Model MESSAGE

Command Line User Manual

Version 0.18, 15. feb. 2001

Februar 2001

Part I

Mathematical formulation of MESSAGE

Chapter 1

Introduction

This part of the document contains the mathematical formulation of MESSAGE. The computer codes of the matrix generator produce equations according to this formulation, the input data determine the form these equations actually take. In its general formulation MESSAGE a dynamic linear programming model with a mixed integer option.[1] This implies that all relations that define the structure of a model are given as linear constraints between continuous variables. The variables of such a model are called "Columns", the equations "Rows". This nomenclature is derived from the usual notation used to write down linear models: in the shape of a matrix.

The variables (columns) of MESSAGE be grouped into three categories:

1. Energy flow variables representing an annual energy flow quantity. The unit is usually MWyr for small regions and GWyr for bigger areas,
2. Power variables representing the production capacity of a technology (usual unit: MW or GW), and
3. Stock-piles representing the quantity of a fuel being cumulated at a certain point in time (usual unit: MWyr or GWyr).

The constraints (rows) generated by MESSAGE be grouped into the following categories:

1. Energy flow balances modelling the flow of energy in the energy chain from resource extraction via conversion, transport, distribution up to final utilization,
2. sum or relational constraints limiting aggregate activities on an annual or overall basis, either absolute or in relation to other activities,
3. dynamic constraints setting a relation between the activities of two consecutive periods, and

[1] Nonlinear constraints or a nonlinear objective function can only be defined for a specific problem, because the nonlinear functions and, if possible, their first derivatives have to be supplied as FORTRAN subroutines for MINOS.

4. counters that are only used for accounting purposes.

This manual gives the mathematical formulation of MESSAGE. It contains a formalized description of all types of variables and equations that the matrix generator generates. The reader of this paper is assumed to be familiar with the theory of linear and mixed integer programming; if he/she wants to apply the nonlinear options, some knowledge about MINOS and access to this code is essential.

Each of the building stones of MESSAGE handled in a separate chapter, which is again subdivided into sections on columns and rows. The notation used for the variable and equation names is the same as in the MPS-file. Uppercase letters are used to indicate predefined identifiers, while lowercase letters represent characters that are chosen by the user or varied over a set of characters.

In order to keep the notation simple and the mathematical description as short as possible the more complex features are omitted from the description of the rows and described in an additional section (see chapter 10 beginning on page 39). Since practically all parameters of MESSAGE be defined as time series (i.e. change over the planning horizon), the index for the period is often omitted in the formulation (e.g., for the efficiencies or the plant factors of conversion technologies).

Chapter 2

Conversion Technologies

2.1 Variables

Energy conversion technologies are modelled using two types of variables, that represent

– the amount of energy converted per year in a period (activity variables) and

– the capacity installed annually in a period (capacity variables).

2.1.1 Activities of Energy Conversion Technologies

zsvd.elt

where

z	is the level identifier of the main output of the technology. $z = U$ identifies the end-use level. This level is handled differently to all other levels: It has to be the demand level and technologies with the main output on this level are defined without load regions.
s	is the main energy input of the technology (supply). If the technology has no input s is set to "." (e.g., solar technologies),
v	additional identifier of the conversion technology (used to distinguish technologies with the same input and output),
d	is the main energy output of the technology (demand),
e	is the level of reduction of demand due to own-price elasticities of demands (does only occur on the demand level; otherwise or if this demand has no elasticities e = "."),
l	identifies the load region, $l \in \{1, 2, 3, ...\}$ or $l =$ ".", if the technology is not modelled with load regions, and
t	identifies the period, $t \in \{a, b, c, ...\}$.

The activity variable of an energy conversion technology is an energy flow variable. It represents the annual consumption of this technology of the main input per period. If a technology has no input, the variable represents the annual production of the main output.

If the level of the main output is *not* U and at least one of the energy carriers consumed or supplied is defined with load regions the technology is defined with load regions. In this case the activity variables are generated separately for each load region, which is indicated by the additional identifier l in position 7. However, this can be changed by fixing the production of the technology over the load regions to a predefined pattern (see section 10.4): one variable is generated for all load regions, the distribution to the load regions is given by the definition of the user (e.g., production pattern of solar power-plants).

If the model is formulated with demand elasticities (see section 10.10), the activity variables of technologies with a demand as main output that is defined with elasticity are generated for each elasticity class (identifier e in position 6).

2.1.2 Capacities of Energy Conversion Technologies

$$Yzsvd..t,$$

where

Y	is the identifier for capacity variables.
z	identifies the level on that the main energy output of the technology is defined,
s	is the identifier of the main energy input of the technology,
v	additional identifier of the conversion technology,
d	is the identifier of the main energy output of the technology, and
t	is the period in that the capacity goes into operation.

The capacity variables are power variables. Technologies can be modelled without capacity variables. In this case no capacity constraints and no dynamic constraints on construction can be included in the model. Capacity variables of energy conversion technologies can be defined as integer variables, if the solution algorithm has a mixed integer option.

If a capacity variable is continuous it represents the annual new installations of the technology in period t, if it is integer it represents either the annual number of installations of a certain size or the number of installations of $1/\Delta t$ times the unit size (depending on the definition; Δt is the length of period t in years).

The capacity is defined in relation to the main output of the technology.

2.2 Constraints

The rows used to model energy conversion technologies limit

- the utilization of a technology in relation to the capacity actually installed (capacity constraint) and
- the activity or construction of a technology in a period in relation to the same variable in the previous period (dynamic constraints).

2.2.1 Capacity Constraints

$Czsvd.lt,$

where

C	is the identifier for capacity constraints,
z	identifies the level on that the main energy output of the technology is defined,
s	is the identifier of the main energy input of the technology,
v	additional identifier of the conversion technology,
d	is the identifier of the main energy output of the technology,
l	identifies the load region, $l \in \{1, 2, 3, ...\}$ or $l =$ ".", if the technology is not modelled with load regions, and
t	is the period in that the capacity goes into operation.

For all conversion technologies modelled with capacity variables the capacity constraints will be generated automatically. If the activity variables exist for each load region separately there will be one capacity constraint per load region (see also section 10.4). If the technology is an end-use technology the sum over the elasticity classes will be included in the capacity constraint.

Additionally the activity variables of different technologies can be linked to the same capacity variable, which allows to leave the choice of the activity variable used with a given capacity to the optimization (see section 10.7).

Technologies without Load Regions

For technologies without load regions (i.e. technologies, where no input or output is modelled with load regions) the production is related to the total installed capacity by the plant factor. For these technologies the plant factor has to be given as the fraction they actually operate per year. All end-use technologies (technologies with main output level "U") are modelled in this way.

$$\epsilon_{svd} \times zsvd...t - \sum_{\tau=t-\tau_{svd}}^{min(t,\kappa_{svd})} \Delta(\tau-1) \times \pi_{svd} \times f_i \times Yzsvd..\tau \leq hc^t_{svd} \times \pi_{svd} .$$

Technologies with Load Regions and "Free" Production Pattern

If a technology has at least one input or output with load regions, the activity variables and capacity constraints will per default be generated separately for each load region. This can be changed by defining the production pattern over the load regions. If the production pattern remains free, the production in each load region is limited in relation to the installed capacity separately for each load region, the capacity is determined by the activity in the load region with the highest requirements. The plant factor has to be given as the fraction the system operates in peak operation mode (in general this is the availability factor).

Maintenance times or minimum operation times could be included by using additional relations, if required (see section 8).

$$\frac{\epsilon_{svd}}{\lambda_l} \times zsvd..lt - \sum_{\tau=t-\tau_{svd}}^{min(t,\kappa_{svd})} \Delta(\tau-1) \times \pi_{svd} \times f_i \times Yzsvd..\tau \leq hc^t_{svd} \times \pi_{svd} .$$

Technologies with Load Regions and "Fixed" Production Pattern

If a technology has at least one input or output with load regions and the production pattern over the load regions is predefined only one activity variable and one capacity constraint is generated per period. The plant factor has, like for technologies with load regions and free production pattern, to be given for the load region with the highest capacity utilization (i.e. the highest power requirement). The capacity constraint is generated for only this load region.

$$\frac{\epsilon_{svd} \times \pi(l_m, svd)}{\lambda_{l_m}} \times zsvd...t -$$

$$\sum_{\tau=t-\tau_{svd}}^{min(t,\kappa_{svd})} \Delta(\tau-1) \times \pi_{svd} \times f_i \times Yzsvd..\tau \leq hc^t_{svd} \times \pi_{svd} .$$

Technologies with Varying Inputs and Outputs

Many types of energy conversion technologies do not have fix relations between their inputs and outputs. MESSAGE has the option to link several activity variables of conversion technologies into one capacity constraint. For the additional activities linked to a capacity variable a coefficient defines the maximum power available in relation to one power unit of the main activity.

