Developing countries' problem of investments endangered by natural catastrophes: Optimal investment and insurance policies

Paul Kenneth Freeman, Leslie Aimée Martin, Georg Christian Pflug

1 Introduction

During the past decade, the cost of natural disasters in developing countries is increasing at an accelerating rate. Since the 1960's, the frequency of severe natural disasters have increased by a factor of three and the direct economic costs have increased by a factor of nine. The annual direct costs to the developing world from natural disasters now averages USD 35 billion a year. Much of the direct costs are in losses to government owned infrastructure. Historically, the international finance institutions have been the source of reconstruction financing. In particular, the World Bank plays an essential role in arranging financing for damage to critical infrastructure. Over the past 20 years, over 50 countries worldwide have relied on post disaster reconstruction lending from the World Bank to rebuild infrastructure. During this period, the World Bank has provided USD 14 billion in reconstruction lending, nearly 2.5 times the amount provided for reconstruction work following civil disturbances. Often, the World Bank plays the role as lead lender for reconstruction lending, setting the terms and conditions for credits provided by other multilateral financial institutions and bilateral lenders. In this way, the role of the World Bank is more important than may be indicated by the size of its post disaster lending. The increasing dependence of countries on post disaster borrowing for reconstruction has stimulated both the international financial institutions and a number of developing countries to explore the viability of alternatives to the existing dependence on post disaster borrowing. The most recent outcome of that exploration is the creation of an earthquake insurance program for residential property in Turkey. This program was created with a major new credit from the World Bank to Turkey as a component of the financing arranged after the devastating 1999 earthquake. Proposals to deal with the risk from natural disasters have also been recently developed for Mexico. An obstacle to examining alternatives to post disaster financing of natural disaster losses is a theory to support government use of risk shifting financial instruments. Generally, governments are viewed as risk neutral economic agents. As such, there is no economic benefit for governments to pay more than expected loss to a third party to assume a portion of a governments risk of loss on its investments. Since any party assuming responsibility for a future loss will expect to be paid more than the expected value of the loss, no basis exists to shift the risk of loss from a government to a third party. Risk shifting only works for risk averse parties willing to pay more than expected loss. For risk averse parties, they measure the benefit of risk shifting to their risk aversion. The higher their risk aversion, the more they should be willing to pay for risk transfer. This paper constructs a decision model for developing countries with a risk of loss from natural disasters to evaluate the potential benefit of insuring a portion of their expected loss. The paper contains two fundamental assumptions. The first assumption is that governments have limited budgets that can be spent on infrastructure. The budgeted amount may be acquired by the country either through internal or external sources. Regardless of how acquired, the amount available is limited and has a cost attached to its acquisition. This assumption varies from existing thinking about risk and government investment policy. Generally, the assumption is that the cost of risk in the hands of governments from investment decisions is zero. This occurs because governments have the ability to pass the cost of risk to each taxpayer through its taxation power. Since the cost of risk in each taxpayers hand is small, approaching zero as the number of taxpayers increase, governments should disregard risk in making investment decisions. It is unlikely that this assumption holds true for developing countries. Rather, the amount that governments can raise through taxes is severely limited. As the size of risk increases, the ability of the government to absorb the risk is constrained. The inability of developing governments to absorb risk through its power of taxation leads to its dependence on post disaster borrowing to finance its risk of loss to infrastructure. Of course, the amount available for governments to borrow is, or at least should be, constrained. The recent highly indebted poor country initiative has demonstrated the consequences of unsustainable external debt for countries. The assumption of constrained budgetary resources for capital expenditures seems reasonable. The second assumption that arises from the analysis is the concept of higher marginal returns from first available funds for post disaster reconstruction. The phenomena has been observed in evaluating the economic consequences of natural disasters in developing countries. The reasons for this phenomena most likely arises from the benefit of "sunk costs". Basically, natural disasters often destroy only a component of essential infrastructure. The cost of restoring full production of the damaged infrastructure is less than the cost of the original installation of the infrastructure. Roads are a good example. Flooding may destroy road surfaces, which need to be replaced. But the cost of land acquisition to originally build the road need not be spent again. It is a "sunk cost" that need not be calculated in determining the productivity of post disaster reconstruction. Consequently, immediate post disaster funds have higher productivity gains than funds invested in new projects. The model created in this paper captures those gains. The paper proceeds by creating a two stage stochastic optimisation problem. The two stages are the ex-ante budget allocation decision and the ex-post reconstruction and budget allocation decision. The model proceeds on the basis that the government has a limited budget available for supporting its infrastructure investment decision. The budget may be spent on new construction, insurance, or post disaster reconstruction. From its portfolio of infrastructure investments, the government anticipates an annual expected return. The infrastructure is in geographically defined regions, and subject to loss from natural disasters. The losses between assets are correlated.

