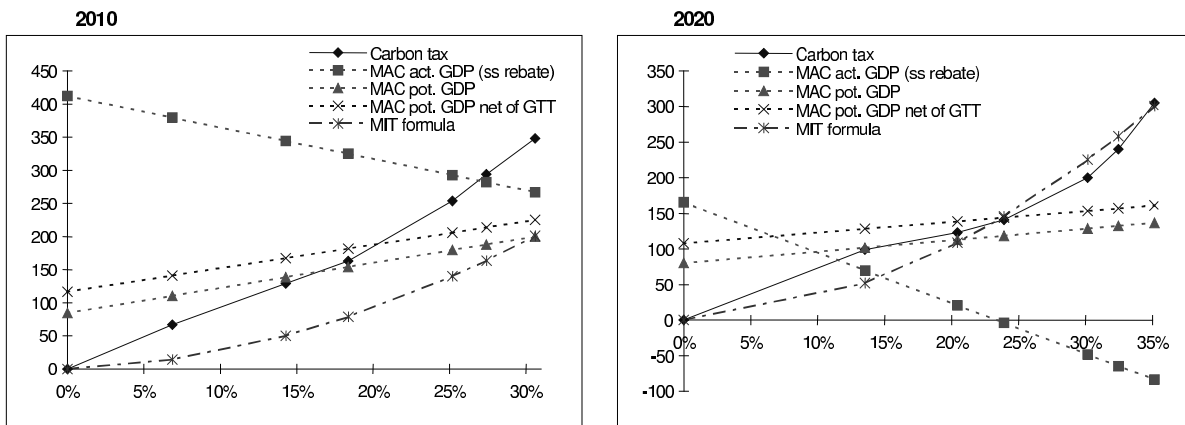


Figure 1

If one considers the long term horizon of 2020 and potential GDP as providing measures of welfare loss closest to theoretical concepts⁷, the graph above clearly shows that the marginal abatement cost differs significantly from the carbon tax. In particular, it is not zero for zero abatement, and becomes smaller than the carbon tax when abatement reaches a high level (about 20 to 25% of baseline emissions).

⁷The change in potential GDP, net of gains from terms of trade (here limited to the decrease of relative price of imported oil and oil products as information on other imports and exports is not available), represents a better measure of the pure abatement cost (see below).

It is evidently possible to question the significance of GDP change as a measure of welfare loss. It is however to be noted that it is considered as a relevant measure by the authors of the study as they consider trade-offs between GDP losses and purchases of pollution permits.

2.2 Appraisals at regional and world levels with general equilibrium models

In these studies, a clear line can be drawn between authors and models considering that carbon tax and marginal abatement cost are equal (eventually synonyms) and those making a difference between them and evaluating both quantities⁸.

In the first camp is the EPPA model of MIT⁹ (and most probably the GREEN model of OECD from which it is derived). Figure 2 shows the values obtained with EPPA for this common quantity also named *shadow price of carbon*¹⁰. In particular, marginal abatement cost is systematically equal to zero for zero abatement.

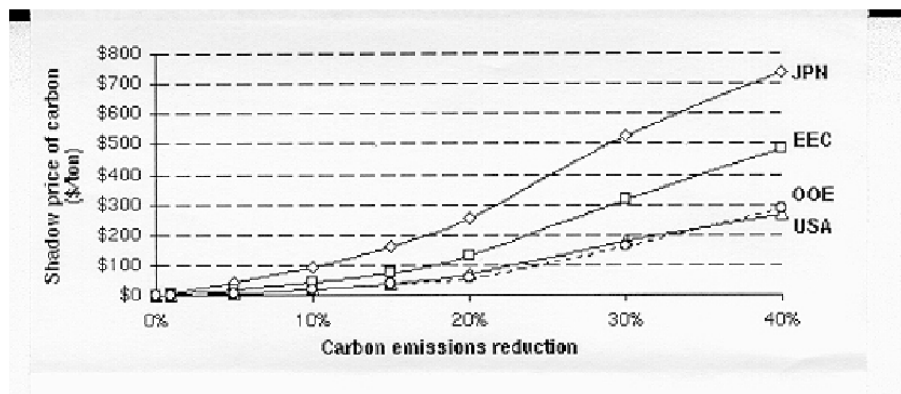


Figure 2

The same approach is followed by the IEA Energy Technology Analysis Programme (Kram, 1998), yielding curves of marginal abatement cost similar to those obtained above, and which in particular exhibit a zero marginal cost in the initial situation.

Measures of the marginal abatement cost, independent of the carbon tax, are derived in several studies, among which the two following. On the one hand, the analysis performed with model CICERO at the Center for International Climate and Environment Research in Oslo (Holtmark, 1998), the results of which are represented in figure 3. Noteworthy is the very big scattering of marginal reduction cost curves, and in particular the "zero value", i. e. the level of abatement for which the marginal abatement cost is zero, which then indicates what should have been the optimal level of energy consumption (and then CO₂ emissions) prior to any environmental concern.

⁸Estimations of costs of abatement policies are reviewed thoroughly in Hourcade, Richels & Robinson, 1996.

⁹Ellerman & Decaux, 1998

¹⁰The formula obtained by MIT for the U.S. has been reported in the two graphs representing the results of the EIA study.

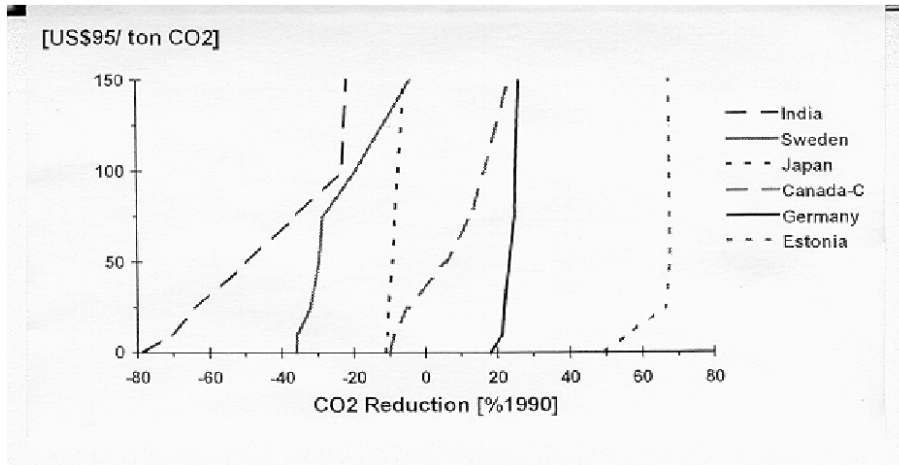


Figure 3

The second appraisal is with the model GEMINI-E3, which determines for each individualized country/region and for each year the marginal abatement cost as the marginal welfare loss or, equivalently, the marginal deadweight loss of taxation (Bernard & Vielle, 1998 and 1999). Results for France, Other European Countries, U. S. and Japan are reproduced in figure 4 relatively to year 2010. Notable -and positive- differences between marginal abatement cost and carbon tax are obtained in particular in France and Japan. As for Other European Countries and United States of America, the difference is much smaller though still positive.

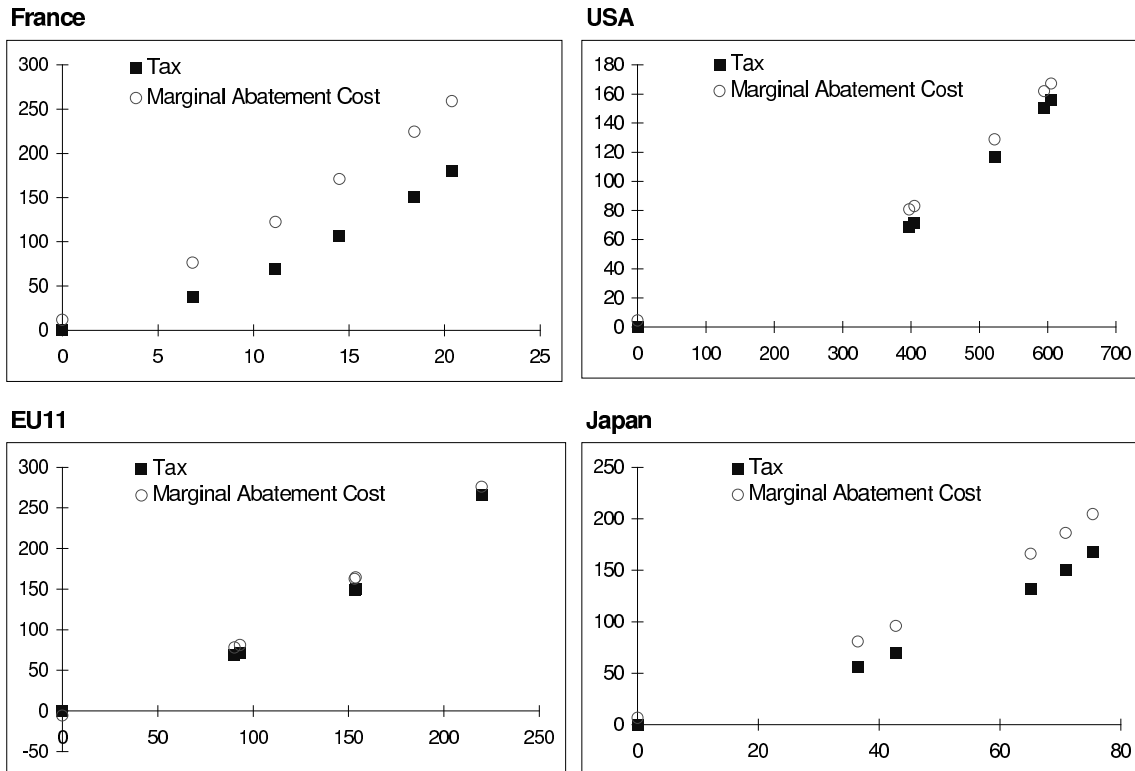


Figure 4

2.3 Theoretical-applied approaches

Possibility of -and reasons for- discrepancy between MAC and pollution tax have been examined by Bernard (1998) in a simple model. Bovenberg and Goulder (1996) have analyzed the effects of fiscal distortions on optimal environmental policy with a theoretical model and implemented numerical simulations. They obtain a difference between pollution tax and marginal abatement cost, which can vary according to the revenue replacement of the environment tax. In the case of "fully optimal tax policies", the discrepancy is small, with a carbon tax lightly higher than the marginal abatement cost, as shows the table below.

Assumed marginal damage	Optimal tax	Marginal abatement cost
25	22	22
50	46	45
75	70	68
100	93	91

On the whole, when they are rigorously calculated, measures of marginal abatement costs show a difference with the carbon tax. This is obvious in real situations which are sub-optimal: when the polluting good is heavily taxed in the initial situation, as is energy in France and Japan in particular for final consumers, a positive marginal abatement cost is to be expected for the first steps of abatement. In the opposite, when energy is highly subsidized, as it was (and still is, concerning households) in former Soviet Union, the marginal abatement cost is negative at the beginning.

What show the simplified theoretical models is that this is also true with an optimized fiscal and environmental policy. It results from the second-best nature of optima, and reflects the effect of non-neutrality of the fiscal system.

3 Pollution Abatement and Permits Trading Within the Diamond & Mirrlees Paradigm

What characterizes the D&M paradigm in second-best theory is the complete disconnection of consumer and producer price-systems and, correlatively, the property of *production efficiency*. The latter requires that pure profits, if not equal to zero (in the case of constant returns to scale), are totally taxed.

Production efficiency has two inter-related meanings: the first is that total production (private and public) is efficient, i. e. that the public sector uses production prices as shadow prices for the provision of public goods. The second is that production prices are equal to social values of goods.

The model determines the optimal tax-system in the Ramsey-Boiteux tradition, and the optimal provision of public goods. Different tax-systems can be considered, linear or non-linear, but general and robust results can be obtained only in the former case. Linear taxation may or may not include a constant (i. e. identical for all consumers) *lump-sum transfer*.

Taxation is obviously necessary for financing public goods, but serves also as a means for welfare redistribution among households. This requires the representation of individual consumers (in this sense, the D&M model is totally micro-economic) and the use of a Social Welfare Function in the Bergson-Samuelson tradition.

Assessing conditions under which pollution permits can be efficiently traded between countries requires a thorough representation of sources of pollution, whether in the production or in the consumption activities. Concerning the former, the most general assumption is that the emitted pollution is, for every firm, an argument in the production function. As for the latter, a reasonable assumption is that emitted pollution is a function of the total vector of final consumption (i. e. not differentiated according to consumers).

Two steps will be successively followed: first determination of optimal taxation in a closed economy; then definition and characterization of a Competitive Exchange Equilibrium, with trade of goods and pollution permits between countries.