In the following this constraint is only described for technologies without load regions; the other types are constructed in analogy (see also section 10.7).

$$\sum_{\sigma v'\delta} rel^{svd}_{\sigma v'\delta} \times \epsilon_{\sigma v'\delta} \times z\sigma v'\delta...t -$$

$$\sum_{\tau=t-\tau_{svd}}^{min(t,\kappa_{svd})} \Delta(\tau-1) \times \pi_{svd} \times f_i \times Yzsvd..\tau \leq hc^t_{svd} \times \pi_{svd} ,$$

The following notation is used in the above equations:

$zsvd..lt$	is the activity of conversion technology v in period t and, if defined so, load region l (see section 2.1.1),
$Yzsvd..t$	is the capacity variable of conversion technology v (see section 2.1.2).
ϵ_{svd}	is the efficiency of technology v in converting the main energy input, s, into the main energy output, d,
κ_{svd}	is the last period in that technology v can be constructed,
π_{svd}	is the "plant factor" of technology v, having different meaning depending on the type of capacity equation applied,
$\Delta\tau$	is the length of period τ in years,
τ_{svd}	is the plant life of technology v in periods,
hc^t_{svd}	represents the installations built before the time horizon under consideration, that are still in operation in the first year of period t,
f_i	is 1. if the capacity variable is continuous, and represents the minimum installed capacity per year (unit size) if the variable is integer,
l_m	is the load region with maximum capacity use if the production pattern over the year is fixed,
$\pi(l_m, svd)$	is the share of output in the load region with maximum production,
$rel^{svd}_{\sigma v'\delta}$	is the relative capacity of main output of technology (or operation mode) svd to the capacity of main output of the alternative technology (or operation mode)$\sigma v'\delta$,
λ_l	is the length of load region l as fraction of the year, and
λ_{l_m}	is the length of load region l_m, the load region with maximum capacity requirements, as fraction of the year.

2.2.2 Upper Dynamic Constraints on Construction Variables

$$MYzsvd.t$$

The dynamic capacity constraints relate the amount of annual new installations of a technology in a period to the annual construction during the previous period.

$$Yzsvd..t \; - \; \gamma y^o_{svd,t} \; \times \; Yzsvd..(t-1) \; \leq \; gy^o_{svd,t},$$

where

$\gamma y^o_{svd,t}$	is the maximum growth rate per period for the construction of technology v,
$gy^o_{svd,t}$	is the initial size (increment) that can be given for the introduction of new technologies,
$Yzsvd..t$	is the annual new installation of technology v in period t.

2.2.3 Lower Dynamic Constraints on Construction Variables

$$LYzsvd.t$$

$$Yzsvd..t \;-\; \gamma y_{svd,t} \;\times\; Yzsvd..(t-1) \;\geq\; -\,gy_{svd,t},$$

where

$\gamma y_{svd,t}$	is the minimum growth rate per period for the construction of technology$v$,
$gy_{svd,t}$	is the "last" size (decrement) allowing technologies to go out of the market, and
$Yzsvd..t$	is the annual new installation of technology v in period$t$.

2.2.4 Upper Dynamic Constraints on Activity Variables

$Mzsvd..t$

The dynamic production constraints relate the production of a technology in one period to the production in the previous period. If the technology is defined with load regions, the sum over the load regions is included in the constraint.

$$\sum_l \epsilon_{svd} \;\times\; \Big[\, zsvd..lt \;-\; \gamma a^o_{svd,t} \;\times\; zsvd..l(t-1) \Big] \;\leq\; ga^o_{svd,t},$$

where

$\gamma a^o_{svd,t}$	and $ga^o_{svd,t}$ are the maximum growth rate and increment as described in section 2.2.2 (the increment is to be given in units of main output), and
$zsvd..lt$	is the activity of technology v in load region l.

If demand elasticities are modelled, the required sums are included for end-use technologies.

2.2.5 Lower Dynamic Constraints on Activity Variables

$Lzsvd..t$

$$\sum_l \epsilon_{svd} \;\times\; [\, zsvd..lt \;-\; \gamma a_{svd,t} \;\times\; zsvd..l(t-1)\,] \;\geq\; -\,ga_{svd,t},$$

where

$\gamma a_{svd,t}$	and $ga_{svd,t}$ are the maximum growth rate and increment as described in section 2.2.3, and
$zsvd..lt$	is the activity of technology v in load region l.

Chapter 3

Storage Technologies

3.1 Variables

Energy storage technologies are modelled with two types of flow variables, and one or two types of capacity variables.

The energy flows in a storage go in two directions: into and out of the storage facility. These two types of flows are represented by two types of activity variables, the input and the output variables of storage technologies.

The capacity variables represent the capacity for energy input and output and the volume capacity of the technology. In this case the relation between I/O capacity and volume capacity is determined by the optimization. Alternatively the relation can be predefined, no volume capacity variables are generated.

3.1.1 Input to Energy Storage Technologies

$$SIzsv.lt,$$

where

SI	identifies storage input variables,
s	is the identifier of the energy carrier to be stored,
z	identifies the level on that the energy carrier is defined,
v	is an additional identifier of the storage technology (this is necessary in the case that several storage technologies for the same energy carrier are defined),
l	identifies the load region in that the energy is put into storage, and
t	is the period identifier.

The storage input variables are energy flow variables and represent the amount of fuel s that is stored in storage with identifier v in load region l and period t (if storing energy entails energy losses, the variable represents the amount *before* the losses apply, i.e. the amount used for storing).

3.1.2 Output from Energy Storage Technologies

$$Ozsv.lmt,$$

where

O	identifies storage output variables,
s	is the identifier of energy carrier stored in the technology,
z	identifies the level on that the energy carrier is defined,
v	is an additional identifier of the storage technology,
l	is the load region in that the energy was put into storage,
m	is the load region in that the energy is retrieved, and
t	is the period identifier.

The storage output variables are energy flow variables and represent the amount of fuel s that was put into storage in load region l and is retrieved in load region m (if retrieving energy entails energy losses, the variable represents the amount *before* the losses apply, i.e. the amount by that the contents of the storage is reduced).

3.1.3 Input/Output Capacity

$$YzGsv..t,$$

where

Y	identifies the capacity variables,
z	is the level on that this energy form is defined,
G	identifies the I/O capacity variables for storage technologies (generation capacity),
s	is the identifier of energy carrier stored in the technology,
v	is an additional identifier of the storage technology, and
t	is the period in that the new capacity is built.

The I/O capacity variables of storage technologies are power variables and represent the annual construction of capacity to fill and empty the storage. They can be defined continuous or integer like the capacity variables of conversion technologies.

3.1.4 Volume Capacity

$$YzVsv..t,$$

where

Y	identifies the capacity variables,
z	is the level on that this energy form is defined,
V	identifies the volume capacity variables for storage technologies,
s	is the identifier of energy carrier stored in the technology,

v	is an additional identifier of the storage technology, and
t	is the period in that the new capacity is built.

The volume capacity variables of storage technologies are stock-pile variables and have an energy- and not power-related unit. They represent the annual new installation of the "container". They can be defined continuous or integer like the other capacity variables. If the relation between I/O and volume capacity in a storage system is predefined, the volume capacity is represented by the I/O capacity with this relation as coefficient.

3.2 Constraints

The flows into and out of a storage technology are balanced by the storage balance constraint.

The input/output capacity of the storage is directly determined from the energy flows in the I/O capacity constraint. The volume capacity is calculated from these variables and either used to determine the volume capacity variables or once more the I/O capacity variables via the relation between volume and I/O capacity.

3.2.1 Storage Balance Constraint

$$Szsv..lt$$

Section 10.5 describes the background of the implementation of energy storage in MESSAGE. In the storage balance constraints the energy flows into and out of the storage technologies are balanced. MESSAGE keeps track of the time that a certain amount is kept in storage by using a separate storage output variable for each pair of input and output load regions. In the following two examples are given; the equations differ for different kinds of storage (e.g., daily, weekly, seasonal).

Daily Storage

The energy can only be balanced over the load regions of one day, not between the seasons.

$$\epsilon_{sv} \times SIzsv.lt - \sum_{m=l+1}^{l+m_{sv}} \frac{1}{\zeta_{l,m}} \times Ozsv.lmt \geq 0 \,,$$

Seasonal Storage

The storage can be used for all types of load defined in the model, since the season is the highest category. An adequate amount of the energy stored at the end of the year is put forward to the next period.

$$\epsilon_{sv} \times SIzsv.lt - \sum_{m=l+1}^{l+m_{sv}} f^1_{l,m} \times \frac{1}{\zeta_{l,m}} \times Ozsv.lmt -$$

$$\sum_{m=l+1}^{l+m_{sv}} f^2_{l,m} \times \frac{1}{\zeta_{l,m}} \times Ozsv.lm(t+1) \geq 0 ,$$

In the above equations the following notation is used

$f_{l,m}$ forwards the appropriate amount of fuel to the next period (this is important for small time steps, for instance $\Delta t = 1$),

$$f^1_{l,m} = \begin{cases} 1 & for\, l < m \\ \frac{\Delta t - 1}{\Delta t} & for\, l > m , \end{cases}$$

$$f^2_{l,m} = \begin{cases} 0 & for\, l < m \\ \frac{1}{\Delta (t+1)} & for\, l > m , \end{cases}$$

$SIzsv.lt$ is the amount of fuel s put into storage v in load region l,
$Ozsv.lmt$ is the amount of fuel s taken out of storage v in load region m, which was put into storage in load region l,
ϵ_{sv} is the efficiency of putting fuel s into storage v (e.g. the pumping losses in pumped hydro storage plants can be accounted for this way),
m_{sv} is the number of load regions that the fuel can be stored. It depends on the kind of storage (for daily storage it is the number of load regions that represent one day, for seasonal storage the whole year, therefore all load regions) and if there is an explicit limit given (e.g., the temperature inside a heat storage can fall below the level where it still can be retrieved after a certain time),
$\zeta_{l,m}$ is the decrease of storage contents from load region$l$ to load region$m$, used for heat storage (exponential decay), and
Δt is the length of period t in years.