2 The model

We model the decision problem for the government as a two-stage stochastic optimization problem. The two stages are

- the ex-ante budget allocation decision and
- the ex-post reconstruction and budget reallocation decision.

Here ex-ante refers to the planning period before a possible cat-event happens and ex-post refers to all recourse actions to be taken after the occurrence of an event.

To make the model simple, we assume that the ex-ante decision is taken at time 0, then an observation period of length 1 follows, in which at most one cat-event may happen and then the ex-post decision is made at time 1. The insured stock is immediately reconstructed after the event.

The initial amount of infrastructure stock is X_0. There is a budget B allocated, which may be used for investment or for insurance or for any combination of both. Let x the amount to be invested and z the amount of stock to be covered by insurance. The cost for this insurance is Pz, where P is the insurance premium factor. This factor equals $P = E(1+V)$, where E is the expected proportion of damage and V is the risk premium. The budget equation is

$$x + Pz = B \tag{1}$$

The total stock after investment is $X_0 + x$. The amount covered by insurance must be less than the total stock, leading to the inequality

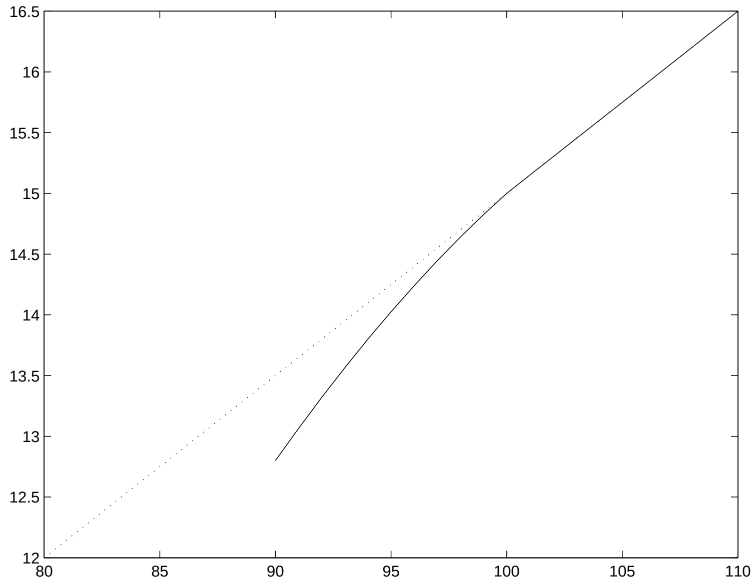
$$0 \leq z \leq X_0 + x. \tag{2}$$

The cat-event destroys a random portion ξ of this stock. The destroyed stock $(X_0 + x)\xi$ is insured for a fraction of $\xi z/(X_0 + x)$. We assume proportional insurance. Therefore, the total stock after the reconstruction by insurance is $(X_0 + x)(1 - \xi) + z\xi$ and the net loss in stock is $D = (X_0 + x - z)\xi$.

We now discuss modeling the return functions. Suppose that stock X gives (discounted future) return of $R_0(X)$. If an amount of D is destroyed as the consequence of a cat-event, then the return drops to a value $R_0(X) - R(D)$. Typically, $R_0(X) - R(D) < R_0(X - D)$ for $D > 0$.

We assume that the return function for undestroyed stock is linear, i.e. $R_0(X) = R_0 \cdot X$, while we assume the R is strictly monotonic, convex, nonlinear, differentiable and satisfies $R(0) = 0$. A typical example is

$$R(D) = R_1 \cdot D + R_2 \cdot D^2. \tag{3}$$



The return function: A cat-event lets the return drop much below the historic return curve

2.1 The single stage model

The objective of the decision problem is to maximize the expected return. This means that we look for the following problem:

$$\begin{aligned} &\text{Maximize } R_0 \cdot (X_0 + x) - \mathbb{E}[R((X_0 + x - z)\xi)] \\ &x + Pz = B \\ &z \leq X_0 + x \\ &x, z \geq 0. \end{aligned} \tag{4}$$