3.1 Taxation of pollution in a closed economy

According to the D&M paradigm, we consider an economy with a unique homogeneous factor (labor), n private goods, \widehat{m} consumers (or households) and \widehat{l} firms. Production by private firms is represented by letter y , private consumption by households by letter x , supply of labor by L , demand for labor by private firms by N , and consumption prices by p . When not exogenously set to zero, lump-sum transfer is represented by R (also the "net income", as profits are totally taxed and labor is a negative consumption).

Public good is produced (by the "public sector") with labor and inputs of private goods as factors of production, and consumed by households.

3.1.1 Notations

x_j^i	: consumption of good j by consumer i	$j = 1, n$
y_j^l	: production of good j by consumer l	$j = 1, n$
L^i	: supply of labor by consumer i	
N^l	: demand of labor by firm l	
R	: lump sum transfer	
w	: wage	
p_j	: consumption price of good j ,	$j = 1, n$
Q	: production of public good	
T	: labor demand by the public sector	
z_j	: demand of good j by the public sector	$j = 1, n$

Pollution emitted by firm l is represented by y_0^l , and pollution emitted by consumers (x_0) as a function h of total consumption:

$$x_0 = h \left[\sum_i (x_j^i) \right]$$

Denoting production functions in the private sector by f^l :

$$f^l(y_j^l, N^l, y_0^l) = 0$$

production function of the public good by g :

$$Q = g(z_k, T)$$

and direct utility functions by V^i :

$$V^i = V^i(x_j^i, L^i, Q)$$

supply of labor and demand for goods by households can be determined as functions of consumption prices, wage, income¹¹ and the quantity of public good:

$$\begin{aligned} L^i &= L^i(p_j, w, R, Q) \\ x_j^i &= x_j^i(p_j, w, R, Q) \end{aligned}$$

and, similarly, the indirect utility function:

$$U^i = U^i(p_j, w, R, Q)$$

Social Welfare Function, which is denoted by W :

$$W = W(U^i)$$

is then also a function of consumption prices, income and consumption of the public good.

In order to simplify formalization of the model, the effects of pollution on the economy are not represented and, correlatively, pollution abatement is supposed to be exogeneously decided. When effects of pollution are to be taken into account, for instance in order to determine the optimal abatement policy or to calculate the marginal pollution damage, an adequate general assumption is that total pollution enters each utility and each production function.

3.1.2 Optimal fiscal policy and provision of public goods

Optimal fiscal policy and provision of public goods are determined by the resolution of the following program where, without loss of generality, w is set equal to 1:

$$\begin{aligned} &\max W(U^i) \\ \text{under the constraints :} \\ (\omega) \quad &\sum_l N^l + T - \sum_i L^i = 0 \\ (\pi_j) \quad &\sum_i x_j^i + z_j - \sum_l y_j^l = 0 \quad j = 1, n \\ (\varphi^l) \quad &f^l(y_j^l, N^l, y_0^l) = 0 \\ (\psi) \quad &Q - g(z_j, T) = 0 \\ (\theta) \quad &\sum_l y_0^l + h [\sum_i (x_j^i)] \leq \bar{E} \end{aligned}$$

Conditions of optimality directly result:

¹¹ Assuming the usual convexity properties which insure unicity and derivability of demand.

$$(p_j) \quad \sum_i \frac{\partial W}{\partial U^i} \frac{\partial U^i}{\partial p_j} = \omega \frac{\partial L^i}{\partial p_j} + \sum_k \left[(\pi_k + \theta h_k) \sum_i \frac{\partial x_k^i}{\partial p_j} \right] \quad j = 1, n$$

$$(R) \quad \sum_i \frac{\partial W}{\partial U^i} \frac{\partial U^i}{\partial R} = \omega \frac{\partial L^i}{\partial R} + \sum_k \left[(\pi_k + \theta h_k) \sum_i \frac{\partial x_k^i}{\partial R} \right]$$

$$(N^l) \quad \omega + \varphi^l \frac{\partial f^l}{\partial N^l} = 0$$

$$(y_j^l) \quad \pi_j - \varphi^l \frac{\partial f^l}{\partial y_j^l} = 0 \quad j = 1, n$$

$$(y_0^l) \quad \theta + \varphi^l \frac{\partial f^l}{\partial y_0^l} = 0 \quad j = 1, n$$

$$(Q) \quad \sum_i \frac{\partial W}{\partial U^i} \frac{\partial U^i}{\partial Q} = \psi - \sum_i \left[\omega \frac{\partial L^i}{\partial Q} - \sum_{k=1, n} (\pi_k + \theta h_k) \frac{\partial x_k^i}{\partial Q} \right]$$

$$(T) \quad \psi \frac{\partial Q}{\partial T} = \omega$$

$$(z_j) \quad \psi \frac{\partial Q}{\partial z_j} = \pi_j \quad j = 1, n$$

Note that, when there is no possible lump sum transfer in the economy, R is to be set to zero and the corresponding dual condition vanishes.

Interpretation of the dual is straightforward:

- shadow prices ω , π_j and θ which measure the social values of the related goods (labor, private good j and pollution) are equal (up to a multiplicative factor) to equilibrium production prices, and the property of production efficiency holds as show the dual conditions (N^l) , (y_j^l) and (y_0^l) ;
- in particular the optimal tax to be set on pollution in the production sector is equal to the social value of pollution, and as in the first best case, there is identity between the tax on pollution and the social marginal cost of abatement (MAC);
- taxes on final consumption are determined by dual conditions (p_j) and (R) , which incorporate the effects of pollution emitted by final consumption. "Additivity property is verified" in the sense that only tax formulas for externality generating goods (i. e. those for which h_k is non null) are altered.

However, the most natural interpretation is to consider production prices incorporating the pollution cost generated by (future) final consumption as *opportunity costs* of the related goods, and then to distinguish in total commodity taxation two components, the pollution tax (equal to the pollution cost) and the pure taxation:

pollution cost of final consumption	:	θh_k
production price including pollution cost	:	$\pi'_k = \pi_k + \theta h_k$
pure commodity tax	:	$p_k - \pi'_k$

Pure tax formulas, when incorporating the pollution cost in production prices, are then identical to those holding in the general case without externality.

The same result holds concerning the social value of the public good:

$$\psi = \sum_i \frac{\partial W}{\partial U^i} \frac{\partial U^i}{\partial Q} + \sum_i \left[\omega \frac{\partial L^i}{\partial Q} - \sum_{k=1,n} (\pi_k + \theta h_k) \frac{\partial x_k^i}{\partial Q} \right]$$

the determination of which is obtained by the usual formula, production prices being replaced by opportunity costs of goods as defined above.

3.2 Competitive Exchange Equilibrium

A Competitive Exchange Equilibrium is classically defined as the result, under the condition of clearing of all exchange markets, of competitive behavior by agents. In the present case, economic agents are countries (governments) who optimize social welfare under a set of constraints, including second-best fiscal constraints.

Competitive behavior means that countries are price-takers, and trade in international markets according to the market prices. The market prices include prices of tradable goods and the price of tradable pollution permits: taking any good (here good 1) as an international numéraire, market prices are prices relative to the numéraire, as defined by notations below:

τ_j	:	relative price of good j in foreign trade	$j = 2, n$
r	:	relative price of pollution permit	
m_j	:	net demand of import for good j	$j = 2, n$
v	:	net sales of tradable permits	

Imports of good 1 are determined by the balance constraint of foreign trade:

$$m_1 = - \sum_{k=2}^n \tau_k m_k + rv$$

3.2.1 Competitive demand by a country

The competitive demand by a country is then defined by an optimization program, which is identical to the case of a closed economy, with the difference that trade possibilities are taken into account in domestic supply-demand equilibria of tradable goods and pollution permits. Constraints (π_1) , (π_j) , and (θ) are then replaced by:

$$(\pi_1) \quad \sum_i x_1^i + z_1 - \sum_l y_1^l + \sum_{k=2}^n \tau_k m_k - rv = 0$$

$$(\pi_j) \quad \sum_i x_j^i + z_j - \sum_l y_j^l - m_j = 0 \quad j = 2, n$$

$$(\theta) \quad \sum_l y_0^l + h \left[\sum_i (x_j^i) \right] + r \leq \bar{E}$$

Previous dual conditions are unchanged, and completed by n additional ones related to variables m_j and v :

$$(m_j) \quad \pi_j - \tau_j \pi_1 = 0 \quad j = 2, n$$

$$(v) \quad \theta - r\pi_1 = 0$$

which show that the considered country aligns the domestic production prices of tradable goods and the domestic pollution tax on international prices.

As for tax formulas, they are determined by the same system of equations than previously.

3.2.2 Competitive exchange equilibria

Excess demand for tradable goods 2 to n and for tradable permits depend on n relative prices. A competitive exchange equilibrium exists if there are relative prices such as all international markets clear. Existence of a competitive exchange equilibrium is not warranted in any circumstance, because of existing non-convexities, but can be expected in "reasonable" situations. Numerous numerical simulations realized in a simple model (see below) did not show any difficulty in obtaining an equilibrium.

Properties of such an equilibrium are of highest interest. In the present case, the main property obtained is *that production efficiency holds at the world level and, correlatively, the same pollution tax is applied in all countries*. This results from the fact that the pollution tax is equal, in each country, to the social marginal abatement cost, and that market competitive mechanisms equalize social marginal abatement costs between countries.

PARETO efficiency is also an important aspect: production efficiency at the world level is a necessary condition for PARETO efficiency, but not a sufficient one. It remains to check that allocation of net production among countries (and in each country among households) is efficient, taking into account existing constraints.

3.2.3 Second-Best PARETO Efficiency

A competitive exchange equilibrium is determined by the tax system set prevailing in each country and by transfers among countries, equal to the net receipts accruing from sales of tradable permits. Such an equilibrium would not be Second Best PARETO Efficient if there existed other domestic tax systems (consistent with constraints put on them) and other transfers such as the SWF of one country could be increased without decreasing the SWF of any other country, under the constraint that world exchanges of tradable goods and of pollution permits are balanced.

A Second Best PARETO Optimum can then be characterized by the following program (with index c denoting the concerned country, $c = 1, \bar{c}$; and d^c the net transfer received by country c):

$$\begin{aligned}
& \max \sum_c \beta^c W^c(U^{ic}) \\
\text{under the constraints :} \\
(\omega^c) \quad & \sum_l N^{lc} + T^c - \sum_i L^{ic} = 0 \quad c = 1, \bar{c} \\
(\pi_1^c) \quad & \sum_i x_1^{ic} + z_1^c - \sum_l y_1^{lc} + \sum_{k=2}^n \tau_k m_k^c - d^c = 0 \quad c = 1, \bar{c} \\
(\pi_j^c) \quad & \sum_i x_j^{ic} + z_j^c - \sum_l y_j^{lc} - m_j^c = 0 \quad c = 1, \bar{c}; j = 2, n \\
(\varphi^{lc}) \quad & f^{lc}(y_j^{lc}, N^{lc}, y_0^{lc}) = 0 \quad c = 1, \bar{c} \\
(\psi^c) \quad & Q^c - g^c(z_j^c, T^c) = 0 \\
(\nu_j) \quad & \sum_c m_j^c = 0 \quad j = 2, n \\
(\mu) \quad & \sum_c d^c = 0 \\
(\theta) \quad & \sum_{l,c} y_0^{lc} + \sum_c [h^c(\sum_i(x_j^{ic}))] \leq \sum_c \bar{E}^c
\end{aligned}$$

This program can be simplified by eliminating variables of imports: reporting $m_j^c = \sum_i x_j^{ic} + z_j^c - \sum_l y_j^{lc}$ in (π_j^c) and $d^c = \sum_i x_1^{ic} + z_1^c - \sum_l y_1^{lc} + \sum_{k=2}^n \tau_k m_k^c$ in (μ) , and taking into account constraints (ν_j) substitutes to constraints (π_1^c) , (π_j^c) , (ν_j) and (μ) the aggregate ones:

$$(\pi_j) \quad \sum_c [\sum_i x_j^{ic} + z_j^c - \sum_l y_j^{lc}] = 0 \quad j = 1, n$$

stating that tradable goods markets clear at the world level.

It is easy to check that optimality conditions split in independent sub-sets identical to the one obtained previously for each country, with the additional condition that social values of the numéraire are equal across countries:

$$\boxed{\bar{\pi}_1^c = \bar{\pi}_1}$$

A Competitive Exchange Equilibrium is then Second-Best PARETO Efficient: the previous condition can effectively be obtained by adjusting relative weights $\frac{\beta^c}{\beta^1}$ (which, at the same time, determines them).

Reciprocally, a Second-Best PARETO Optimum is a Competitive Exchange Equilibrium. Imports of goods and sales of permits are determined by the relations:

$$\begin{aligned}
m_j^c &= \sum_i x_j^{ic} + z_j^c - \sum_l y_j^{lc} \\
d^c &= \sum_i x_1^{ic} + z_1^c - \sum_l y_1^{lc} + \sum_{k=2}^n \tau_k m_k^c
\end{aligned}$$

Sales of permits are equal to the difference between pollution rights and effective emissions:

$$v^c = \bar{E}^c - y_0^{lc} - h^c \left(\sum_i x_j^{ic} \right)$$

and it can be checked that, in every country:

$$d^c = \frac{\theta}{\pi_1} v^c$$

which states that transfers are equal to the receipts accruing from the sale of tradable permits with a price:

$$r = \frac{\theta}{\pi_1}$$

i.e. the marginal abatement cost.