3.2.2 Input/Output Capacity

$CzGsv.lt$

This equation defines the capacity of storing or releasing energy per unit of time in a certain storage technology.

$$\frac{\epsilon_{sv}}{\lambda_l} \times \left[SIzsv.lt + \sum_{m=l-m_{sv}}^{l-1} Ozsv.mlt \right] -$$

$$\sum_{\tau=t-\tau_{sv}}^{min(t,\kappa_{sv})} \pi_{sv} \times \Delta(\tau - 1) \times f_i \times YzGsv..\tau \le hc_{sv,G}^t \times \pi_{sv} ,$$

where

$SIzsv.lt$	and $Ozsv.mlt$ are the flows into and out of the storage technology v, as described in sections 3.1.1 and 3.1.2
$YzGsv..\tau$	is the generation capacity of storage v as described in section 3.1.3
ϵ_{sv}	is the efficiency of storage technology v,
λ_l	is the length of load region l as fraction of the year,
κ_{sv}	is the last period in that technology v can be constructed,
π_{sv}	is the plant factor of technology v,
$\Delta(\tau - 1)$	is the length of period $\tau - 1$ in periods,
τ_{sv}	is the plant life of technology v in years,
$hc_{sv,G}^t$	represents the installations built before the time horizon under consideration, that are still in operation in period t,
f_i	is 1. if the capacity variable is continuous, and equal to the minimum installed capacity per year (unit size) if the variable is integer.

3.2.3 Volume Capacity

$CzVsv.lt$

The amount of energy that can be stored (the maximum content at a time) can either be linked to the I/O capacity or evaluated during optimization. Thus either a predefined storage technology like batteries can be modelled or the model can have the choice to optimize the relation between I/O capacity and storage volume.

$$\sum_{m=l-m_{sv}^l}^{l} \zeta_{m,l} \times \left[\epsilon_{sv} \times SIzsv.mt - \sum_{n=m_{sv}+1}^{l} \frac{1}{\zeta_{m,n}} \times Ozsv.mnt \right] \times$$

$$\frac{1}{nl \times \lambda_l} - \left\{ \begin{array}{l} \sum_{\tau=t-\tau_{sv}}^{min(t,\kappa_{sv})} \Delta(\tau-1) \times \pi_{sv} \times f_i \times YzVsv..\tau \\ \sum_{\tau=t-\tau_{sv}}^{min(t,\kappa_{sv})} \Delta(\tau-1) \times \pi_{sv} \times f_{gv} \times f_i \times YzGsv..\tau \end{array} \right\} \leq hc^t_{sv,V} \times \pi_{sv} ,$$

where

$SIzsv.lt$	and $Ozsv.mlt$ are the flows into and out of the storage technology v, as described in sections 3.1.1 and 3.1.2,
$YzGsv..\tau$	is the generation capacity of storage v as described in section 3.1.3,
f_{gv}	is the relation of I/O to volume capacity,
$YzVsv..\tau$	is the volume capacity variable as described in section 3.1.4,
nl	is the number of occurences per year (1 for seasonal, 365 for daily, etc.),
$hc^t_{sv,V}$	represents the installations built before the time horizon under consideration, that are still in operation in period t,
f_i	is 1. if the capacity variable is continuous, equal to the minimum installed capacity per year (unit size) if the variable is integer,
$\zeta_{m,l}$	is the decrease parameter as described in section 10.5,
m^l_{sv}	is described in section 10.5,

Chapter 4

Domestic Resources

4.1 Variables

Extraction of domestic resources is modelled by variables that represent the quantity extracted per year in a period. A subdivision into cost categories (which are called "grades" in the model) and further into elasticity classes can be modelled.

4.1.1 Resource Extraction Variables

$$Rzrgp..t,$$

where

R	identifies resource extraction variables,
z	is the level on that the resource is defined (usually = R),
r	is the identifier of the resource being extracted,
g	is the grade (also called cost category) of resource r, $g \in \{a, b, c, ...\}$.
p	is the class of supply elasticity, which is defined for the resource and grade, or "." , if no elasticity is defined for this resource and grade, and
t	identifies the period.

The resource variables are energy flow variables and represent the annual rate of extraction of resource r. If several grades are defined, one variable per grade is generated (identifier g in position 4). Supply elasticities can be defined for resource extraction as described in section 10.11; in this case one variable per elasticity class (identifier p in position 5) is generated.

4.2 Constraints

The overall availability of a resource is limited in the availability constraint per grade, annual resource consumption can be constrained per grade (sum of the elasticity classes) and total. Additionally resource depletion and dynamic resource extraction constraints can be modelled.

4.2.1 Resource Availability per Grade

$RRrg.g..$

Limits the domestic resource available from one cost category (grade) over the whole time horizon. Total availability of a resource is defined as the sum over the grades.

$$\sum_p \sum_t \Delta t \times RRrgp..t \leq Rrg - \Delta t_0 R_{rg,0} ,$$

where

Rrg	is the total amount of resource r, cost category g, that is available for extraction,
$RRrgp..t$	is the annual extraction of resource r, cost category (grade) g and elasticity class p in period t,
Δt	is the length of period t.
Δt_0	is the number of years between the base year and the first model year, and
$R_{rg,0}$	is the extraction of resource r, grade g in the base year.

4.2.2 Maximum Annual Resource Extraction

$RRr....t$

Limits the domestic resources available annually per period over all cost categories.

$$\sum_g \sum_p RRrgp..t \leq Rrt ,$$

where

$Rrgt$	is the maximum amount of resource r, grade g, that can be extracted per year of period t, and
$RRrgp..t$	is the annual extraction of resource r, cost category (grade) g and elasticity class p in period t.

4.2.3 Resource Depletion Constraints

$RRrg.d.t$

The extraction of a resource in a period can be constrained in relation to the total amount still existing in that period. For reasons of computerization these constraints can also be generated for imports and exports, although they do not have any relevance there (they could, e.g., be used for specific scenarios in order to stabilize the solution).

$$\Delta t \sum_{p} RRrgp..t \;\leq\; \delta_{rg}^{t} \left[Rrg \;-\; \Delta t_0 R_{rg,0} \;-\; \sum_{\tau=1}^{t-1} \Delta\tau \;\times\; RRrgp..\tau \right] ,$$

where

Rrg	is the total amount of resource r, cost category g, that is available for extraction,
$RRrgp..t$	is the annual extraction of resource r, cost category (grade) g and elasticity class p in period t,
δ_{rg}^{t}	is the maximum fraction of resource r, cost category g, that can be extracted in period t,
Rrg	is the total amount available in the base year,
Δt	is the length of period t in years,
Δt_0	is the number of years between the base year and the first model year, and
$R_{rg,0}$	is the extraction of resource r, grade g in the base year.

4.2.4 Maximum Annual Resource Extraction per Grade

$RRrg.a.t$

Limits the domestic resources available from one cost category per year.

$$\sum_{p} RRrgp..t \;\leq\; Rrgt\,.$$

where

Rrg	is the total amount of resource r, cost category g, that is available for extraction, and
$RRrgp..t$	is the annual extraction of resource r, cost category (grade) g and elasticity class p in period t.

4.2.5 Upper Dynamic Resource Extraction Constraints

$MRRr...t$

The annual extraction level of a resource in a period can be related to the previous one by a growth parameter and an increment of extraction capacity resulting in upper dynamic extraction constraints. For the first period the extraction is related to the activity in the baseyear.

$$\sum_{g,p} RRrgp..t \; - \; \gamma^{o}_{rt} \sum_{g,p} RRrgp..(t-1) \; \leq \; g^{o}_{rt} \, ,$$

where

γ^{o}_{rt}	is the maximum growth of extraction of resource r between period $t-1$ and t,
g^{o}_{rt}	is the initial size (increment) of extraction of resource r in period t, and
$RRrgp..t$	is the annual extraction of resource r, cost category (grade) g and elasticity class p in period t.

4.2.6 Lower Dynamic Resource Extraction Constraints

$LRRr...t$

The annual extraction level of a resource in a period can also be related to the previous one by a decrease parameter and a decrement resulting in lower dynamic extraction constraints. For the first period the extraction is related to the activity in the baseyear.

$$\sum_{g,p} RRrgp..t \; - \; \gamma_{rt} \sum_{g,p} RRrgp..(t-1) \; \geq \; - \, g_{rt} \, ,$$

where

γ_{rt}	is the maximum decrease of extraction of resource r between period $t-1$ and t,
g_{rt}	is the "last" size (decrement) of extraction of resource r in period t, and
$RRrgp..t$	is the annual extraction of resource r, cost category (grade) g and elasticity class p in period t.

4.2.7 Dynamic Extraction Constraints per Grade

$MRRrg..t$, and
$LRRrg..t$

The same kind of relations as described in sections 4.2.5 and 4.2.6 can be defined per grade of the resource.

Chapter 5

Imports and Exports

5.1 Variables

Imports and exports are modelled by variables that represent the quantity imported per year in a period. A subdivision into countries and further into elasticity classes can be modelled.

5.1.1 Import Variables

$$Izscp.lt,$$

where

I	identifies import variables,
z	is the level on that the imported energy form is defined (usually primary energy and secondary energy),
s	identifies the imported energy carrier,
c	is the identifier of the country or region the imports come from,
p	is the class of supply elasticity, which is defined for the energy carrier and country, or ".", if no elasticity is defined for this energy carrier and country,
l	is the load region identifier if s is modelled with load regions, otherwise ".", and
t	identifies the period.

The import variables are energy flow variables and represent the annual import of the identified energy carrier from the country or region given. If supply elasticities are defined for the import of this energy carrier and country one variable per elasticity class (identifier p in position 5) is generated.

5.1.2 Export Variables

$$Ezrcp.lt,$$

where

E	is the identifier for export variables, and
z	is the level on that the exported energy form is defined (usually primary energy and secondary energy),
s	identifies the exported energy carrier,
c	is the identifier of the country or region the exports go to,
p	is the class of supply elasticity, which is defined for the energy carrier and country, or ".", if no elasticity is defined for this energy carrier and country,
l	is the load region identifier if s is modelled with load regions, otherwise ".", and
t	identifies the period.