We introduce the following quantities: The maximal possible stock A

$$A = X_0 + B \tag{5}$$

and the unprotected stock w

$$w = X_0 + x - z = A - (P + 1)z. \tag{6}$$

The unprotected stock w is the new decision variable. We may express x and z in terms of w:

$$z = (A - w)/(P + 1), \tag{7}$$

$$x = B - P(A-w)/(P+1) = \frac{P}{P+1}\left[w - (X_0 - \frac{B}{P})\right]. \quad (8)$$

The constraints $x \geq 0$, $z \geq 0$ and $z \leq (X_0 + x)$ translate into

$$[X_0 - B/P]^+ \leq w \leq A. \quad (9)$$

The optimization problem reads

$$\begin{aligned} &\text{Maximize } (R_0 \cdot [A/(P+1) + wP/(P+1)] - \mathbb{E}[R(w\xi)] \\ &[X_0 - B/P]^+ \leq w \leq A. \end{aligned} \quad (10)$$

Let $S(w) = \mathbb{E}[R'(w \cdot \xi)\xi]$. S is monotonic and satisfies $S(0) = R'(0)E$. Let

$$\bar{w} = \begin{cases} 0 & \text{if } S(0) > R_0 P/(P+1) \\ \infty & \text{if } S(\infty) < R_0 P/(P+1) \end{cases}$$

In all other cases let $\bar{w}$ such that $S(\bar{w}) = R_0 P/(P+1)$. The solution of the optimization problem (10) is

$$w^* = [\bar{w}]^A_{[X_0 - B/P]^+} \quad (11)$$

where

$$[x]^b_a = \begin{cases} a & \text{if } x < a \\ x & \text{if } a \leq x \leq b \\ b & \text{if } b < x \end{cases}$$

One sees immediately that the amount x^* which should be invested in stock increases with increasing P and decreasing R_0, but decreases with increasing R'.

Let us look for the case that R is quadratic, i.e.

$$R(D) = R_1 \cdot D + R_2 \cdot D^2. \quad (12)$$

We get $S(u) = R_1 E + 2R_2 u \mathbb{E}[\xi^2]$. If $R_1 E \geq \frac{R_0 P}{P+1}$, then $\bar{w} = 0$. Otherwise $\bar{w} = \frac{PR_0 - R_1 E(P+1)}{2R_2 \mathbb{E}[\xi^2](P+1)}$. The optimal w is

$$w^* = \begin{cases} [X_0 - B/P]^+ & \text{if } \bar{w} < [X_0 - B/P]^+ \\ \min(\frac{PR_0 - R_1 E(P+1)}{2R_2 \mathbb{E}[\xi^2](P+1)}, A) & \text{otherwise} \end{cases}$$

If $R_2 = 0$, then only the two extreme cases $w^* = [X_0 - B/P]^+$ (insure all) or $w^* = A$ (insure nothing) may happen.

Example:

We suppose that ξ has a the power distribution $p = 49$ (see later), resulting in $\mathbb{E}(\xi) = 1/(p+1) = 0.02$ and $\mathbb{E}(\xi^2) = 1/(2p+1) = 0.0101$. Moreover, we use the following parameter settings

R_0	0.15
R_1	0.17
R_2	0.005
X_0	100
B	20
E	0.02
P	0.024

The above formula results in $w^* = 1.14$. That gives $z^* = (A - w^*)/(P + 1) = 116.07$ and $x^* = w^* - X_0 + z^* = 17.21$. Thus the best decision is to invest 17.21 of the available budget of 20 in investments and the remaining sum of 2.79 for insurance of the stock of 117.21.

2.2 The two-stage model

The single stage model does not consider possible corrective actions to be taken after the occurrence of an event. We design therefore a two stage model for a more realistic representation of the decision makers options.

In the second stage, i.e. after the possible occurrence of an disaster, the decision is to be made whether some sources of money other than the insurance will be used to do more reconstructions.

Suppose that y is the amount to be used for reconstruction. This money may come from donations or from the taxpayer or from other sources. If it comes from donations, it comes at no costs. In all other cases, there is some cost associated with it. These costs have to be put in relation to the benefits.

Of course, we cannot reconstruct more than was damaged, i.e.

$$y \leq D.$$

The allocation of additional money causes (discounted future) costs of magnitude $K(y)$. We assume that K is a convex strictly monotonic function. The second-stage cost-benefit analysis tells us that $y = y(D)$ has to be chosen in order to minimize the following quantity

$$\text{Minimize } \{R_1(D - y) + K(y) : 0 \leq y \leq D\} \tag{13}$$

Let $y^*(D)$ be the minimizer of (13). We assume that it is unique.