3.2.4 Marginal environmental damage and optimal abatement policy

The essence of the Pigovian rule being that the pollution tax equalizes the marginal damage, the latter has to be clearly calculated. The expression of marginal damage results from the condition of optimality related to total abatement \bar{E} , then considered as an endogenous variable.

With the general assumptions above-mentioned on the effects of pollution on economic activity, the optimality condition can be written, in the case of a closed economy, under the form:

$$\theta = - \sum_i \frac{\partial W}{\partial U^i} \frac{\partial U^i}{\partial \bar{E}} + \sum_i \left[\sum_{k=1, n} (\pi_k + \theta h_k) \frac{\partial x_k^i}{\partial \bar{E}} - \omega \frac{\partial L^i}{\partial \bar{E}} \right] + \sum_l \varphi^l \frac{\partial f^l}{\partial \bar{E}}$$

which then yields, in the second-member of the relation, the expression of the marginal environmental damage.

Then the Pigovian rule applies in its full scope: pollution tax is equal to the marginal environmental damage (adequately defined) and to the marginal abatement cost.

In the case of an open economy, it is straightforward to verify that, in a second-best PARETO efficient situation, the marginal abatement cost θ is equal to the sum of willingness to pay by countries. Then, a Lindhal-type market for the environmental public good "pollution abatement" would be second-best PARETO efficient.

The implementation of such a market, together with the market of tradable permits, would require that pollution rights are allocated among countries.

3.3 Summary of main conclusions

The main theoretical results obtained in the section can be summarized by the following theorems.

Theorem 1. *When there is a sufficiently large array of fiscal tools, the Pigovian rule applies to the production sector. The pollution tax is equal to the marginal abatement cost, and represents the willingness to pay by the given country for pollution rights in a market of tradable permits.*

Theorem 2. *When there is a sufficiently large array of fiscal tools, pollution emitted by households' final consumption does not change the pure tax formulas, but only the reference prices for representing the opportunity costs of consumption goods. The latter are equal to the sum of production price and of marginal cost of pollution, the final consumption is responsible of. Total tax on commodity is the sum of the marginal cost of pollution and of the pure tax.*

Theorem 3. *When in every country there is a sufficiently large array of fiscal tools, a competitive exchange equilibrium with tradable pollution permits is Second-Best PARETO Efficient, and production efficiency holds in every country and at the world level. Efficient abatement at the world level can be reached through a Lindhal-type market for the environmental public good "pollution abatement", where demand by each country is determined consistently to his willingness to pay (equal to marginal environmental damage).*

Thus, within the D&M paradigm, the optimal environmental policy closely parallels (at the domestic and at the world level) the first-best strategy, at least for what concerns the production sector. Differences appear relatively to commodity taxation, and to the determination of the marginal environmental damage, but they are consistent with general results and formulas of the D&M paradigm.

Efficiency of the market mechanism does not necessarily mean that the global cost of abatement is "equitably" shared between countries. This would require either compensative transfers or an adequate allocation of pollution rights between countries.

In particular, with the rule of "proportional abatement", which tends to be more or less followed (at least among industrialized countries) it is possible that, in comparison to the initial situation, some countries benefit while others bear a heavy burden. This results from the fact that changes in the structure of final demand (less demand for polluting goods and more for non-polluting goods) modify the structure of relative prices in international trade and generate gains or losses from changes in the terms of trade.

Turning back to the model of competitive demand, we can express the change in welfare¹² of a country when the level of abatement is increased and the (relative) prices of international trade are modified with the formula:

$$\delta W = \theta \delta \bar{E} + \sum_{k=2}^n m_k \delta \tau_k$$

The first term is negative (and measures the Deadweight Loss of Taxation), while the second is positive for some countries and negative for other countries¹³. If the gain from terms of trade is high enough, it can counterbalance the DWL and then increase welfare in the considered country.

Another aspect of welfare allocation is the domestic one: households/consumers can "inequally" (but not "inequitably" since the change is consistent with the Social Welfare Function supposed to incorporate equity targets) bear the cost of abatement, some benefiting and some loosing.

3.4 Numerical simulations

Numerical simulation have been performed with a two tradable commodities, two categories of households, two countries model. What differentiates: i) *the goods*, is that the first is polluting (in final consumption, both being polluting in production); ii) *the households*, is that they have different skills in labor, and different elasticities of demand with respect to the polluting good; and iii) *the countries*, is that the first is net importer of the polluting good and the second net exporter.

Specifications of production and utility functions and numerical values of the initial situation are given in Appendix. They don't aim at representing precisely a real situation, but only at exhibiting

¹²In practise, such a relation can be used to determine social marginal abatement costs of different countries in a World General Equilibrium Model.

¹³In applied models this formula, which is general, can be used to calculate easily marginal abatement costs.

some (non obvious) results which can be obtained. Simulations are realized with different scenarios regarding availability of fiscal tools (with or without lump sum transfer) and behavioral assumptions:

- scenario type I: highly skilled households have a high price elasticity of demand with respect to the polluting good;
- scenario type II: highly skilled households have a low price elasticity of demand with respect to the polluting good.

3.4.1 Pollution tax and pure commodity taxation in a closed economy

Results are given in tables 22 and 23 of Appendix, the first corresponding to the case with lump sum transfer and the second to the case without. They are represented graphically in figures 5 and 6. We can observe that:

- the relative production price of the polluting good decreases rapidly with the rate of abatement;
- the *total rate of taxation* on the polluting good increases rapidly with the rate of abatement, but this is not necessarily the case for the *pure rate of taxation* which may decrease faster than the rate of taxation on the other good;
- abatement benefits, in the early stages of abatement, to households who have the higher price-elasticity of demand for the polluting good.

We can also note that, with the assumptions made on pollution generated by production (proportional to production for each good, with different rates of emissions), it is possible that there exists no equilibrium with a non-null wage in the production sector. This is the case without lump sum transfer, when the rate of abatement reaches a high level (exceeds 16% to 18% according to the scenarios). From then on, only fiscal equilibria with unemployment are possible.

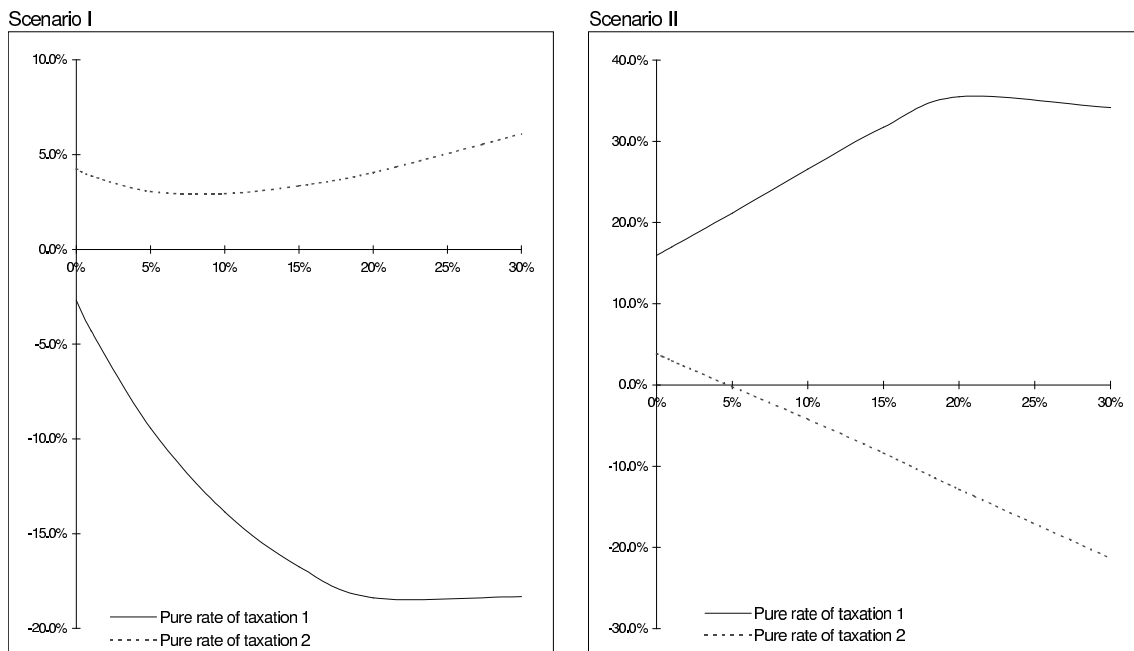


Figure 5

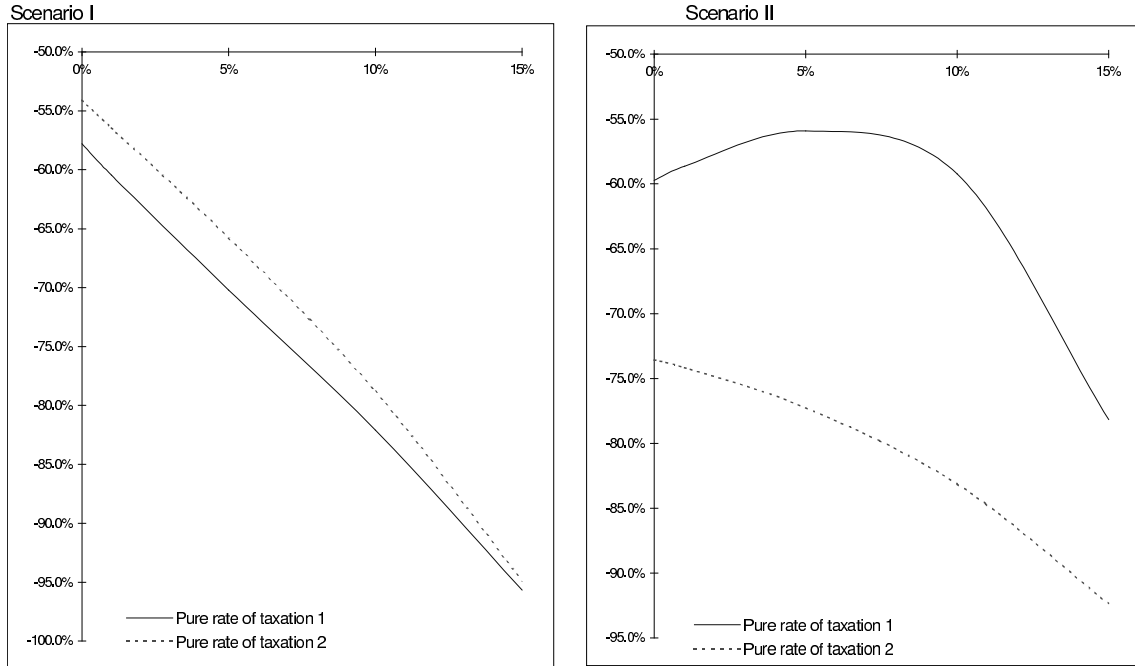


Figure 6

3.4.2 Competitive equilibria with and without trade of pollution permits

Table 24 of Appendix yields the numerical results of simulating a 5% proportional abatement of pollution emissions, without and with tradable permits, compared to the initial situation and to a situation in which the global abatement is allocated between countries in order to equalize social marginal costs of abatement (iso-MAC allocation). Only the simulation concerning the case without lump sum transfer is presented, the results of the other case being very similar. The results are represented graphically in figure 7.

It can be noted that country 1, which is net importer of the polluting good, benefits from pollution abatement: this reflects the effect of terms of trade, and the corresponding gain accruing to country 1.

Another interesting result is that trade of pollution permits (compared to the situation without trade) does not necessarily benefit to all countries. In the two scenarios, country 1 loses while country 2 gains.

Finally comparison of the equilibrium with tradable permit (and proportional allocation of pollution rights) and the iso-MAC allocation shows that effective abatements by countries are approximatively the same: the difference is in the transfer represented by the sales of permits and the associated gain and loss of welfare.