The export variables are energy flow variables and represent the annual export of the identified energy carrier to the country or region given. If supply elasticities are defined for the export of this energy carrier and country one variable per elasticity class (identifier p in position 5) is generated.

5.2 Constraints

5.2.1 Imports per Country

$$Izrc.g..$$

Limits the imports of a fuel from a specific country c over the whole horizon.

$$\sum_p \sum_t \Delta t \times Izrcp..t \leq Irc \; ,$$

where

Irc	is the total import limit for r from country c,
$Izrcp..t$	is the annual import of r from country c, elasticity class p in period t, and
Δt	is the length of period t in years.

5.2.2 Maximum Annual Imports

$$Izr....t$$

Limits the annual imports of a fuel from all countries per period.

$$\sum_c \sum_p Izrcp..t \leq Irt \; ,$$

where

Irt	is the annual import limit for r in period t, and
$Izrcp..t$	is the annual import of r from country c, elasticity class p in period t.

5.2.3 Maximum Annual Imports per Country

$Izrc.a.t$

Limits the imports from one country per year.

$$\sum_{p} Izrcp..t \;\leq\; Irct\,,$$

where

$Irct$	is the limit on the annual imports from country c, period t of fuel r, and
$Izrcp..t$	is the annual import of r from country c, elasticity class p in period t.

5.2.4 Upper Dynamic Import Constraints

$MIzr...t$

The annual import level of a fuel in a period can, like the resource extraction, be related to the previous one by a growth parameter and an increment resulting in upper dynamic constraints.

$$\sum_{c,p} Izrcp..t \;-\; \gamma^{o}_{rt} \sum_{c,p} Izrcp..(t-1) \;\leq\; g^{o}_{rt}\,,$$

where

$Izrcp..t$	is the annual import of r from country c, elasticity class p in period t,
γ^{o}_{rt}	is the maximum increase of import of r between period $t-1$ and t, and
g^{o}_{rt}	is the initial size (increment) of import of r in period t.

5.2.5 Lower Dynamic Import Constraints

$LIzr...t$

The annual import level of a fuel in a period can also be related to the previous one by a decrease parameter and a decrement resulting in lower dynamic import constraints.

$$\sum_{c,p} Izrcp..t \;-\; \gamma_{rt} \sum_{c,p} Izrcp..(t-1) \;\geq\; -\,g_{rt}\,,$$

where

$Izrcp..t$	is the annual import of r from country c, elasticity class p in period t,
γ_{rt}	is the maximum decrease of import of r between period $t-1$ and t, and
g_{rt}	is the "last" size (decrement) of import of r in period t.

5.2.6 Dynamic Import Constraints per Country

$MIzrc..t$ and

$LIzrc..t$

The same kind of relations can be defined per country from that the fuel is imported.

5.2.7 Constraints on Exports

The exports of fuels can principally be limited in the same way as the imports. In the identifiers of the variables and constraints the "I" is substituted by an "E".

Chapter 6

Energy Flows

6.1 Constraints

Energy flows are modelled solely by linking the activity variables of the different conversion technologies and the resource extraction, import and export variables in balance constraints. These constraints ensure that only the amounts of energy available are consumed. There are no further variables required to model energy flows.

Energy demands are also modelled as part of a balance constraint: it is the right hand side and defines the amount to be supplied by the technologies in this constraint.

The following description of the energy flow constraints in MESSAGE is given for the following set of level identifiers:

U Useful energy (demand level),

F Final energy (after distribution),

T Final energy (after transmission),

X Secondary energy,

A Primary energy, and

R Energy resources.

The identifier of the demand level (U) which gives it a special meaning (see section 2.1.1) and imports and exports are given for primary energy. Clearly any other combination of technologies is also possible.

6.1.1 Demand Constraints

$Ud.....t$

Out of the predefined levels each one can be chosen as demand level. However, level "U" has a special feature. This is related to the fact that useful energy is usually produced on-site, e.g., space heat is produced by a central heating system, and the load variations over the year are all covered by this one system. Thus, an allocation of production technologies to the different areas of the load curve, like the model would set it up according to the relation between investment and operating costs would ignore the fact that these systems are not located in the same place and are not connected to each other. MESSAGE represents the end-use technologies by one variable per period that produces the required useful energy in the load pattern needed and requires the inputs in the same pattern. For special technologies like, e.g., night storage heating systems, this pattern can be changed to represent the internal storage capability of the system.

This representation of end-use technologies has the advantage of reducing the size of the model, because the demand constraints, the activity variables and the capacity constraints of the end-use technologies do not have to be generated for each load region.

If another level is chosen as demand level or the demand level is not named "U", all demand constraints for energy carriers that are modelled with load regions are generated for each load region. The general form of the demand constraints is

$$\sum_{svd} \epsilon_{svd} \times \sum_{e=0}^{e_d} k_e \times Usvd.e.t + \sum_{sv\delta} \beta^d_{sv\delta} \times \sum_{e=0}^{e_\delta} k_e \times Usv\delta ue.t \ \geq \ Ud.t \, ,$$

where

$Ud.t$ is the annual demand for d in period t,

$Usvd.e.t$ is the activity of end-use technology v in period t, elasticity class e and period t (see section 2.1.1),

ϵ_{svd} is the efficiency of end-use technology v in converting s to d,

$\beta^d_{sv\delta}$ is the efficiency of end-use technology v in producing by-product d from s (δ is the main output of the technology),

e_d is the number of steps of demand reduction modelled for own-price elasticities of demand d, and

k_e is the factor giving the relation of total demand for d to the demand reduced to level e due to the demand elasticity. ($k_e \times Usvd.e.t = Usvd.0.t$, $k_0 = 1$, k_e is increasing monotonously.)

6.1.2 Distribution Balance

$Fs....lt$

This constraint, the final energy balance, matches the use of final energy needed in the end-use technologies and the deliveries of the distribution systems. It is generated for each load region, if energy form s is modelled with load regions.

$$\sum_{svs} \epsilon_{svs} \times Fsvs..lt - \sum_{svd} \eta_{d,l} \times \sum_{e=0}^{e_d} Usvd.e.t -$$

$$\sum_{\sigma vd} \beta^s_{\sigma vd} \times \eta_{d,l} \times \sum_{e=0}^{e_d} U\sigma vd.e.t \geq 0 ,$$

where

$Fsvs..lt$	is the activity of the distribution technology in load region l and period t (see section 2.1.1),
ϵ_{svs}	is the efficiency of technology v in distributing s,
$Usvd.e.t$	is the activity of end-use technology v in period t and elasticity class e,
$\beta^s_{\sigma vd}$	is the use of fuel s relative to fuel σ (the main input) by technology v, and
$\eta_{d,l}$	is the fraction of demand for d occurring in load region l.

6.1.3 Transmission or Transportation Balance

$$Ts....lt$$

This constraint gives the simplest form of an energy balance equation of MESSAGE. It matches the output of transmission to the requirements of distribution systems. The difference to other levels (F, X, A) is not built-in, but emerges from the simplicity of energy transportation (i.e., transportation technologies do usually not have by-products and only one input). Also big industrial consumers that are directly connected to the transmission system would have to be included in this constraint. Like level F it does usually exist for all load regions if they are defined for the fuel.

$$\sum_{svs} \epsilon_{svs} \times Tsvs..lt - \sum_{svs} Fsvs..lt \geq 0 .$$

where

$Tsvs..lt$	is the activity of the transportation technology v (see section 2.1.1), and

all the other entries to the equation are the same as in section 6.1.2.

6.1.4 Central Conversion Balance

$Xs....lt$

In principle the secondary energy balance is built up in the same way as the two previous ones (sections 6.1.2 and 6.1.3). It matches the production of central conversion technologies to the requirements of the transmission systems. Secondary energy imports and exports of secondary energy are usually assigned to level X.

$$\sum_{rvs} \epsilon_{rvs} \times Xrvs..lt + \sum_{rv\sigma} \beta^s_{rv\sigma} \times Xrv\sigma..lt - \sum_{svs} Tsvs..lt +$$

$$\sum_{c,p} IXscp.lt - \sum_{c,p} EXscp.lt \geq 0$$

where

$Xrvs..lt$ is the activity of central conversion technology v in load region l and period t (see section 2.1.1); if the secondary energy form s is not defined with load regions (i.e. $l = "."$) and the activity of technology v exists for each load region, this equation will contain the sum of the activity variables of technology v over the load regions.

ϵ_{rvs} is the efficiency of technology v in converting energy carrier r into secondary energy form s,

$\beta^s_{rv\sigma}$ is the efficiency of technology v in converting energy carrier r into the by-product s of technology v,

$Tsvs..lt$ is explained in section 6.1.3, and

$IXscp.lt$ and $EXscp.lt$ are the import and export variables explained in sections 5.1.1 and 5.1.2, respectively.