Using the previous notation, we formulate the two-stage stochastic optimization problem as a single stage program

$$\begin{array}{l} \text{Maximize } R_0 \cdot (X_0 + x) - \mathbb{E}[R_1(D - y^*(D)) + K(y^*(D))] \\ D = (X_0 + x - z)\xi \\ x + Pz = B \\ x, z \geq 0 \end{array} \tag{14}$$

Substituting again x and z by w we arrive at a simple, one dimensional optimization problem

$$\begin{array}{l} \text{Maximize } R_0[A/(P+1) + wP/(P+1)] - \mathbb{E}[R_1(w\xi - y^*(w\xi)) + K(y^*(w\xi))] \\ [X_0 - B/P]^+ \leq w \leq A. \end{array} \tag{15}$$

2.3 Piecewise linear K and quadratic R_1

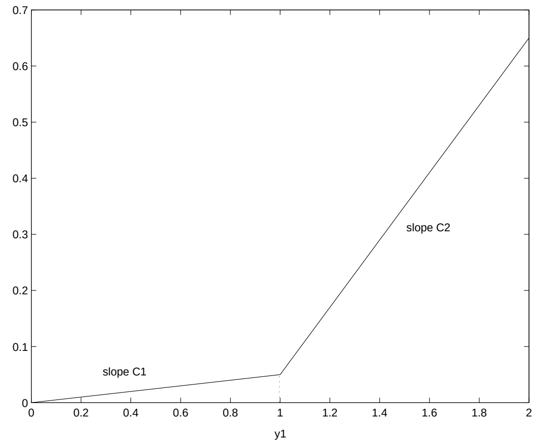
In order to get more concrete results, we assume in this section that R is quadratic, i.e.

$$R(D) = R_1 \cdot D + R_2 \cdot D^2.$$

like before and that K is piecewise linear with two pieces

$$K(y) = \begin{cases} C_1 y & \text{for } 0 \leq y \leq y_1 \\ C_2 y + (C_1 - C_2) y_1 & \end{cases} .$$

This models a situation, where some extra money may come for no ($C_1 = 0$), or low costs, but beyond this limit raising extra money is quite expensive ($C_2 > C1$). The function K is shown below



The ex-post cost function K

We calculate the optimal value of the second stage problem (13). Assume that $C_1 < R_1 < C_2$. Let

$$y_2 = y_1 + \frac{C_2 - R_1}{2R_2}.$$

After an easy calculation one finds

$$y^*(D) = \begin{cases} D & \text{if } 0 \le D \le y_1 \\ y_1 & \text{if } y_1 \le D \le y_2 \\ D - y_2 + y_1 & \text{if } y_2 \le D. \end{cases}$$

or – in different notation –

$$y^*(D) = D - [D - y_1]^+ + [D - y_2)^+.$$

It follows that $K(y^*(D)) = C_1 \min(D, y_1) + C_2[D - y_2]^+$.

Therefore setting

$$L(D) = R(D - y^*(D)) + K(y^*(D)) \tag{16}$$

we get

$$L(D) = \begin{cases} C_1 D & \text{if } 0 \le D \le y_1 \\ C_1 y_1 + R_1(D - y_1) + R_2(D - y_2)^2 & \text{if } y_1 \le D \le y_2 \\ C_2 D + C_3 & \text{if } y_2 \le D. \end{cases}$$

where

$$C_3 = C_1 y_1 - C_2 y_2 + R_1(y_2 - y_1) + R_2(y_2 - y_1)^2.$$

The full optimization problem is

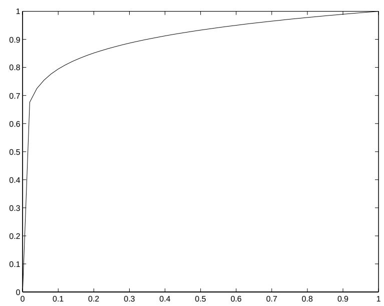
$$\begin{array}{l} \text{Maximize } R_0[A/(P+1) + wP/(P+1)] - \mathbb{E}[L(w\xi)] \\ [X_0 - B/P]^+ \le w \le A. \end{array} \tag{17}$$

2.4 The power distribution family

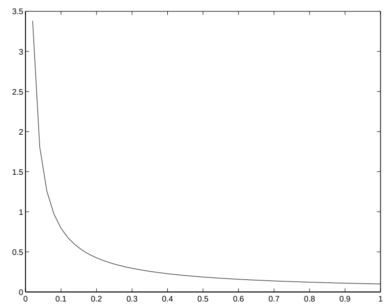
Consider the following class of distribution functions:

$$F(v) = \begin{cases} v^{1/p} & \text{if } 0 \le v \le 1 \\ 0 & \text{if } v < 0 \\ 1 & \text{if } v > 1 \end{cases}$$

and call this class "power distributions". Another name is the Beta($1/p$,1)-distributions. The expectation of F is $\frac{1}{p+1}$ and one may set $p = \frac{1-E}{E}$, if only E is given.