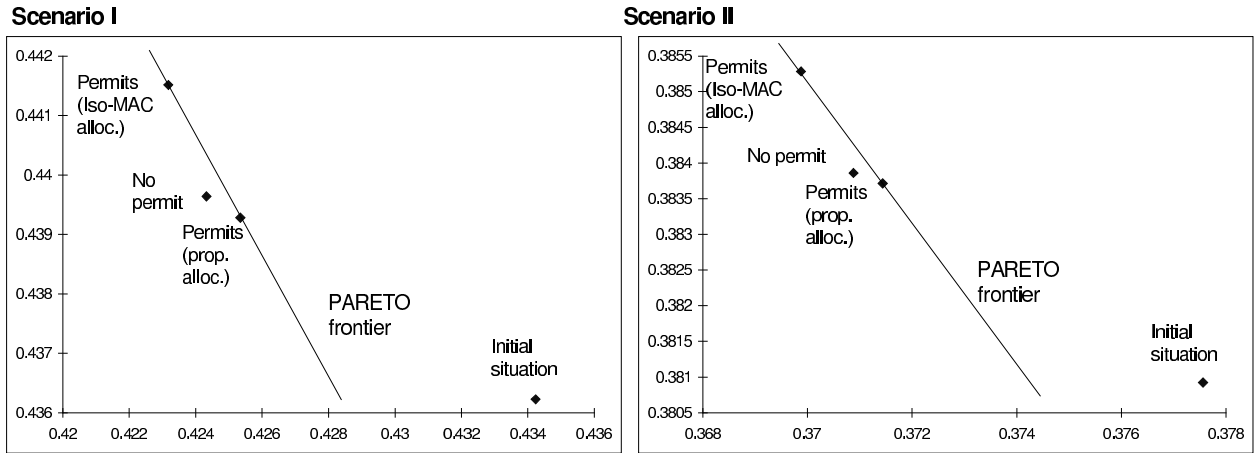


Figure 7

4 Pollution Abatement and Permits Trading With Constraints on Taxation

Compared to the pure D&M paradigm, different types of constraints can be put on taxation in real situations, either for technical reasons or for political -eventually ethical- considerations. They prevent realization of production efficiency and, correlatively, the desirability of taxing externalities according to their (social) marginal abatement costs.

One constraint which appears realistic is a limitation put on the rate of profit taxation (or eventually zero profit taxation). In such a model, which is in the BOITEUX tradition with an explicit representation of the government budget constraint, all prices for private agents - whether consumption or production prices - are explicit variables. Except for the public good(s), provision of which is under control of the government, supply by firms is expressed in function of production prices. The assumption that firms are price-takers allows to aggregate them under a single production function. Then, in the case of an open economy, production prices of tradable goods are set by international markets.

Concerning households, they are also aggregated under a single utility function, which can be considered either as the utility function of a representative consumer/household, or as a Social Welfare Function.

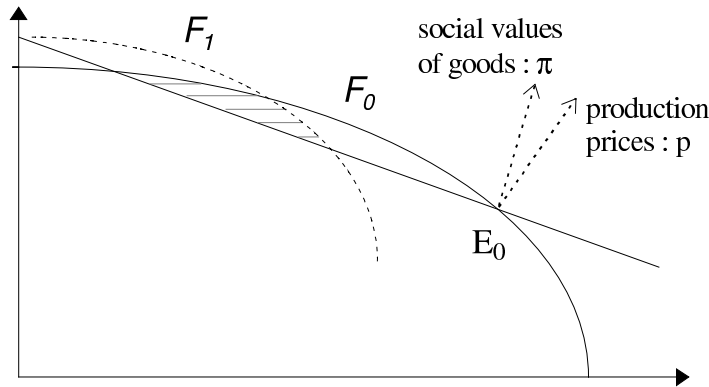
As for taxation of profits, it is unambiguously defined only once the numéraire (and untaxed good) has been selected; consistently to tradition, it is convenient to choose labor. As in the previous section, the cases of a closed economy and of a competitive exchange world economy will be examined successively.

4.1 Constraints on taxation and pollution internalizing in a closed economy

Imposing an additional constraint to the economy should normally -and is usually expected to- result in a welfare loss. It is however one paradox (among several) of second-best theory that this is not necessarily true. The reason is directly linked to the property of production efficiency: the fact that this property does not hold (in most circumstances) means that some production plans may be more desirable than the second-best optimal one, even though they are not efficient (i. e. on the produc-

tion frontier). If the additional constraint push in the direction of one of these plans, welfare will be increased and not decreased.

Effectively, when there is no environmental concern, the behavior of firms is to maximize profit with a zero opportunity cost on pollution emissions. At the optimum, emissions are such that the marginal productivity (marginal change in profits) is zero. This determines the corresponding production frontier F_0 of the economy as represented in figure 8.



The hatched area contains production plans which are less profitable than E_0 relatively to p but more profitable relatively to π

Figure 8

But as the system of social values of goods differs from production prices, it may be desirable that the economy works along another production frontier F_1 , corresponding to a different level of emissions¹⁴. Thus, if the government is able to enforce a different level of emissions, in particular through taxation, and even though this is not dictated by any environmental concern, it is generally possible to obtain a welfare gain.

If this level is smaller, we then observe a phenomenon of "double-dividend", *i. e. an increase in welfare with simultaneously a decrease in emissions*. The fundamental reason of such a result is that abating emissions is a biased way to put the economy closer to the production plan which is most desirable with respect to social values of goods.

A simulation has been performed with specifications and data identical to those which will be considered in the next section (except that foreign trade is zero). The results are presented in the graphs of figure 9 which clearly show that welfare (not taking into account any environmental gain) is maximized for a level of abatement of 2.5%, with a positive pollution tax and evidently a zero marginal abatement cost¹⁵.

¹⁴Which may eventually be dominated by the production set F_0 , as it will be the case in the numerical application.

¹⁵Condition of optimality is effectively than the marginal cost of change in "pollution" emissions be equal to zero.

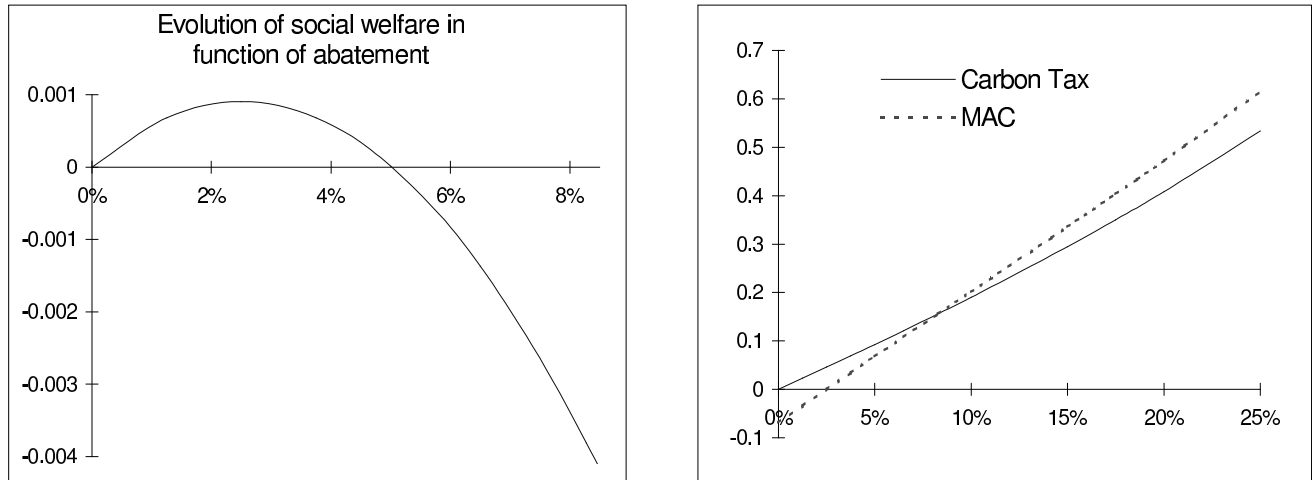


Figure 9

4.2 Constraints on taxation and pollution internalizing in an open economy

An important technical difference with the D&M paradigm is that now the production prices are explicit variables of the model, not just dual variable resulting from optimization. Additional notations are then:

- q_j : producer price of good j $j = 1, n$
- q_0 : pollution tax for the private production sector
- Π : profits of the private production sector
- F : rate of taxation on profits

However, the approach for determining and characterizing the market equilibrium between second-best economies mimics the previous one, consisting first in determining competitive demands by countries, then in defining competitive exchange equilibria and checking their PARETO efficiency.

4.2.1 Competitive demands by countries

Similarly to the previous case, the competitive net demand of tradable goods and pollution permits by a given country is determined by the following optimization program:

max U
under the constraints :

$$(\psi) \quad Q - g(z_k, T)$$

$$(\omega) \quad N + T - L = 0$$

$$(\pi_1) \quad x_1 + z_1 - y_1 + \sum_{j=2,n} \tau_j m_j - rv = 0$$

$$(\pi_j) \quad x_j + z_j - y_j - m_j = 0 \quad j = 2, n$$

$$(\rho_j) \quad q_j - \tau_j q_1 = 0 \quad j = 2, n$$

$$(\varphi) \quad \sum_{k=1,n} (p_k - q_k)x_k - q_0 y_0 + F\Pi + q_1 rv - \sum_{k=1,n} q_k z_k - T = 0$$

$$(\theta) \quad y_0 + h(x_k) + v \leq \bar{E}$$

Utility and net demand of households is function of consumption prices p_j , income R and consumption of the public good Q . Similarly supply by the private sector is function of production prices q_j and pollution tax q_0 (and evidently of the wage, which is set to unity, and then is not an explicit variable).

Π represent profits of the private sector, and is equal to:

$$\Pi = \sum_{k=1,n} q_k y_k + q_0 y_0 - wN$$

4.2.2 Conditions of optimality

They can be written in the following form:

$$(p_j) \quad \frac{\partial U}{\partial p_j} = -\omega \frac{\partial L}{\partial p_j} + \sum_{k=1,n} \pi_k \frac{\partial x_k}{\partial p_j} + \varphi \left[x_j + \sum_{k=1,n} (p_k - q_k) \frac{\partial x_k}{\partial p_j} \right] + \theta \sum_{k=1,n} h_k \frac{\partial x_k}{\partial p_j} \quad j = 1, n$$

$$(R) \quad \frac{\partial U}{\partial R} = -\omega \frac{\partial L}{\partial R} + \sum_{k=1,n} \pi_k \frac{\partial x_k}{\partial R} + \varphi \left[\sum_{k=1,n} (p_k - q_k) \frac{\partial x_k}{\partial R} \right] + \theta \sum_{k=1,n} h_k \frac{\partial x_k}{\partial R}$$

$$(q_1) \quad \omega \frac{\partial N}{\partial q_1} - \sum_{k=1,n} \pi_k \frac{\partial y_k}{\partial q_1} - \sum_{l=2,n} \tau_l \rho_l - \varphi(x_1 + z_1) - \varphi q_0 \frac{\partial y_0}{\partial q_1} + \varphi F \frac{\partial \Pi}{\partial q_1} + \varphi r v + \theta \frac{\partial y_0}{\partial q_1} = 0$$

$$(q_j) \quad \omega \frac{\partial N}{\partial q_j} - \sum_{k=1,n} \pi_k \frac{\partial y_k}{\partial q_j} + \rho_j - \varphi(x_j + z_j) - \varphi q_0 \frac{\partial y_0}{\partial q_j} + \varphi F \frac{\partial \Pi}{\partial q_j} + \theta \frac{\partial y_0}{\partial q_j} = 0 \quad j = 2, n$$

$$(q_0) \quad \omega \frac{\partial N}{\partial q_0} - \sum_{k=1,n} \pi_k \frac{\partial y_k}{\partial q_0} - \varphi y_0 - \varphi q_0 \frac{\partial y_0}{\partial q_0} + \varphi F \frac{\partial \Pi}{\partial q_0} + \theta \frac{\partial y_0}{\partial q_0} = 0$$

$$(Q) \quad \frac{\partial U}{\partial Q} = \psi - \omega \frac{\partial L}{\partial Q} + \sum_{k=1,n} \pi_k \frac{\partial x_k}{\partial Q} + \varphi \sum_{k=1,n} (p_k - q_k) \frac{\partial x_k}{\partial Q} + \theta \sum_{k=1,n} h_k \frac{\partial x_k}{\partial Q}$$

$$(T) \quad \psi \frac{\partial Q}{\partial T} = \omega - \varphi$$

$$(z_j) \quad \psi \frac{\partial Q}{\partial z_j} = \pi_j - \varphi q_j \quad j = 1, n$$

$$(m_j) \quad \tau_j \pi_1 - \pi_j = 0 \quad j = 2, n$$

$$(v) \quad -r \pi_1 + q_1 r \varphi + \theta = 0$$

This set of conditions of optimality appears fairly complex, as it involves several price systems, market prices and social values of commodities. It can however be split in four groups:

- relations (m_j) , (q_1) , (q_j) and (q_0) which determine production prices;
- relations (p_j) and (R) which determine the consumption prices, and then taxes on final demand;
- relations (Q) , (T) and (z_j) which determine the provision and the factors of production of the public good;
- relation (v) , the resolution of which determines the relation between the social marginal abatement cost and the market price of permits.

Market prices and social values Besides market prices of production and consumption, the model exhibits dual prices of goods and factors, ω for labor and π_j for private goods.

Dual prices do not represent exactly social values of goods, which have to be incremented by a correcting factor taking into account the marginal cost of the public funds φ . Effectively, any good which

is received from outside (for instance in payment of pollution permits) increases available resources - as constrained in relations (π_j) -, but also alleviates the budget constraint. Then social values of goods are:

$$s_j = \pi_j - q_j\varphi$$

and similarly, the social value of labor is $\omega - \varphi$.