6.1.5 Resource Extraction, Export and Import Balance

$Ar.....t$

This equation matches production and import of primary energy to the requirements of central conversion, transport and for export. In the general case primary energy does not have load regions. Some technologies, like, e.g., nuclear reactors need inventories of primary energy and also leave a last core that is available at the end of the lifetime. It may be necessary to model by-products of extraction technologies, for instance the availability of associated gas at oil production sites.

$$\sum_{rvr} \epsilon_{rvr} \times Arvr...t - \sum_{l} \left[\sum_{rvs} Xrvs..lt + \sum_{\rho vs} \beta^r_{\rho vs} \times X\rho vs..lt \right] + \sum_{c,p} IArcp..t -$$

$$\sum_{c,p} EArcp..t \; + \; \sum_{fvs} \Big[\; \frac{\Delta(t-\tau_{fvs})}{\Delta t} \; \times \; \rho(fvs,r) \; \times \; YXfvs..(t-\tau_{fvs}) \; -$$

$$\frac{\Delta(t+1)}{\Delta t} \; \times \; \iota(fvs,r) \; \times \; YXfvs..(t+1) \Big] \; \geq \; 0 \,,$$

where

$Arvr...t$	is the activity of technology v extracting resource r,
ϵ_{rvr}	is the efficiency of technology v in extracting fuel r (this is usually 1.),
$\beta^r_{\rho vs}$	is the efficiency of technology v in producing secondary energy form s from the by-input ρ,
$IArcp..t$	and $EArcp..t$ are the import and export variables described in section 5.1.1 and 5.1.2, respectively,
τ_{fvs}	is the plant life of technology v in periods (depending on the lengths of the periods covered),
$YXfvs..t$	is the annual new installation of technology v in period t (see section 2.1.2),
$\iota(fvs,r)$	is the amount of fuel r that is needed when technology v goes into operation (usually this is the first core of a reactor). It has to be available in the period before technology v goes into operation, the normal unit is kWyr/kW,
$\rho(fvs,r)$	is the amount of fuel r that becomes available after technology v goes out of operation (for a reactor this is the last core that goes to reprocessing). The unit is the same as for $\iota(fvs,r)$, and
Δt	is the length of period t in years.

6.1.6 Resource Consumption

$Rr.....t$

The resources produced by the extraction technologies in a period can come from different cost categories (also called grades), which can, e.g., represent the different effort to reach certain resources. Short-term variations in price due to steeply increasing demand can be represented by an elasticity approach (see section 10.11).

$$\sum_{g,p} RRrgp..t \; - \; \sum_{rvr} Arvr...t \; \geq \; 0 \,,$$

where

$RRrgp..t$	is the annual extraction of resource r, cost category (grade) g and elasticity class p in period t, and
$Arvr...t$	is the activity of extraction technology v in period t (as described in section 2.1.1).

Chapter 7

Stock-piles

7.1 Variables

Generally MESSAGE does not generate any variables related to an energy carrier alone. However, in the case of man-made fuels, that are accumulated over time, a variable that shifts the quantities to be transferred from one period to the other is necessary.

7.1.1 Stock-pile Variables

$$Qfb....t,$$

where

Q	identifies stock-pile variables,
f	identifies the fuel with stock-pile,
b	distinguishes the variable from the equation, and
t	is the period identifier.

The stock-pile variables represent the amount of fuel f that is transferred from period t into period $t+1$. Note that these variables do not represent annual quantities, they refer to the period as a whole. These variables are a special type of storage, that just transfers the quantity of an energy carrier available in one period into the next period. Stock-piles are defined as a separate level. For all other energy carriers any overproduction that occurs in a period is lost.

7.2 Constraints

7.2.1 Stock-piling Constraints

$$Qf.....t$$

Q is a special level on that energy forms can be defined that are accumulated over time and consumed in later periods. One example is the accumulation of plutonium and later use in fast breeder reactors.

The general form of this constraint is:

$$Qfb....t \;-\; Qfb....(t-1) \;+\; \sum_v \left[\sum_l \Delta t \;\times\; (\; zfvd..lt \;+\; \beta^f_{\phi vd} \;\times\; z\phi vd..lt \;-\right.$$

$$\epsilon_{svf} \;\times\; zsvfu.lt \;-\; \beta^f_{sv\phi} \;\times\; zsv\phi..lt \;) \;+\; \Delta t \;\times\; \iota(svd,f) \;\times\; Yzsvd..(t+1) \;-$$

$$\left. \Delta(t-\tau_{svd}-1) \;\times\; \rho(svd,f) \;\times\; Yzsvd..(t-\tau_{svd}) \;\right] \;=\; 0\,,$$

where

f	is the identifier of the man-made fuel (e.g. plutonium, U_{233}),
τ_{svd}	is the plant life of technology v in periods,
$\iota(svd,f)$	is the "first inventory" of technology v of f (relative to capacity of main output),
$\rho(svd,f)$	is the "last core" of f in technology v, see also section 6.1.5,
Δt	is the length of period t in years,
$zfvd..lt$	is the annual input of technology v of fuel f in load region l and period t (l is "." if v does not have load regions), and
$Yzfvd..t$	is the annual new installation of technology v in period t.

Chapter 8

User-defined Relations

8.1 Constraints

The user-defined relations allow the user to construct constraints that are not included in the basic set of constraints. For each technology the user can specify coefficients with that either the production variables (see section2.1.1), the annual new installation variables (see section 2.1.2) or the total capacity in a year (like it is used in the capacity constraints, see section 2.2.1) can be included in the relation. The relations can be defined with and without load regions, have a lower, upper or fix right hand side or remain free (non-binding) and be related to an entry in the objective function, i.e., all entries to this relation are also entered to the objective function with the appropriate discount factor. There are two types of user-defined constraints, for which the entries to the objective function–without discounting–are summed up under the cost accounting rows $CAR1$ and $CAR2$ (see chapter 9).

The formulation of the user-defined relations is given for relations, that are related to the main output of the technologies. It is also possible (e.g., for emissions) to relate the constraint to the main input of the technology, i.e. the amount of fuel used. In this case the efficiencies (ϵ) would be omitted from the formulation.

8.1.1 Relation without Load Regions

$$Nm.....t \text{ or } Pm.....t$$

Relations without load regions just sum up the activities (multiplied with the given coefficients) of all variables defined to be in this constraint. If a technology has load regions, the activity variables for all load regions of this technology are included. If the total capacity of a technology is included, all new capacities from previous periods still operating are included, if new capacities are included, the annual new installation of the current period is taken.

$$\sum_{svd} \left[ro_{svd}^{mt} \times \sum_{e=0}^{e_d} Usvd.e.t \times \epsilon_{svd} + \sum_{\tau=t-ip}^{t} rc_{svd}^{mt} \times YUsvd..\tau \right] +$$

$$\sum_{rvs} \left[ro^{mlt}_{rvs} \times \sum_{l} zrvs..lt \times \epsilon_{rvs} + ro^{mt}_{rvs} \times zrvs...t \times \epsilon_{rvs} + \right.$$

$$\left. \sum_{\tau=t-ip}^{t} rc^{mt}_{rvs} \times Yzrvs..\tau \right] \begin{cases} free \\ \geq rhs^t_m \\ = rhs^t_m \\ \leq rhs^t_m \end{cases} ,$$

where

$Usvd.e.t$	and $YUsvd..t$ are the activity and capacity variables of the end-use technologies,
$zrvs..lt$,	$zrvs...t$ and $Yzrvs..t$ are the activity variables of technologies with and without load regions and the capacity variables of the technologies,
ϵ_{rvs}	and ϵ_{svd} are the efficiencies of the technologies; they are included by the code,
ro^{mt}_{svd}	is the relative factor per unit of output of technology v (coefficient) for relational constraint m,
rc^{mt}_{svd}	is the same per unit of new built capacity,
ro^{mlt}_{rvs}	is the relative factor per unit of output of technology v (coefficient) for relational constraint m, load region l,
rc^{mlt}_{rvs}	is the same per unit of new built capacity,
tl	is 1 for relations to construction and $\Delta\tau$ for relations to total capacity,
ip	is 1 for accounting during construction and the plant life on periods for accounting of total capacity, and
rhs^t_m	is the right hand side of the constraint.

8.1.2 Relation with Load Regions

$Nm....lt$ or $Pm....lt$

The user defined relations can be defined with load regions. Then all entries of activities of technologies with load regions are divided by the length of the according load region resulting in a representation of the utilized power.

$$\sum_{svd} \left[ro^{mlt}_{svd} \times \sum_{e=0}^{e_d} Usvd.e.t \times \epsilon_{svd} + \sum_{\tau=t-ip}^{t} rc^{mlt}_{svd} \times YUsvd..\tau \right] +$$

$$\sum_{rvs} \left[\frac{ro^{mlt}_{rvs}}{\lambda_l} \times zrvs..lt \times \epsilon_{rvs} + ro^{mt}_{rvs} \times zrvs...t \times \epsilon_{rvs} + \right.$$

$$\sum_{\tau=t-ip}^{t} rc_{rvs}^{mlt} \times tl \times Yzrvs..\tau\Bigg] \begin{cases} free \\ \geq rhs_{ml}^{t} \\ = rhs_{ml}^{t} \\ < rhs_{ml}^{t} \end{cases},$$

where

$Usvd.e.t$	and $YUsvd..t$ are the activity and capacity variables of the end-use technologies,
$zrvs..lt$,	$zrvs...t$ and $Yzrvs..t$ are the activity variables of technologies with and without load regions and the capacity variables of the technologies,
ϵ_{rvs}	and ϵ_{svd} are the efficiencies of the technologies; they are included by the code,
ro_{svd}^{mlt}	is the relative factor per unit of utilized capacity of technology v (coefficient) for relational constraint m in load region l, period t (this constraint is adapted to represent the utilized power, as stated above),
rc_{svd}^{mlt}	is the same per unit of new built or installed capacity,
ro_{rvs}^{mlt}	is the relative factor per unit of output of technology v (coefficient) for relational constraint m, load region l,
rc_{rvs}^{mlt}	is the same per unit of new built capacity,
tl	is 1 for relations to construction and $\Delta\tau$ for relations to total capacity,
ip	is 1 for accounting during construction and the plant life on periods for accounting of total capacity, and
rhs_{ml}^{t}	and is the right hand side of the constraint.

8.1.3 Construction of Relations between Periods

$Nm.....t$ or $Pm.....t$

The change of activities over time can either be limited or included in the objective by constructing relations between periods: The relations expresses the difference between the annual activity in a period and the following period. This difference can either be limited or included in the objective function.

$$\sum_{svd}\Bigg[ro_{svd}^{mt} \times \sum_{e=0}^{e_d} Usvd.e.t \times \epsilon_{svd} - ro_{svd}^{m(t-1)} \times$$

$$\sum_{e=0}^{e_d} Usvd.e.(t-1) \times \epsilon_{svd}\Bigg] + \sum_{rvs}\Big[ro_{rvs}^{mt} \times zrvs...t \times \epsilon_{rvs} - ro_{rvs}^{m(t-1)} \times$$

$$zrvs...(t-1) \times \epsilon_{rvs}] + \sum_{rvs} \left[ro^{mlt}_{rvs} \times \sum_{l} zrvs..lt \times \epsilon_{rvs} - ro^{ml(t-1)}_{rvs} \times \right.$$

$$\left. \sum_{l} zrvs..l(t-1) \times \epsilon_{rvs} \right] \begin{cases} free \\ \geq rhs^{t}_{m} \\ = rhs^{t}_{m} \\ < rhs^{t}_{m} \end{cases} ,$$

where

$Usvd.e.t$	is the activity variable of the end-use technologies,
$zrvs..lt$	and $zrvs...t$ are the activity variables of technologies with and without load regions,
ϵ_{rvs}	and ϵ_{svd} are the efficiencies of the technologies; they are included by the code,
ro^{mt}_{svd}	is the relative factor per unit of output of technology v (coefficient) for relational constraint m, period t,
ro^{mlt}_{rvs}	is the relative factor per unit of output of technology v (coefficient) for relational constraint m, load region l, and
rhs^{t}_{m}	and is the right hand side of the constraint.

For this type of constraints only the ro-coefficients have to be supplied by the user, the rest is included by the model. It can be defined with and without load regions.

8.1.4 Special Handling of Demand Elasticities

$Pm.....t$

The second type of user defined relations differs from the first one in the fact that the activity of the end-use technologies is multiplied by k_e and therefore represents the production without reduction by demand elasticities.

Thus this constraint can be applied to force a certain reduction level due to the elasticities reached in one period to be also reached in the following period, allowing the interpretation of the reduction as investments in saving. The coefficient of the technologies supplying a demand have to be the inverse of this demand in the current period, then. This constraint has the following form:

$$\sum_{sv} \sum_{e=0}^{e_d} Usvd.e.t \times \epsilon_{svd} \times \frac{\kappa_e}{Ud.t} -$$

$$\sum_{sv} \sum_{e=0}^{e_d} Usvd.e.(t-1) \times \epsilon_{svd} \times \frac{\kappa_e}{Ud.(t-1)} \leq 0 ,$$

where

the coefficients are supplied by MESSAGE. The user can additionally define multiplicative factors for these coefficients.

Chapter 9

Objective and Cost Counters

9.1 Constraints

9.1.1 Cost Accounting Rows

The different types of costs (i.e. entries for the objective function) can be accumulated over all technologies in built-in accounting rows. These rows can be generated per period or for the whole horizon and contain the sum of the undiscounted costs. They can also be limited. The implemented types are:

$CCUR$ – fix (related to the installed capacity) and variable (related to the production) operation and maintenance costs,
$CCAP$ – investment costs; if the investments of a technology are distributed over the previous periods, also the entries to this accounting rows are distributed (if the capital costs are levellized, the total payments in a period can be taken from $CINV$; $CCAP$ shows the share of investments in the according period, then),
$CRES$ – domestic fuel costs,
$CAR1$ – costs related to the user defined relations of type 1 (see section 8),
$CAR2$ – costs related to the user defined relations of type 2 (see section 8),
$CRED$ – costs for reducing demands due to demand elasticities, only related to technologies supplying the demands directly,
$CIMP$ – import costs,
$CEXP$ – gains for export, and
$CINV$ – total investments (in case of levellized investment costs, see $CCAP$)

9.1.2 The Objective Function

$$FUNC$$

In its usual form the objective function contains the sum of all discounted costs, i.e. all kinds of costs that can be accounted for. All costs related to operation (i.e. resource use, operation costs, costs of demand elasticities,...) are discounted from the middle of the current period to the first year. Costs related to construction are by default discounted from the beginning of the current period to the first year. By using the facility of distributing the investments or accounting during construction these costs can be distributed over some periods before or equal to the current one (see section 10.2). This distribution can also be performed for user defined relations.

The objective function has the following general form:

$$\sum_t \left[\beta_m^t \, \Delta t \left\{ \sum_{svd} \sum_l zsvd..lt \times \epsilon_{svd} \times \left[ccur(svd,t) + \sum_i \sum_m ro_{svd}^{mlt} \times cari(ml,t) \right] + \right.\right.$$

$$\sum_{svd} \epsilon_{svd} \times \sum_{e=0}^{e_d} Usvd.e.t \times \epsilon_{svd} \times \left[\kappa_e \times (ccur(svd,t) + \sum_m ro_{svd}^{mt} \times car2(m,t)) + \right.$$

$$\left. cred(d,e) + \sum_m ro_{svd}^{mt} \times car1(m,t) \right] + \sum_{svd} \sum_{\tau=t-\tau_{svd}}^{t} \Delta\tau \times Yzsvd..\tau \times cfix(svd,\tau) +$$

$$\sum_r \left[\sum_g \sum_l \sum_p Rzrgp.lt \times cres(rgpl,t) + \right.$$

$$\left.\left. \sum_c \sum_l \sum_p Izrcp.lt \times cimp(rcpl,t) - \sum_c \sum_l \sum_p Ezrcp.lt \times cexp(rcpl,t) \right] \right\} +$$

$$\beta_b^t \times \left\{ \sum_{svd} \sum_{\tau=t}^{t+t_d} \Delta(t-1) \times Yzsvd..\tau \times \left[ccap(svd,\tau) \times fri_{svd}^{t_d-\tau} + \right.\right.$$

$$\left.\left.\left. \sum_i \sum_m rc_{svd}^{mt} \times cari(m,t) \times fra_{svd,m}^{t_d-\tau} \right] \right\} \right] ,$$

where

Δt is the length of period t in years,

$$\beta_b^t = \prod_{i=1}^{t-1} \left[\frac{1}{1+\frac{dr(i)}{100}}) \right]^{\Delta i} ,$$

$$\beta_m^t = \beta_b^t \times \left[\frac{1}{1+\frac{dr(t)}{100}}) \right]^{\frac{\Delta t}{2}} ,$$

$dr(i)$ is the discount rate in period i in percent,

$zsvd..lt$ is the annual consumption of technology v of fuel s load region l and period t; if v has no load regions, $l =$ ”.”.

ϵ_{svd} is the efficiency of technology v in converting s to d,

$ccur(svd,t)$ are the variable operation and maintenance costs of technology v (per unit of main output) in period t,

ro^{mlt}_{svd} is the relative factor per unit of output of technology v for relational constraint m in period t, load region l,

$car1(m,t)$ and $car2(m,t)$ are the coefficients for the objective function, that are related to the user defined relation m in period t,

$car1(ml,t)$ and $car2(ml,t)$ are the same for load region l, if relation m has load regions,

$Usvd.e.t$ is the annual consumption of fuel s of end-use technology v in period t and elasticity class e,

κ_e is the factor giving the relation of total demand for d to the demand reduced due to the elasticity to level e,

ro^{mt}_{svd} is the relative factor per unit of output of technology v for relational constraint m in period t,

$cred(d,e)$ is the cost associated with reducing the demand for d to elasticity level e,

$Yzsvd..t$ is the annual new built capacity of technology v in period t,

$cfix(svd,t)$ are the fix operation and maintenance cost of technology v that was built in period t,

$ccap(svd,t)$ is the specific investment cost of technology v in period t (given per unit of main output),

fri^{n}_{svd} is the share of this investment that has to be paid n periods before the first year of operation,

rc^{mt}_{svd} is the relative factor per unit of new built capacity of technology v for user defined relation m in period t,

$fra^{n}_{svd,m}$ is the share of the relative amount of the user defined relation m that occurs n periods before the first year of operation (this can, e.g., be used to account for the use of steel in the construction of solar towers over the time of construction),

$Rzrgp.lt$ is the annual consumption of resource r, grade g, elasticity class p in load region l and period t,

$cres(rgpl,t)$ is the cost of extracting resource r, grade g, elasticity class p in period t and load region l (this should only be given, if the extraction is not modelled explicitly),

$Izrcp.lt$ is the annual import of fuel r from country c in load region l, period t and elasticity class p; if r has no load regions l=".",

$cimp(rcpl,t)$ is the cost of importing r in period t from country c in load region l and elasticity class p,

$Ezrcp.lt$ is the annual export of fuel r to country c in load region l, period t and elasticity class p; if r has no load regions l=".", and

$cexp(rcpl,t)$ is the gain for exporting r in period t to country c in load region l and elasticity class p.

Chapter 10

Special Features of the Matrix Generator

The mathematical formulation of MESSAGE as presented in the previous sections shows the structure of all constraints as the matrix generator builds them up. The background of the more complicated features is given here for a better understanding.

10.1 The Time Horizon–Discounting the Costs

The whole time horizon of the calculations is divided into periods of optional length. All variables of MESSAGE are represented as average over the period they represent, resulting in a step-function. All entries in the objective function are discounted from the middle of the respective period to the first year, if they relate to energy flow variables and from the beginning of that period if they represent power variables. The function to discount the costs has the following form:

$$c_t = \frac{C_t^r}{\prod_{k=1}^{t-1} (1 + \frac{dr_k}{100})^{\Delta k} \times f_i} ,$$

where

C_t^r is the cost figure to be discounted,

c_t is the objective function coefficient in period t,

$$f_i = \begin{cases} 1 & \text{for costs connected to investments,} \\ (1 + \frac{dr_t}{100})^{\frac{\Delta t}{2}} & \text{else, and} \end{cases}$$

dr_k is the discount rate in period k.