Relations (ρ_j) together with (m_j) show that production prices of tradable goods are proportional to social values. Setting:

$$\pi_j = \pi q_j$$

it comes:

$$s_j = (\pi - \varphi)q_j$$

Then $(\pi - \varphi)$ is the factor of proportionality between production prices and net social values. When $\omega = \pi$, the same factor of proportionality holds for labor, and this is the *necessary and sufficient condition for production efficiency*.

Marginal social cost of abatement Relation (v) can be written:

$$r = \frac{\theta}{q_1(\pi - \varphi)}$$

showing that the social marginal abatement cost (deflated by the net social value of the numéraire) exactly adjusts to the market price of permit (deflated by the price of the numéraire): *countries are then willing to exchange permits at a price which is exactly the marginal abatement cost*.

Production prices and pollution tax Detailed resolution needs to simplify, taking into account the relations existing between derivatives of supply. Reporting $\pi_j = \pi q_j$ in (q_1) , (q_j) and (q_0) , and taking into account the following identities:

$$\sum_{k=1,n} q_k \frac{\partial y_k}{\partial q_j} + q_0 \frac{\partial y_0}{\partial q_j} = N_j$$

$$\sum_{k=1,n} q_k N_k + q_0 N_0 = -N_w$$

with:

$$N_k = \frac{\partial N}{\partial q_k}, \quad k = 0, n$$

$$N_w = \frac{\partial N}{\partial w}$$

and:

$$\frac{\partial \Pi}{\partial q_j} = y_j, \quad k = 0, n$$

yields:

$$(\omega - \pi)N_1 - \varphi(x_1 + z_1 - Fy_1 - rv) + [q_0(\pi - \varphi) + \theta] \frac{\partial y_0}{\partial q_1} - \sum_{l=2,n} \tau_l \rho_l = 0$$

$$(\omega - \pi)N_j - \varphi(x_j + z_j - Fy_j) + [q_0(\pi - \varphi) + \theta] \frac{\partial y_0}{\partial q_j} + \rho_j = 0$$

$$(\omega - \pi)N_0 - \varphi(1 - F)y_0 + [q_0(\pi - \varphi) + \theta] \frac{\partial y_0}{\partial q_0} = 0$$

Multiplying respectively by q_1 , q_j and q_0 and summing up gives the following relation:

$$-(\omega - \pi)N_w - \varphi \left[\sum_{k=1,n} q_k(x_k + z_k - Fy_k) + (1 - F)q_0y_0 - q_1rv \right] + [q_0(\pi - \varphi) + \theta] N_0 = 0$$

i. e., taking into account relations (π_1) and (π_j) :

$$-(\omega - \pi)N_w - \varphi(1 - F)V + [q_0(\pi - \varphi) + \theta] N_0 = 0$$

with:

$$V = \sum_{k=1,n} q_k y_k + q_0 y_0$$

V is the value of production (or value added).

The production price-system is then determined by the two relations:

$$\begin{array}{l} -(\omega - \pi)N_w - \varphi(1 - F)V + [q_0(\pi - \varphi) + \theta] N_0 = 0 \\ (\omega - \pi)N_0 - \varphi(1 - F)y_0 + [q_0(\pi - \varphi) + \theta] y_{00} = 0 \end{array}$$

If profits are totally taxed, the solution of this system is:

$$\begin{array}{l} \omega = \pi \\ \theta = -q_0(\pi - \varphi) \end{array}$$

(under the condition that $N_w y_{00} \neq (N_0)^2$, i. e. the production function is not degenerate). *This is exactly the D&M case (with a single consumer) and then production efficiency holds, the second relation stating that the pollution tax is equal to the social marginal abatement cost.*

Production efficiency also holds when φ is null. This eventually happens for high levels of pollution abatement: carbon taxes then become sufficient for financing total public outlays, without any distortionary taxation.

A contrario, in the general case, and particularly when initial taxation is highly distortionary (with an high value of the MCPF), production efficiency does not hold and, consequently, the domestic pollution tax differs from the price of permits.

Implementation of the optimal abatement policy thus requires:

- either that governments are the traders in the market of pollution permits (they buy or sell permits, according to their endowment of pollution rights and to their marginal abatement cost). They can then allocate pollution domestic rights through domestic taxation or a domestic market of permits;
- either that firms are the traders in the market of pollution permits, domestic governments adjusting to the optimal cost to be borne by firms through a corrective tax or subsidy, specific to each country.

Consumption prices and commodity taxation As usual in the BOITEUX style approach, a first transformation is to switch to compensated demand derivatives. Multiplying (R) by x_j and adding to (p_j), and taking into account the identity:

$$\frac{\partial U}{\partial p_j} + x_j \frac{\partial U}{\partial R} = 0$$

allows to write optimality conditions (p_j) in the following way:

$$-\omega L_j + \sum_{k=1,n} \pi_k x_{kj} + \varphi \left[x_j + \sum_{k=1,n} (p_k - q_k) x_{kj} \right] + \theta \sum_{k=1,n} h_k x_{kj} = 0$$

with:

$L_j = \frac{\partial L}{\partial p_j} + x_j \frac{\partial L}{\partial R}$, $j = 1, n$ (compensated derivative of labor supply with respect to price of good j)

$x_{kj} = \frac{\partial x_k}{\partial p_j} + x_j \frac{\partial x_k}{\partial R}$, $k, j = 1, n$ (compensated derivative of demand in good k with respect to price of good j)

Reporting $\pi_j = \pi q_j$ and taking into account the relations between derivatives of demand:

$$\begin{aligned} \sum_{k=1,n} p_k x_{kj} &= L_j \\ \sum_{k=1,n} p_k \frac{\partial x_k}{\partial R} &= 1 + \frac{\partial L}{\partial R} \end{aligned}$$

tax formulas become:

$$\begin{aligned} (p_j) \quad \sum_{k=1,n} [(\omega - \varphi)p_k - ((\pi - \varphi)q_k + \theta h_k)] x_{kj} &= \varphi x_j \quad j = 1, n \\ (R) \quad \sum_{k=1,n} [(\omega - \varphi)p_k - ((\pi - \varphi)q_k + \theta h_k)] x_{kR} &= \omega - \lambda \end{aligned}$$

with: $\lambda = \frac{\partial U}{\partial R}$ and : $x_{kR} = \frac{\partial x_k}{\partial R}$, $k = 1, n$

As in the case of the D&M model, *pure tax formulas are the same than those prevailing without externality, social values of commodities being incremented with the social value of pollution generated by subsequent use of the good in final demand.*

Provision of public goods Relation (Q) can be rewritten under the form:

$$\psi = \frac{\partial U}{\partial Q} + \omega \frac{\partial L}{\partial Q} - \sum_{k=1,n} \pi_k \frac{\partial x_k}{\partial Q} - \varphi \sum_{k=1,n} (p_k - q_k) \frac{\partial x_k}{\partial Q} - \theta \sum_{k=1,n} h_k \frac{\partial x_k}{\partial Q}$$

Reporting $\pi_j = \pi q_j$ and taking into account the identity:

$$\sum_{k=1,n} p_k \frac{\partial x_k}{\partial Q} = \frac{\partial L}{\partial Q}$$

completes the system of shadow prices for public production:

shadow value of output	: $\psi = \frac{\partial U}{\partial Q} + (\omega - \varphi) \frac{\partial L}{\partial Q} - \sum_{k=1,n} [(\pi - \varphi)q_k + \theta h_k] \frac{\partial x_k}{\partial Q}$
shadow price of labor	: $\omega' = \omega - \varphi$
shadow price of good j	: $\pi'_j = \pi_j - \varphi q_j$

Shadow prices of factors are equal to social values. As in the case of commodity taxation, *shadow value of output is defined by the same formula than without externality, social values of commodities being incremented by the social value of pollution generated by final consumption.*

4.2.3 Competitive Exchange Equilibria and Second-Best PARETO Efficiency

As previously, a competitive exchange equilibrium is determined by the existence of an exchange price system (τ_j, r) such that competitive markets of tradable goods and tradable permits clear up. As previously, the existence of a competitive exchange equilibrium is not warranted, due to non convexities arising from second-best constraints and externalities. However, if budget constraints are not too tight (marginal cost of public funds not too high), and abatement of pollution not too stringent, then existence of a competitive exchange equilibrium can be reasonably expected.

Second-Best PARETO Efficiency characterizes, as in the previous section, a situation such that it is not possible to change policy decisions by governments (tax systems, provision of public goods), reallocate pollution abatement between countries and compensate by transfer payments in order to increase global welfare (increase welfare in one country without decreasing welfare in all other countries). A Second-Best PARETO Optimum is then solution of the following program:

max $\sum_c U^c$
under the constraints :

$$\begin{aligned}
(\psi^c) \quad & Q^c - g^c(z_k^c, T^c) && c = 1, \widehat{c} \\
(\omega^c) \quad & N^c - L^c = 0 && c = 1, \widehat{c} \\
(\pi_1^c) \quad & x_1^c + z_1^c - y_1^c + \sum_{j=2, n} \tau_j m_j^c - d^c = 0 && c = 1, \widehat{c} \\
(\pi_j^c) \quad & x_j^c + z_j^c - y_j^c - m_j^c = 0 && c = 1, \widehat{c}; j = 2, n \\
(\rho_j^c) \quad & q_j^c - \tau_j q_1^c = 0 && c = 1, \widehat{c}; j = 2, n \\
(\varphi^c) \quad & \sum_{k=1, n} (p_k^c - q_k^c) x_k^c - q_0^c y_0^c + F^c \Pi^c \\
& \quad \quad \quad + q_1^c d^c - \sum_{k=1, n} q_k^c z_k^c - T^c = 0 && c = 1, \widehat{c} \\
(\theta) \quad & \sum_c [y_0^c + h^c(x_k^c)] \leq \sum_c \overline{E}^c \\
(\mu) \quad & \sum_c d^c = 0 \\
(\nu_j) \quad & \sum_c m_j^c = 0 && j = 2, n
\end{aligned}$$

Conditions of optimality can be split in sub-sets, a sub-set of conditions linking countries together, and the others each specific to a given country.

Dual conditions linking countries together They are related to the prices of foreign trade and to the transfers between countries, and are the following:

$$\begin{aligned}
(\tau_j) \quad & \sum_c [\pi_1^c m_j^c - \rho_j^c q_1^c] = 0 && j = 2, n \\
(d^c) \quad & -\pi_1^c + q_1^c \varphi^c + \mu = 0 && c = 1, \widehat{c}
\end{aligned}$$

Dual conditions specific to each country Conditions specific to each country are identical to the ones obtained above, with the difference that proportionality between production prices and social values of tradable goods is not anymore verified in the general case. Effectively, relations dual to net imports are now written:

$$(m_j^c) \quad \tau_j \pi_1^c - \pi_j^c + \nu_j = 0 \quad c = 1, \widehat{c}; j = 2, n$$

Proportionality of production prices and social values of tradable goods is verified only when the dual variable ν_j are all equal to zero. This is not true in the general case, and then a competitive exchange equilibrium is not, in general, second-best PARETO efficient.

PARETO efficiency conditions do not necessarily boil down to nullity of all the dual variables ν_j and, in particular, when pollution is generated by the consumption of a single good (production itself being free from pollution), efficiency holds with the ν_j proportional to the τ_j , which is verified under the two conditions that: i) utility functions are homothetically separable with respect to tradable goods;

and ii) production functions are homothetically separable with respect to all tradable goods except the polluting good, with an identical aggregate in all countries.

These conditions are very restrictive and more general conditions of PARETO efficiency are to be sought in the nullity of the ν_j .

Conditions for PARETO efficiency of a competitive exchange economy Efficiency holds if a solution with all ν_j null is feasible and this requires that relations (τ_j) are verified with this assumption.