10.2 Distributions of Investments

In order to support short term applications of MESSAGE the possibility to distribute the investments for a new built technology over several periods was implemented. The same type of distributions can be applied to entries in user defined relations if they relate to construction. The distribution of investments can be performed in several ways. There is one common parameter that is needed for all of these possibilities, the construction time of the technology [*ct*].

The implemented possiblilities are: 1. Explicit definition of the different shares of investments for the years of construction. The input are *ct* figures that will be normalized to 1 internally. 2. The investment distribution is given as a polynomium of 2nd degree, the input consists of the three coefficients:

$$y = a + bx + cx^2 \quad , x = 1(1)ct,$$

where *ct* is the construction time. The values of the function are internally normalized to 1, taking into account the construction time. 3. Equal distribution of the investments over the construction period. 4. A distribution function based on a logistic function of the type

$$f = \frac{100}{1 + e^{-\alpha(x-x_0)}} \ ,$$

where

$$x_0 = \tfrac{ct}{2} \ ,$$

and

$$\alpha = \tfrac{2}{ct} \ln\left(\tfrac{100}{\epsilon} - 1\right) .$$

This function is expanded to a normalized distribution function of the following type:

$$g = \left[\frac{100}{1 + e^{-\frac{ln(\frac{100}{\epsilon} - 1)(x-50)}{50}}} - \epsilon \right] \times \frac{1}{1 - \frac{\epsilon}{50}} \ .$$

g gives the accumulated investment at the time x, x is given in percent of the construction time. The parameter ϵ describes the difference of the investment in the different years. ϵ near to 50 results nearly in equal distribution, an ϵ close to 0 indicates high concentration of the expenditures in the middle of the construction period.

In order to shift the peak of costs away from the middle of the construction period the function is transformed by a polynomium:

$$x = az^2 + bz \quad , 0 < z < 100 ,$$

where

$$b = \frac{5000 - d^2}{100d - d^2} \quad , 0 < d < 100 \, ,$$

and

$$a = \frac{1 - b}{100} \; .$$

d denotes the time at that the peak of expenditures occurs in percent of ct.

The distribution of these yearly shares of investments is done starting in the first period of operation with a one years share, the expenditures of the remaining $ct - 1$ years are distributed to the previous periods.

The coefficients of the capacity variables of a technology in a relational constraint can be distributed like the investments.

10.3 The Load Curve

The years representing a period can be subdivided into so-called load regions. This can be done by either ordering the whole year according to the power requirements for the most important energy carriers like, e.g., electricity, or by grouping the year into load regions with similar characteristics (hereafter called characteristic loads), like, e.g., winter days and nights and summer days and nights. The first option results in an interpolation of the usual representation of the load curve by a step function (see Figure 10.1), the second one in a step-function where the time is still ordered in a historic way.

Figure 10.1: Example of an ordered (left) and a semi-ordered load curve (right) (WD stands fpr winter day, WN for winter night, SD for summer day and SN for summer night.)

10.4 Consideration of Load Variations in Conversion Technologies

The activity of a conversion technology is generated for each load region, if the main in- or output energy form is defined to have load regions. In this case the relation of these activities between the load regions is freely chosen by the model. The relations can be fixed by the user to reflect a certain fixed production pattern. In this case the activity will only be generated once and written to the energy flow balances with coefficients reflecting the chosen pattern. A powerplant operating in baseload mode would for instance have the shares of the load regions in the year as coefficients in the balances of energy forms with load regions.

For end-use technologies (output level "U") the production is assumed to meet the demand pattern, the input of the technology is fixed to reflect the according demand variations. This can also be changed into to a different pattern. This would, e.g., model night storage heating systems that meet the heat demand of a houshold, but generate a final electricity demand with a different load distribution, namely at night.

10.5 The Implementation of Energy Storage

MESSAGE contains a quite complex model of energy storage. Section 3 contains the mathematical formulation. In order to allow for different types of storage like daily and seasonal the distribution of demands over the year has to be depicted in a semi ordered load curve: The user has to define the load regions in a physical order. Daily storage would for instance need the definition of several parts of the day that are ordered like in an actual day. The model can then store energy in one part of the day and release it in one of the following load regions, keeping track of the storage contents in each load region. This loop of storage is closed for all but seasonal storage, where an appropriate part of the energy stored in the last load region is delivered to the next period.

The length of time that the content of a storage can be held can also be limited to some fraction of the time it is dedicated to. An example would be a daily heat storage that can only keep the heat 80% of the day, after that time it could have too low a temperature to be used. The loss of energy in the case of heat storage can be modeled by a decay function:

$$co_{l+1} \;=\; co_l \;\times\; e^{\zeta \,\times\, \delta l},$$

where

co_l	is the content of storage in load region l,
ζ	is the decay constant of storage [unit: $\frac{1}{k}$],
k	is 1 day for daily storage, 1 year for seasonal storage, etc. and
δl	is the fraction of k that lies between load regions l and $l+1$ [unit: k].

The amount of energy available from storage is reduced over time according to this function.

If several types of load regions are defined, e.g. weekly and seasonal, (it should rather be named yearly for reasons of consistency) they are ordered according to the length of the time period they span. The "bigger" one (the seasonal) can then work like the smaller one (weekly), too (see Figure 10.2). The decay of content and limitation on time is only applied to the biggest type of load the storage works in.

Figure 10.2: Flows of energy in daily and seasonal storage

The two basic parts of a storing device, namely the input/output part (for a pumped hydro storage the generator/turbine/pump part) and the real storage (dam and reservoir) can be handled in two different ways. One of them is to link them in size, i.e. to fix the content (in MWyrs or GWyrs) in relation to the generation capacity (in MW or GW), as it is usually the case with batteries.

The other possibility, which could, e.g., be useful for pumped hydropower storage plants, is to keep them separate with their own costs and leave the relation of the two open for the optimization process.

10.6 Lag Times Between Input and Output of a Technology

Since MESSAGE can be used for very short time steps, even for steps of 1 year per period, the implementation of lag-times between input and output of a conversion technology seemed to be appropriate. One possible application are the reprocessing units for nuclear fuels, which usually keep the fuels for several years.

The lag time for a technology (keyword `lag`) is given in years and the period in which the output is available is calculated beginning from the middle of the period when the input is required.

10.7 Variable Inputs and Outputs

A lot of power plants can use different fuels for electricity generation, the highest variability occurs between oil products and natural gas as fuel. This can be modeled by having two or even more energy conversion variables with different inputs, efficiencies and variable operation and maintainance costs linked to one capacity in one capacity equation (definition using keywords `adda` and `activity`; see also section 2.2.1).

The same link of different conversion activities can be used to model co-generation of electricity and heat with a variable output pattern. In this case one of the conversions would be to electricity (with an efficiency ϵ_e) and the other one producing a mix of electricity and the maximal possible share of district heat (producing ϵ_c electricity and δ_c district heat from one unit of input). In the latter case the efficiency to electricity (ϵ_c) is lower than in the first case (ϵ_e), but the overall efficiency is naturally much higher. The two conversion variables have to be related to the same capacity by a factor giving the relative production of the main product possible with one unit of installed capacity, which is always related to the first operation mode. In the terms used above this would mean that the plant can produce ϵ_e electricity in the first operation mode, while it can produce ϵ_c with the same capacity in the second operation mode. For the model this means that the electric capacity is not utilized fully in the second mode, the relation has to be defined by the user. (It would be ϵ_e / ϵ_c in the described case, but could also be independent from the efficiencies for other technologies.)

10.8 The Contribution of Capacities Existing in the Base Year

The possible contribution of an installation that exists in the base year is kept track of over time. There are two possibilities to give the necessary information to MESSAGE.

1. Define the capacities that were built in the years $iyr, ..., iyr - \tau + 1$, with iyr = base year and τ = plant life in years explicitly. These capacities are then distributed to historic periods of the length ν.

2. Define the total capacity, c_0, that exists in iyr and the rate at that it grew in the last τ years, γ. This information is then converted to one similar to 1. by using the function:

$$y_0 = c_0 \frac{\gamma^{-\nu} - 1}{\nu(\gamma^{-\tau} - 1)} ,$$

$$y_t = y_0 \, \gamma^{-t \times \nu} \quad , t = 1(1)\frac{\tau}{\nu} ,$$

where

y_t	is the annual construction in period $-t$, (0 = base year),
γ	is the annual growth of new installations before the base year,
c_0	is the total capacity in the base year,

τ is the plant life, and

ν is the length of the periods in that the time before the base year is divided.

The right hand sides in the capacity constraints are derived by summing up all the old capacities that still exist in a certain period (according to the plant life). If the life of a technology expires within a period, MESSAGE takes the average production capacity in this period as installed capacity (this represents a linear interpolation between the starting points of this and the following period).

10.9 Capacities which Operate Longer than the Time Horizon

If a capacity of a technology is built in one of the last periods its life time can exceed the calculation horizon. This fact is taken care of by reducing the investment costs by the following formula:

$$C_t^r = C_t \times \frac{\sum_{k=1}^{\tau_p-\nu} \prod_{\tau=t}^{t+k-1} \frac{1}{1+dr_\tau}}{\sum_{k=1}^{\tau_p} \prod_{\tau=t}^{t+k-1} \frac{1}{1+dr_\tau}}$$

where

ν is the number of years the technology exists after the end of the calculation horizon,

dr_τ is the discount rate for year τ,

τ_p is the plant life in years,

C_t is the investment cost in year t, and

C_t^r is the reduced investment.