Variables ρ_j^c can be drawn from dual relations q_j^c :

$$\rho_j^c = (\pi^c - \omega^c)N_j^c + \varphi^c(x_j^c + z_j^c - F^c y_j^c) - [q_0^c(\pi^c - \varphi^c) + \theta] y_{0j}$$

and reported in (τ_j) , taking into account (ν_j) and (d^c) :

$$\sum_c q_1^c [-(\pi^c - \omega^c)N_j^c + \varphi^c(m_j^c - x_j^c - z_j^c + F^c y_j^c) + (q_0^c(\pi^c - \varphi^c) + \theta) y_{0j}^c] = 0$$

i. e., as $m_j^c = x_j^c + z_j^c - y_j^c$:

$$\sum_c q_1^c [-(\pi^c - \omega^c)N_j^c - \varphi^c(1 - F^c)y_j^c + (q_0^c(\pi^c - \varphi^c) + \theta) y_{0j}^c] = 0$$

As production variables related to countries are independent, the above condition is satisfied if (and, in all generality, only if) we have for every country (dropping index c):

$$-(\pi - \omega)N_j - \varphi(1 - fF)y_j + [q_0(\pi - \varphi) + \theta] y_{0j} = 0 \quad j = 2, n$$

But we have already obtained the same relations for $j = 0$ and in the aggregate:

$$(\omega - \pi)N_0 - \varphi(1 - F)y_0 + [q_0(\pi - \varphi) + \theta] y_{00} = 0$$

$$-(\omega - \pi)N_w - \varphi(1 - F)V + [q_0(\pi - \varphi) + \theta] N_0 = 0$$

Substraction relations relative to $j = 0$ and $j = 2, n$ pre-multiplied by q_0 and q_j from the aggregate relation, and substituting $-y_{wj}$ to N_j yields the same relation for $j = 1$, and then the set of conditions:

$$\boxed{(\pi - \omega)y_{wj} - \varphi(1 - F)y_j + [q_0(\pi - \varphi) + \theta] y_{0j} = 0 \quad j = 1, n}$$

They are verified in all generality when supply, its derivative with respect to wage and its derivative with respect to pollution emissions are proportional for all tradable goods:

$$\frac{y_{wj}}{y_{wl}} = \frac{y_{0j}}{y_{0l}} = \frac{y_j}{y_l} \quad \forall j, l$$

This condition means that the *production function is homothetically separable with respect to tradable goods*, which can be written:

$$f [H(y_1, y_2, \dots, y_n), y_0, N] = 0$$

or, resolving with respect to labor and considering the "cost of production function":

$$N = N[y_0, H(y_1, \dots, y_n)]$$

with H linear homothetic function.

This condition is of course stringent, as it involves that in the production sector *pollution emissions are not specific to outputs, for instance bigger for some than for other, but related to total output as represented by the aggregate H* . It is interesting to note that, when production is free from pollution, and even when there is no pollution and no pollution abatement at all, the condition subsists: the competitive exchange equilibrium is not second-best PARETO efficient unless the cost of production in labor N is a function of an homothetic aggregate of outputs. Then, inefficiency of competitive exchange markets with second-best economies is not linked to pollution as it pre-exists to it.

Theorem 4. *When constraints are put on taxation, preventing production efficiency to hold in each country, a competitive exchange equilibrium is not, under all generality, second-best PARETO efficient. When there are constraints only on profit taxation, PARETO efficiency holds when the production function in each country is homothetically separable with respect to tradable goods.*

It can be easily checked that, if the above condition is verified, a Lindhal-type market of pollution abatement is also second-best PARETO efficient.

4.3 Numerical simulations

Appendix presents the specifications which were retained for production and utility functions, and the calibration data for the initial situation. In particular, the specification retained for production functions is such that the above condition for PARETO efficiency does not hold.

Let us recall that, at the difference of simulations performed in the previous section, only one category of households is considered in each country. Then taxation does not serve for welfare redistribution, but for financing a public good, the demand for which is assumed constant (a "fixed cost" for the economy).

4.3.1 General results of simulations

Results of simulations, without and with tradable permits (based on MACs), for different rates of abatement (identical in both countries) are given in tables 28 and 29 of Appendix. Interpretation is complex because the initial situation, and equilibria corresponding to different levels of pollution abatement, are not PARETO efficient.

Assessing the importance of the inefficiency of "decentralized" policies of pollution abatement (i. e. without a coordination aiming at reaching a PARETO efficient situation) requires to use of a relevant index, the definition of which is given in Appendix.

4.3.2 Initial steps of abatement: effects of inefficiency of Competitive Exchange Equilibria

The initial situation is a competitive exchange equilibrium, without taxation of pollution and then without voluntary abatement, which is not PARETO efficient for reasons indicated at the beginning of the

section. Then it is possible that a "double-dividend" exists, and that global welfare increases in the first steps of abatement. This, of course, depends on the specification of the model, and in particular on the form of the production functions (in particular, it does not exist if the separability condition for second-best PARETO efficiency holds).

With the specifications and numerical values retained here, the phenomenon effectively appears. Graphs in figure 10 represent, in the plan of utilities¹⁶, situations corresponding to equilibria (right side without permits and left side with tradable permits) obtained in the initial steps of abatement (from 0% to 5%), and the PARETO frontier of the world economy (corresponding to no constraint on pollution abatement). Inefficiency decreases when pollution abatement increases, till approximately 1.5% in the case without permits and 2.5% in the case with tradable permits. The latter equilibrium is even PARETO superior to the initial situation, which confirms that pollution abatement (up to a limited level) may increase welfare in all countries and then yield a "double-dividend".

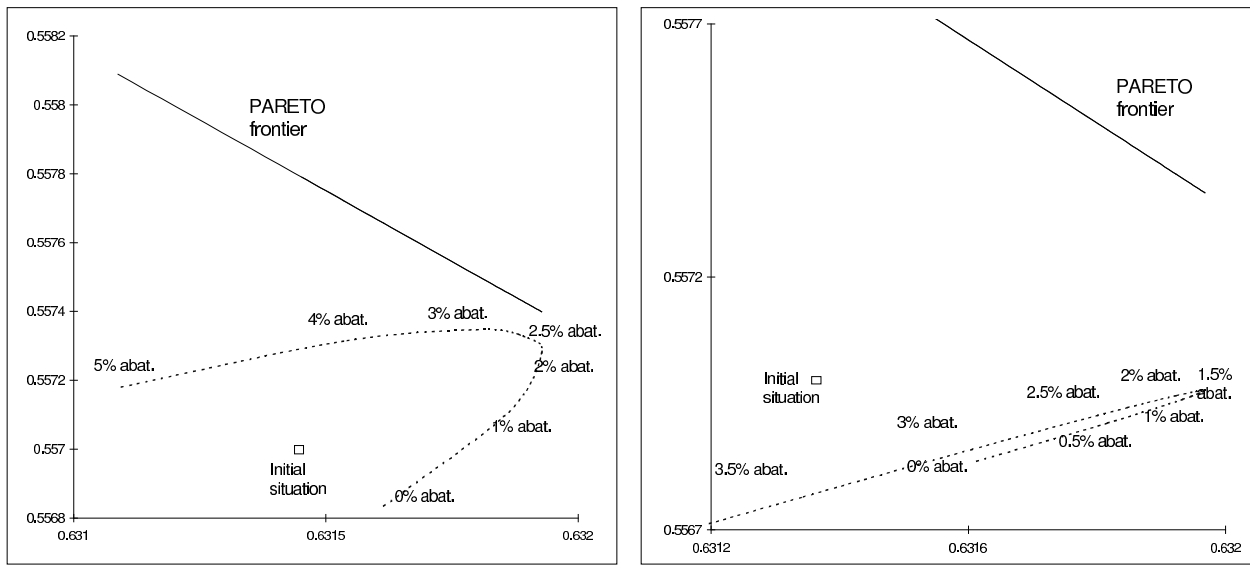


Figure 10

Such a result is however of limited importance and should not be overemphasized, as interesting situations correspond to levels of abatement significantly bigger than few percents. They are commented below.

4.3.3 Taxation without tradable permits

Without tradable permits, carbon taxes and marginal abatement costs differ from one country to the other. Figure 11 shows that marginal abatement cost is smaller than carbon tax till the marginal cost of public funds becomes null (case for the second country with 6.5% abatement, see figure 12).

¹⁶Country 1 in y -axis, country 2 in x -axis.

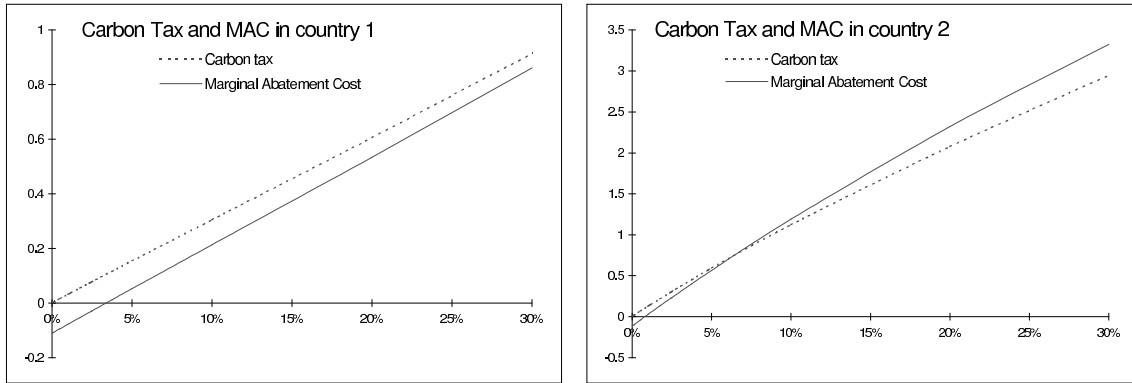


Figure 11

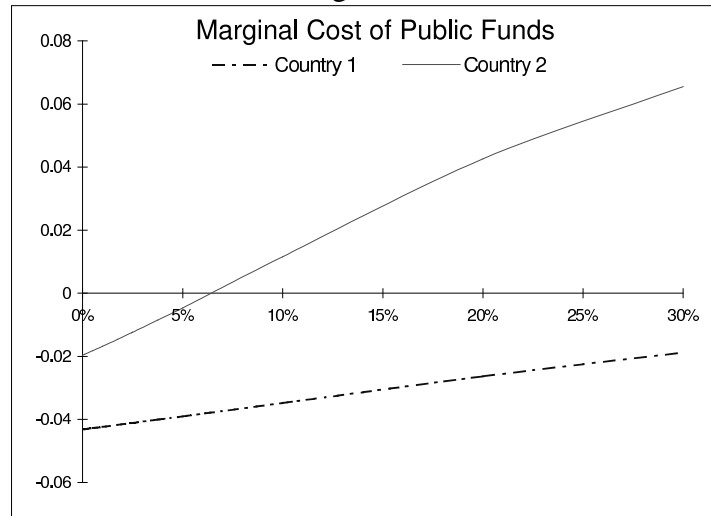


Figure 12

An interesting numerical result concerns the pure rates of taxation of final demand. While total rates of taxation diverge rapidly, as intensities of pollution emissions in final consumption differ from one commodity to the other, the pure rates of taxation remain fairly close to each other (figure 13). They evidently decrease as receipts accruing from carbon taxes are substituted to pure taxes, and become negative when the MCPF become positive (case of country 2 for a rate of abatement superior to 6.5%).

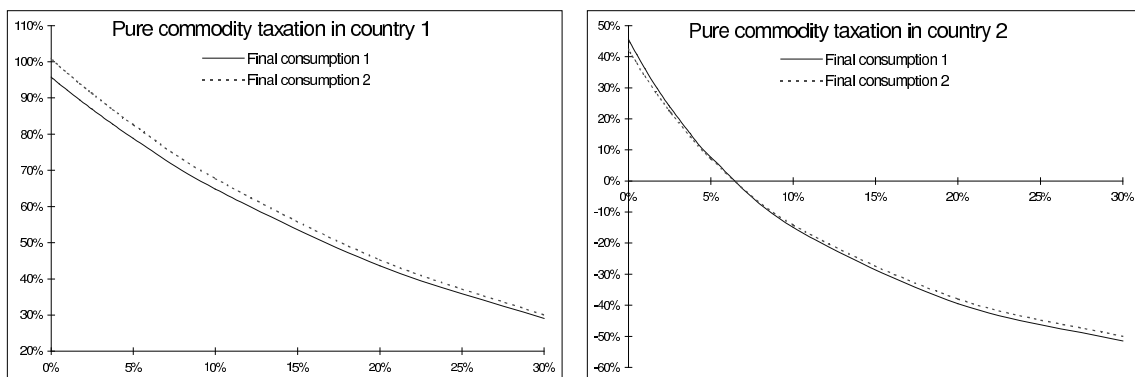


Figure 13

4.3.4 Taxation with tradable permits

With tradable permits, marginal abatement costs are equalized in both countries, but the price of permits still differs from domestic carbon taxes (figures 14 and 15). In the present numerical simulation, domestic carbon taxes are bigger than the international price of pollution permits, and this subsists for both countries till fairly high levels of abatement (MCPF remaining negative till high levels of abatement in country 2, because its optimal policy is to buy permits rather than abate pollution, which requires fiscal receipts from taxation).

Implementation of an international market of pollution permits where traders are firms instead of governments is possible with corrective (pure) taxation of pollution in the production sector, so that the total price paid by firms becomes equal to the required level of carbon tax.

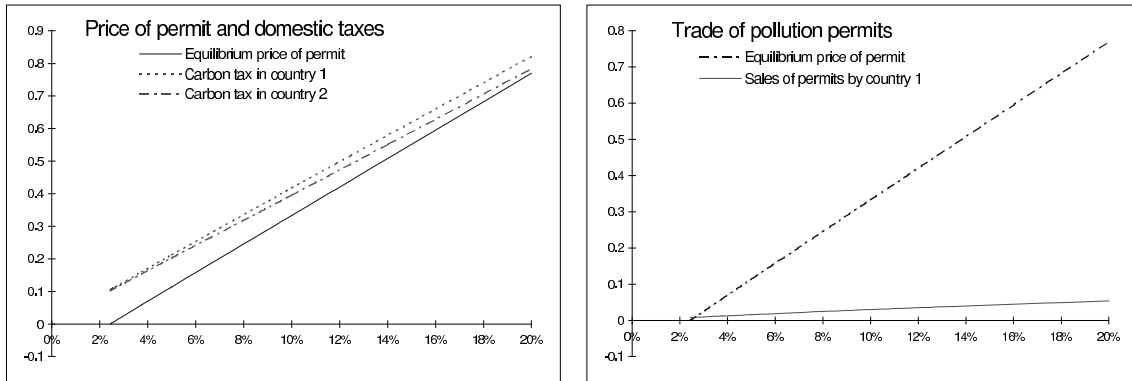


Figure 14

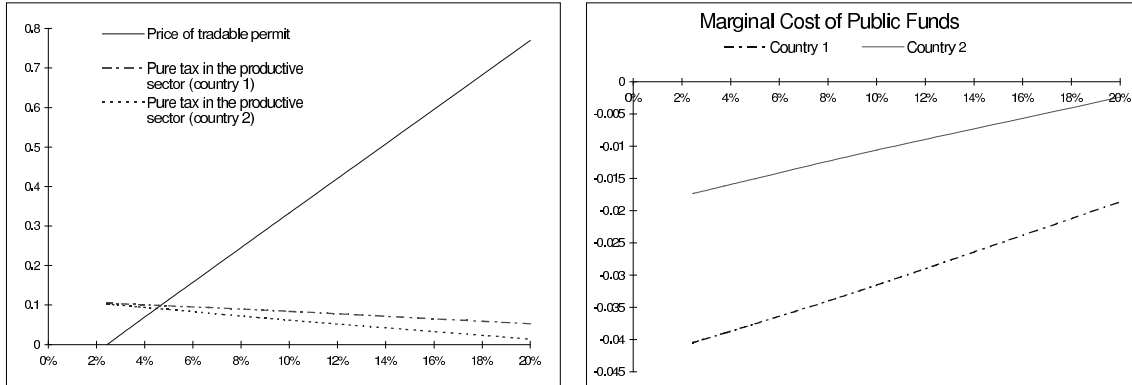


Figure 15

Concerning taxation of final consumption, the same result than previously is obtained, with pure rates close to proportional taxation (figure 16).

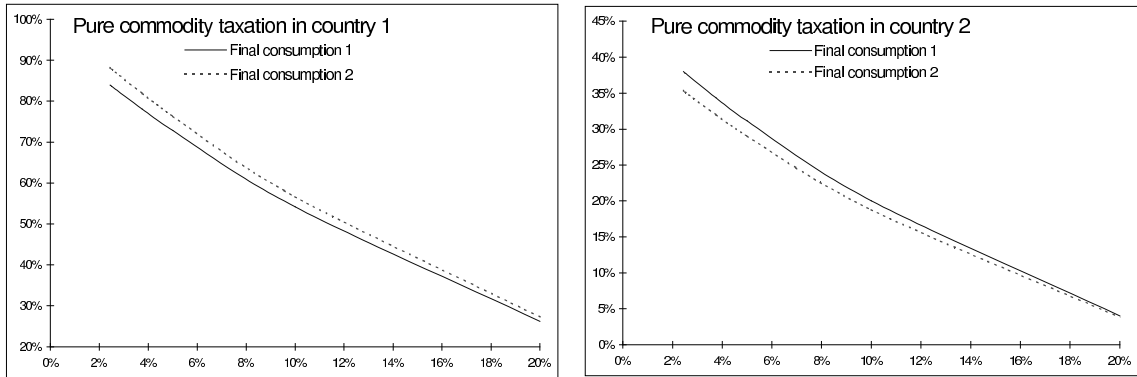


Figure 16

4.3.5 Welfare gains from tradable permits

Tradable permits allow to limit the total cost of pollution abatement, in particular when pollution abatement reaches high levels. Figure 17 shows that inefficiency of equilibria without tradable permits is rapidly increasing, while inefficiency of equilibria with tradable permits tends to zero.

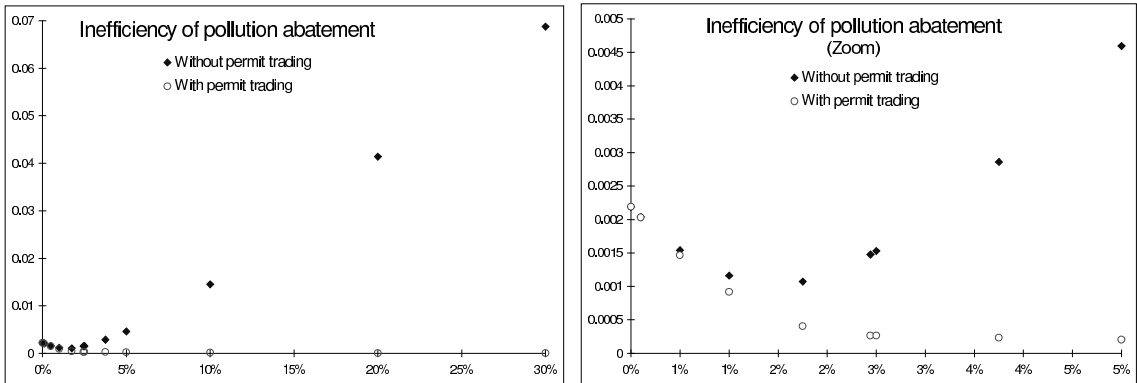


Figure 17

4.3.6 Comparison of permits based on MACs and permits based on taxes

Simulations have been performed with tradable permits based on carbon taxes (then equalized across countries), instead of MACs, and proportional pure taxation of final demand. Figure 18 represents the comparative results of the two systems of permits for initial steps of abatement. Tradable permits based on carbon taxes do not appear significantly less efficient than permits based on MACs, the main difference being the relative situations of the two countries: permits based on carbon taxes are more favorable to country 1, which is net seller of permits, and then benefits from an higher transaction price.

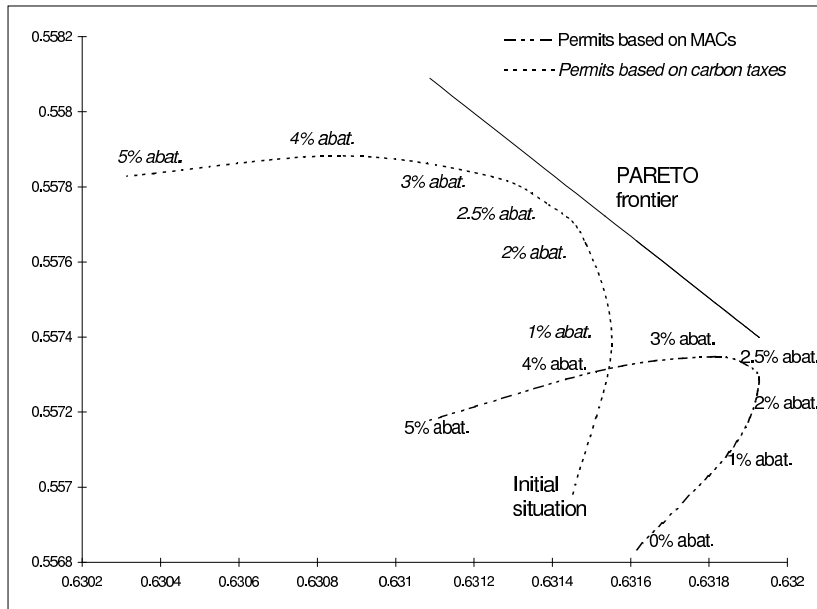


Figure 18

The same comments can be made with higher values of pollution abatement, as show the graphs of figure 19.

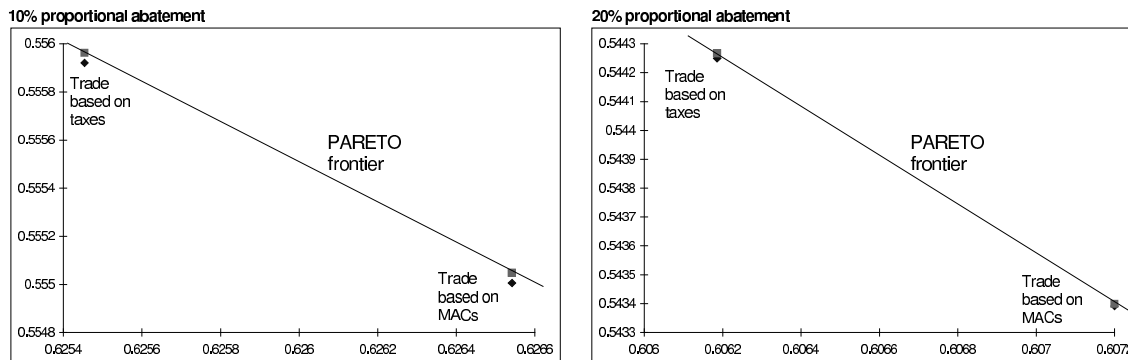


Figure 19

5 Conclusions and further developments

Even assuming that governments optimize their fiscal policies, i. e. that they have a clear formulation of the constraints and the targets of their fiscal system, incorporating environmental concerns raises several problems.

- The most notable is that the equilibrium pollution tax, the one instituted to reach a given target of pollution abatement, differs from the (social) marginal cost of abatement, i. e. the social welfare loss of a unit increase of pollution abatement, and that it is not a suitable basis for trading pollution rights. This divergence is probably much bigger in real situations than in theoretical models, because tax-systems are not likely to be close to optimality.

- For pollutions which have a world scale, such as Greenhouse Effect, sources are both industries and households. Then taxing only firms or only households is not efficient, and taxation has to be adjusted to incorporate the relevant incentives to both firms and households. Change in commodity taxation depends heavily on the structure of the tax-system, and in particular its targets. It is not the same when taxation has mainly a redistributive purpose, and when it serves essentially for financing public outlays.
- Pollution abatement decided at a world scale has obvious effects on the international trade of goods and services, and in particular of those which are at the source of pollution. Change in the relative prices of international trade may generate important losses or gains from terms of trade and benefit to some countries (mainly net importers) and penalize others (mainly net exporters). This is an aspect which cannot be concealed in a political agreement.

Better understanding of the issues raised by the implementation of a world policy of pollution abatement is to be expected from applied appraisals and applied models such as general equilibrium models. It is important, for a rigorous comparison of their results, that they exhibit the same summary indicators, such as marginal abatement costs, gains and losses from terms of trade, and total welfare losses.

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Appendix

Numerical simulations have been performed with a world economy consisting of two countries, each producing two tradable goods with labor as factor of production and non tradable good.

In simulations related to the D&M model, two households/consumers (or groups of identical households/consumers) are considered, differing from skills in labor and utility functions. In simulations related to the general model with constraints on taxation, a single (representative) household/consumer is considered.

Complete determination and calibration of the numerical models result from the specification of production and utility functions, representation of pollution emissions by firms and households, and data characterizing the initial situation.

1. General specification of production and utility functions

Utility and production functions are chosen of the following forms:

$$U = f(x_1, x_2, \bar{L} - \frac{L}{e})$$

$$g(y_1, y_2, \bar{N} - N) = A$$

with f and g being nested C.E.S. constant returns to scale functions, with a nest between goods 1 and 2 in the utility function and a nest between good 1 and labor in the production function, coefficient e representing the skill in labor of the considered consumer. When there is a single consumer, $e = 1$.

As the utility functions are homothetic (with respect to tradable goods and leisure), the surplus measured by Equivalent Variation of Income (minus effective variation of income) is proportional to the change in utility. Then, surpluses calculated relatively to a given reference situation (here the initial situation) allow to compare directly any couple of situations¹⁷.

To measure the inefficiency of a given situation, i. e. the "distance" from the PARETO frontier, it has been retained an index which is the potential surplus of country 1 at constant utility of country 2.

2. Representation of pollution emissions

Pollution generated by the production sector can be represented in several ways, and notably the two following:

- pollution is an argument in the production function, as other inputs and outputs;
- pollution is a function of net production of the economy, i. e. of the quantities of production of each good:

$$E = E(y_1, y_2)$$

¹⁷This is not true in all generality: if $S^1 > S^2$, with S^1 and S^2 surpluses calculated relatively to a given reference situation, 1 is not necessarily better than 2.

The latter representation has been retained for the D&M model, with emissions proportional to productions of each good:

$$E = c_1 y_1 + c_2 y_2$$

c_1 and c_2 being coefficients of "pollution intensity".