10.10 Own-Price Elasticities of Demand

Own-price elasticities of demand can be interpreted either as short-term elasticities resulting in reduced demand due to sharp price increases (they have to relate to a reference price- and demand level and represent renunciation of services) or as long-term elasticities reached by substituting capital for energy. In the latter case the user has to asure that the relatively decreased demand level is maintained over the calculation horizon by applying user-defined relations (see section 8). The costs and levels of demand reduction can be derived from the investments and savings that are associated to certain additional installations, like, e.g., three-glass windows to save in space heating.

The form of the own price elasticity function of demand is

$$\frac{Q}{Q_r} = \left[\frac{P}{P_r}\right]^{\epsilon} ,$$

where

Q_r	is the reference demand level,
P_r	is the reference price level, and
ϵ	is the elasticity, (assumed to be < 0).

It says that the demand will decrease by a factor of x^{ϵ} if the price rises by x. This function is approximated by a step-function of the following form:

The demands (Q) and prices (P) are normalized to the reference levels:

$$Q = q \times Q_r ,$$

and

$$P = p \times P_r ,$$

the normalized values follow the function

$$q = p^{\epsilon} ,$$

or

$$p(q) = q^{\frac{1}{\epsilon}} .$$

To reduce the demand to the level q_i the supply has to have the cost

$$c(q_i) = \int_{q_i}^{1} q^{\frac{1}{\epsilon}} \, dq = \frac{1}{1+\frac{1}{\epsilon}} \times \left[1 - {q_i}^{1+\frac{1}{\epsilon}}\right] ,$$

a function increasing monotonously with decreasing q_i (see also Figure 10.3). In absolute terms this means that the cost would be higher by an absolute value of

$$R(Q_i) = c(\frac{Q_i}{Q_r}) \times Q_r \times P_r$$

compared to the cost at the reference level.

The step-function is then defined by choosing certain levels of demands and prices (Q_i, P_i), $i = 1(1)n$ with $Q_i < Q_r$, that fulfil the elasticity function. The code can choose, which demand level it supplies, but if it supplies a level $Q_i < Q_r$ it has to pay addititionally $R(Q_i)$, the cost of reducing the demand to level i.

Figure 10.3: Representation of Demand Elasticities

10.11 Supply Elasticities

The reaction of the market prices to changes in demand can be expressed as elasticities:

$$\frac{P}{P_r} = \left[\frac{S}{S_r}\right]^{\alpha},$$

where

P_r	is the reference price level,
S_r	the reference supply level, and
α	the elasticity.

The normalized form of this equation is

$$c = s^{\alpha},$$

where

$c = \frac{P}{P_r}$,

and

$$s = \frac{S}{S_r}.$$

The relationship is converted to a step-function with n steps, which is shown in Figure 10.4. $f(s_1)$ is the cost of supplying amount s_1 relative to supplying s_r, while $f(s_1) + (s_2)$ ist the relative cost of supplying the amount s_2. The marginal costs are then defined as

$$\mu(s) = \frac{\int_{s_{i-1}}^{s_i} \sigma^\alpha \, d\sigma}{s_i - s_{i-1}} ,$$

where
$s_{i-1} < s \leq s_i$

Figure 10.4: Representation of Supply Elasticities

According to the normalized function the total price of buying the amount s is then

$$tc(s) = \sum_{j=1}^{i-1} \mu(s_j) \times (s_j - s_{j-1}) + \mu(s_i) \times (s - s_{i-1}) .$$

The price of the amount S, $S_{i-1} < S < S_{i,}$ is defined as

$$TC(S) = P_r \times S_r \times tc(\frac{S}{S_r}) .$$

In the matrix this function is implemented as $n+1$ additive elasticity classes for resources and imports ($R_0 = S_r, R_i = S_i - S_{i-1}$, $i = 1(1)n$), which have increasing costs. The code takes these classes as supply one after the other and has to pay incrasing prices, then.

10.12 The Mixed Integer Option

If the LP-package used to solve a problem formulated by MESSAGE has the capability to solve mixed integer problems, this can be used to improve the quality of the formulated problems, especially for applications to small regions.

The improvement consists in a definition of unit sizes for certain technologies that can only be built in large units. This avoids for instance the installation of a 10 kW nuclear reactor in the model of the energy system of a city or small region (it can only be built in units of e.g., 700 MW). Additionally this option allows to take care of the "economies of scale" of certain technologies.

This option is implemented for a technology by simply defining the unit size chosen for this technology (keyword `cmix`). The according capacity variable is then generated as integer in the matrix, its value is the installation of one powerplant of unit size.

If a problem is formulated as mixed integer it can be applied without this option by changing just one switch in the general definition file (keyword `mixsw`). Then all capacity variables are generated as real variables.

10.13 Nonlinear Objective Functions

In combination with MINOS MESSAGE can be applied to problems with a partly nonlinear objective function or with nonlinear constraints. The requirements are that the functions are differentiable and convex with respect to the solution space.

In order to use a nonlinear objective or nonlinear constraints the user has to identify the variables that are to be included with nonlinear coefficients in the input file (keyword `nonl`; they will be written to the matrix as first entries in the columns section–as required by MINOS) and to supply MINOS with an additional subroutine (`Funcon` for nonlinear constraints and `Funobj` for nonlinear objective gradients), which yields the nonlinear part of the constraints or objective and the first derivatives as required by MINOS.

In order to start a nonlinear problem it can be solved as linear problem in the beginning. The nonlinear variables can be fixed to user-defined estimates by specifying "initial bounds" in the bounds section.

The order in that the nonlinear variables appear in the input file is essential, because the same order is used for identifying them in `Funcon` and in `Funobj`. MESSAGE generates the activity variables first, then the capacity variables (both of them in the order in that the technologies appear in the input file). The loops in producing the columns are nested in the following order:

– load regions,

– demand elasticity classes, and

– time periods.

10.14 Multiobjective Optimization

MESSAGE is capable of handling two types of multiobjective optimization: It can generate weights on different types of activities or prepare the MPS-file for using the Reference Trajectory Optimization Method.

Weights on Activities

The common way to optimize several objectives at the same time is to define weights for the different types of activities in the model. MESSAGE provides an easy way to do this: It is possible to define costs that are added to the objective gradients of all technologies that have coefficients in a specific additional relation (see chapter 8), e.g. all technologies emitting SO_2 could get some addition to the objective gradient, if this addition is defined for the relation accounting for the emissions of SO_2.

Alternatively additive and multiplicative weights for all activities considered in the "Cost Accounting Rows" (see section 9.1.1) can be defined. As an example, additional costs (taxes) put on energy imports could be imposed this way.

The Reference Trajectory Method

The "Reference Trajectory Optimization Method"[1] is an approach to optimize more than one objective function for a problem in a way that circumvents the necessity to define weights on the single objectives. It allows to define *reference trajectories* for all objectives; the solution will lie on the "pareto"-optimal border of the feasible region and be as close as possible to all reference trajectories.

The way in which the pareto-optimal border is approached can either be problem oriented with an egalitarian approach between the objectives, or aspiration oriented, meaning that objectives with a reference point that is closer to the overall optimum (the UTOPIA[2] point), get higher influence on the solution.

The objectives can be grouped to nodes, for each of which a multiobjective approach is taken. The nodes are just summed up in the objective function. This feature is useful if interconnected models with separate objectives are depicted in one physical model.

[1] This method is based on the "Reference Point Optimization Method, that has been developed at IIASA and is implemented in the DIDASS system.

[2] The UTOPIA point is the combination of all single-objective optima into one overall optimum, which is usually not in the feasible region.

Mathematically the objective functions are summed into a single function that minimizes the maximum difference between the reference trajectory and the actual value of the function for each time step. The difference is calculated using the Chebychev norm of the two points (reference point and actual value). The time steps are usually handled like the nodes, i.e. each point in time has a single objective. Alternatively the time steps can be included in one objective, which means that the compromise solution is searched over all objectives and time steps at once. This algorithm may lead to unrealistic results, since the dynamics of the model may not be handled adequately.

The aggregated objective function has the following general form:

$$\min \sum_n \sum_t \left[\left(\max_{j \in J_n} \sigma_j \frac{y_{j,t} - \bar{y}_{j,t}}{\alpha_{j,t}} \right) + \epsilon \sum_{j \in J_n} \sigma_j \frac{y_{j,t} - \bar{y}_{j,t}}{\alpha_{j,t}} \right]$$

where

n	is the index for the nodes,
t	is the index for the time steps,
J_n	is the set of objectives in node n,
j	is the index for the objectives,
σ_j	is a scaling factor to improve numerical stability (all objectives should have the same order of magnitude to avoid rounding-off errors),
$y_{j,t}$	is the actual value of objective j in time step t,
$\hat{y}_{j,t}$	is the reference point for objective j in time step t,
$\alpha_{j,t}$	is the scaling factor for objective j in time step t, it represents the way the pareto-optimal border is approached, and
ϵ	is a small number to drive th solution algorithm in the right direction.

If all time steps are to be aggregated into one objective, the sum over t is added to the sums over the variables instead of being outside the maximum.

The scaling factors are generated depending on a criterion regarding the way in which the absolutely optimal (and probably infeasible) point is approached:

Problem oriented scaling: $\alpha_{i,t} = \frac{1.}{\bar{y}_{i.t}}$

Aspiration oriented scaling: $\alpha_{i,t} = \frac{1.}{u_{i,t} - \bar{y}_{i.t}}$

where

$u_{j,t}$	is the optimal value for objective j in time step t with single-objective optimization. The respective values for all objective constitute the UTOPIA point.