For the general model, the specification is of the first type, and pollution is assumed to affect the second member A , which is of the form:

$$A = \gamma_0 + \gamma_1 E + \gamma_2 E^2$$

with $\gamma_1 > 0$ and $\gamma_2 < 0$. Then, when there is no incentive to abate pollution, i. e. in the initial situation, A is at its maximum:

$$E = -\frac{\gamma_1}{2\gamma_2}$$

$$A = \gamma_0 - \frac{(\gamma_1)^2}{4\gamma_2}$$

As for pollution generated by final consumption, the same formulation than above with pollution intensities is retained:

$$h(x_1, x_2) = c'_1 x_1 + c'_2 x_2$$

3. Case of the D&M model

3.1 Calibration

Elasticities of substitution and other structural parameters are given in table 20, for scenarios of type I (highly skilled households having a high price elasticity of demand for the polluting good). For scenarios of type II, elasticities of substitution in utility function are permuted between household 1 and household 2.

	Country 1	Country 2
Elasticities of substitution		
Production		
Nest	-0.205	-0.205
Function	-0.98	-0.98
Household 1		
Nest	0.75	0.75
Function	0.25	0.25
Household 2		
Nest	0.25	0.25
Function	0.75	0.75
Skills in labor		
Household 1	4	4
Household 2	0.625	0.625
Intensities of pollution emissions		
Production		
good 1	0.1	0.3
good 2	0.3	0.1
Households		
good 1	1	1
good 2	0	0

Figure 20

As for the social welfare function, the specification retained is the sum of logarithms of individual utilities as defined above (i. e. in the linear-homothetic cardinal form).

Calibration data for economic variables (quantities and prices) are given in table 21. They do not represent the "initial situation" because the corresponding situation is not a competitive exchange equilibrium, as taxation is not optimized in each country.

	Country 1	Country 2
Production		
Quantity		
good 1	0.24	0.48
good 2	1.395	1.155
Price		
good 1	1	1
good 2	1	1
Consumption		
Household 1		
good 1	0.18	0.18
good 2	0.6375	0.6375
Household 2		
good 1	0.18	0.18
good 2	0.6375	0.6375
Price of consumption		
good 1	5/6	5/6
good 2	1 1/3	1 1/3
Labor supply		
Household 1	0.125	0.125
Household 2	0.8	0.8

Figure 21

Results of the optimization process depend on the tools available (and in particular possibility of lump sum transfer), and then initial situations differ from one scenario to the other.

3.2 Numerical results

Table 22 yields the results corresponding to the case with lump sum transfers, and table 23 without. Table 24 compares the equilibria obtained, in the case of a 5% proportional abatement, without and with tradable permits, to the situation where pollution rights are allocated so as to equalize marginal abatement costs between countries (iso-MAC allocation).

Rate of abatement	0%	1%	5%	10%	15%	20%	30%
Scenario type I							
Pollution tax (= SMAC) (relatively to price of good 2)	0	0.149	0.629	1.090	1.488	1.855	2.474
relative price of production (good 1 / good 2)	0.829	0.729	0.460	0.298	0.236	0.224	0.254
pure rate of taxation good 1	-2.7%	-4.3%	-9.4%	-13.8%	-16.7%	-18.4%	-18.3%
pure rate of taxation good 2	4.2%	3.9%	3.1%	2.9%	3.3%	4.1%	6.1%
total rate of taxation good 1	-2.7%	15.2%	114.3%	301.1%	507.8%	656.4%	776.0%
Lump sum transfer	0.357	0.376	0.458	0.581	0.738	0.943	1.552
Surplus households 1	0	0.00002	-0.00425	-0.01772	-0.03982	-0.07216	-0.18134
Surplus households 2	0	-0.00039	-0.00748	-0.02596	-0.05269	-0.08621	-0.16934
Scenario type II							
Pollution tax (= SMAC) (relatively to price of good 2)	0	0.115	0.472	0.788	1.048	1.306	1.892
relative price of production (good 1 / good 2)	0.771	0.681	0.429	0.264	0.191	0.168	0.195
pure rate of taxation good 1	16.0%	17.0%	21.2%	26.6%	31.7%	35.5%	34.1%
pure rate of taxation good 2	3.8%	3.0%	-0.2%	-4.2%	-8.4%	-12.8%	-21.3%
total rate of taxation good 1	16.0%	36.8%	154.6%	403.6%	755.0%	1090.3%	1336.1%
Lump sum transfer	0.404	0.412	0.441	0.479	0.521	0.573	0.730
Surplus households 1	0	-0.00153	-0.01322	-0.03863	-0.07538	-0.12365	-0.24841
Surplus households 2	0	0.00004	-0.00373	-0.01477	-0.03064	-0.05063	-0.10668

Figure 22

Rate of abatement	0%	1%	5%	10%	15%
Scenario type I					
Pollution tax (= SMAC) (relatively to price of good 2)	0	0.234	1.015	1.905	2.974
relative price of production (good 1 / good 2)	1.001	0.855	0.487	0.310	0.311
pure rate of taxation good 1	-57.8%	-60.5%	-70.2%	-82.1%	-95.7%
pure rate of taxation good 2	-54.1%	-56.4%	-65.8%	-78.8%	-94.9%
total rate of taxation good 1	-57.8%	-49.6%	-8.0%	27.9%	-54.2%
Surplus households 1	0	0.00074	-0.00098	-0.01033	-0.02520
Surplus households 2	0	-0.00054	-0.00917	-0.03220	-0.06870
Scenario type II					
Pollution tax (= SMAC) (relatively to price of good 2)	0	0.192	0.807	1.458	2.381
relative price of production (good 1 / good 2)	1.044	0.905	0.525	0.305	0.270
pure rate of taxation good 1	-59.7%	-58.6%	-55.9%	-59.2%	-78.2%
pure rate of taxation good 2	-73.6%	-74.2%	-77.3%	-83.1%	-92.3%
total rate of taxation good 1	-59.7%	-49.9%	11.9%	135.5%	114.2%
Surplus households 1	0	-0.00581	-0.04195	-0.11797	-0.24209
Surplus households 2	0	0.00060	-0.00003	-0.00495	-0.01226

Figure 23

	Initial situation		No tradable permit		Tradable permit		Iso-MAC Allocation	
	Country 1	Country 2	Country 1	Country 2	Country 1	Country 2	Country 1	Country 2
Scenario I								
Pollution tax (= SMAC) (relatively to price of good 2)	0	0	0.7448	0.4069	0.5413	0.5413	0.5370	0.5370
Surplus households 1	0	0	0.0120	-0.0118	0.0085	-0.0080	0.0114	-0.0111
Surplus households 2	0	0	0.0030	-0.0125	0.0030	-0.0116	0.0058	-0.0142
Social Welfare Function	-0.8296	-0.8342	-0.8218	-0.8573	-0.8226	-0.8549	-0.8175	-0.8600
Effective abatement	0	0	5%	5%	3.39%	7.21%	3.30%	7.33%
Scenario II								
Pollution tax (= SMAC) (relatively to price of good 2)	0	0	0.5423	0.3229	0.4017	0.4017	0.4003	0.4003
Surplus households 1	0	0	-0.0266	-0.0210	-0.0122	-0.0314	-0.0089	-0.0346
Surplus households 2	0	0	0.0090	-0.0078	0.0065	-0.0053	0.0086	-0.0074
Social Welfare Function	-0.9652	-0.9740	-0.9575	-0.9919	-0.9579	-0.9904	-0.9538	-0.9946
Effective abatement	0	0	5%	5%	3.03%	7.87%	2.93%	8.01%

Figure 24

4. "General" model

4.1 Calibration

Structural coefficients are given in table 25.

	Country 1	Country 2
Elasticities of substitution		
Production		
Nest	-0.205	-0.706
Function	-0.98	-1.993
Households		
Nest	0.444	0.534
Function	0.45	0.225
Intensities of pollution emissions		
Households		
good 1	0.3	0.1
good 2	0.1	0.3
Emissions by the production sector in the initial situation	0.75	0.25

Figure 25

Elasticities of substitution have been determined in order to obtain given values of price elasticities of supply and (compensated) demand, as given in table 26.

	Country 1	Country 2
elasticity of demand for good 1 ("energy")	-0.4	-0.5
elasticity of supply for "energy"	0.3	0.9
elasticity of supply for labor	0.3	0.15
elasticity of demand for labor	-0.3	-0.6

Figure 26

Table 27 characterizes the initial situation, which is a competitive exchange equilibrium, with optimized taxation¹⁸. It can be noted that, as the utility functions are homothetically separable with respect to tradable goods, proportional taxation holds in each country (respectively 100% and 42.86% taxation).

	Country 1	Country 2
Production		
Quantity		
good 1	0.1583	0.0538
good 2	0.6666	0.7265
Price		
good 1	0.5572	0.8205
good 2	0.6039	0.8892
Final consumption by households		
good 1	0.0589	0.1532
good 2	1.5710	0.9674
Price of consumption		
good 1	1.1144	1.1721
good 2	1.2077	1.2702
Labor supply	0.4998	0.5034

¹⁸Determined, as previously, from a given calibration situation, after fiscal optimization.

Figure 27

Receipts accruing from taxation finance a public good, the demand of which is constant ("fixed cost" for the economy).

4.2 Numerical results

Results of simulations, without and with tradable permits (based on MACs), for different rates of abatement (identical in both countries) are given in tables 28 and 29 below.

	Initial situation	Percentage of abatement					
		0%		1%		2.44%	
		0%	1%	2.44%	5%	10%	20%
Without permit							
Carbon tax	0	0.00079	0.03177	0.07615	0.15432	0.30536	0.60588
	0	0.00457	0.12810	0.30061	0.59308	1.12364	2.07650
MAC		-0.11048	-0.07757	-0.03039	0.05282	0.21383	0.53416
		-0.11162	0.02912	0.22710	0.56623	1.19043	2.32086
MCPF	-0.05603642	-0.04315	-0.04234	-0.04116	-0.03903	-0.03478	-0.02636
	-0.019504	-0.01962	-0.01678	-0.01253	-0.00461	0.01159	0.04263
Households' surplus	0	-0.00039	-0.00006	-0.00013	-0.00178	-0.01046	-0.04841
	0	0.00038	0.00113	0.00086	-0.00308	-0.02121	-0.08731
Total rate of taxation on HFC							
-commodity 1	1	95.8%	94.0%	91.6%	87.5%	80.4%	69.4%
-commodity 2	1	100.8%	97.5%	92.9%	85.4%	72.8%	54.0%
-commodity 1	0.42857143	45.6%	38.0%	28.3%	14.4%	-5.1%	-27.0%
-commodity 2	0.42857143	42.5%	38.8%	33.8%	26.2%	14.8%	0.7%
Pure rate of taxation							
-commodity 1	1	95.7%	92.1%	87.0%	78.8%	64.8%	43.6%
-commodity 2	1	100.8%	96.8%	91.4%	82.6%	67.7%	45.2%
-commodity 1	0.42857143	45.6%	36.1%	24.3%	7.6%	-14.9%	-39.5%
-commodity 2	0.42857143	42.3%	33.6%	22.8%	7.2%	-14.2%	-38.0%
Index of inefficiency	0.00214991	0.0021879	0.0011623	0.00147714	0.00459494	0.01451897	0.04140936

Figure 28

Initial situation	Percentage of abatement						
	(first line: country 1, second line: country 2)						
	0%	1%	2.44%	5%	10%	20%	
With tradable permits							
Sales of permits by country 1	-0.00003	0.00322	0.00781	0.01564	0.02981	0.05360	
Price of tradable permit	-0.11062	-0.06504	0.00000	0.11400	0.33309	0.76953	
Carbon tax	0.00067 0.00545	0.04368 0.04551	0.10496 0.10260	0.21210 0.20257	0.41702 0.39503	0.82189 0.78338	
Pure tax in the productive sector	0.11128 0.11607	0.10872 0.11054	0.10496 0.10260	0.09810 0.08858	0.08393 0.06194	0.05235 0.01385	
MCPF	-0.04315 -0.01960	-0.04208 -0.01869	-0.04049 -0.01738	-0.03758 -0.01506	-0.03156 -0.01061	-0.01865 -0.00241	
Households' surplus	-0.00039 0.00038	0.00034 0.00096	0.00091 0.00105	0.00052 -0.00085	-0.00526 -0.01153	-0.03615 -0.05694	
Total rate of taxation on HFC							
-commodity 1	95.8%	93.4%	90.1%	84.5%	74.7%	58.9%	
-commodity 2	100.8%	96.3%	90.1%	80.1%	63.2%	37.8%	
-commodity 1	45.6%	43.0%	39.5%	34.0%	25.1%	12.6%	
-commodity 2	42.5%	41.2%	39.5%	36.9%	32.9%	28.2%	
Pure rate of taxation							
-commodity 1	95.7%	90.7%	83.9%	72.8%	54.2%	26.2%	
-commodity 2	100.8%	95.4%	88.2%	76.3%	56.6%	27.3%	
-commodity 1	45.5%	42.3%	38.0%	31.1%	20.0%	4.0%	
-commodity 2	42.2%	39.3%	35.3%	29.0%	18.8%	3.8%	
Index of inefficiency	0.00219	0.00092	0.00026	0.00020	0.00011	0.00002	

Figure 29