The cumulative d.f.($p = 10$)



The density ($p = 10$)

If ξ is distributed according to F, then the $\mathbb{E}(L(w\xi))$ can be calculated, where L is given by (16). After some tedious calculation, we get for $\mathcal{L}(w) = \int L(w \cdot u)\, dF(u)$.

If $w \le y_1$, then

$$\mathcal{L}(w) = \frac{wC_1}{p+1}.$$

If $y_1 \le w \le y_2$, then

$$\begin{aligned} \mathcal{L}(w) &= \frac{C_1 y_1}{p+1}\left(\frac{y_1}{w}\right)^{1/p} + C_1 y_1 [1 - \left(\frac{y_1}{w}\right)^{1/p}] \\ &+R_1[\frac{w}{p+1}\left(1 - \left(\frac{y_1}{w}\right)^{(p+1)/p}\right) - y_1\left(1 - \left(\frac{y_1}{w}\right)^{1/p}\right)] \\ &+R_2[\frac{w^2}{2p+1}\left(1 - \left(\frac{y_1}{w}\right)^{(2p+1)/p}\right) - \frac{2wy_1}{p+1}\left(1 - \left(\frac{y_1}{w}\right)^{(p+1)/p}\right) \\ &+y_1^2\left(1 - \left(\frac{y_1}{w}\right)^{1/p}\right)]. \end{aligned}$$

If $y_2 \leq w$, then

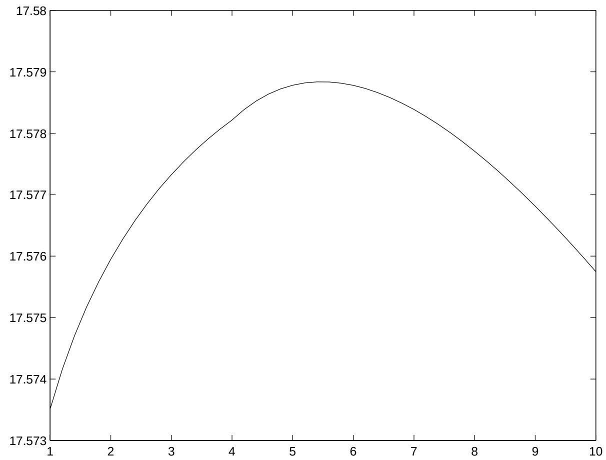
$$\begin{aligned}
\mathcal{L}(w) &= \frac{C_1 y_1}{p+1}\left(\frac{y_1}{w}\right)^{1/p} + C_1 y_1 [\left(\frac{y_2}{w}\right)^{1/p} - \left(\frac{y_1}{w}\right)^{1/p}] \\
&+R_1[\frac{w}{p+1}\left(\left(\frac{y_2}{w}\right)^{(p+1)/p} - \left(\frac{y_1}{w}\right)^{(p+1)/p}\right) - y_1\left(\left(\frac{y_2}{w}\right)^{1/p} - \left(\frac{y_1}{w}\right)^{1/p}\right)] \\
&+R_2[\frac{w^2}{2p+1}\left(\left(\frac{y_2}{w}\right)^{(2p+1)/p} - \left(\frac{y_1}{w}\right)^{(2p+1)/p}\right) - \frac{2wy_1}{p+1}\left(\left(\frac{y_2}{w}\right)^{(p+1)/p} - \left(\frac{y_1}{w}\right)^{(p+1)/p}\right) \\
&+y_1^2\left(\left(\frac{y_2}{w}\right)^{1/p} - \left(\frac{y_1}{w}\right)^{1/p}\right)] \\
&+\frac{C_2 w}{p+1}[1 - \left(\frac{y_2}{w}\right)^{(p+1)/p}] + C_3[1 - \left(\frac{y_2}{w}\right)^{1/p}].
\end{aligned}$$

Here $C_3 = C_1 y_1 - C_2 Y_2 + R_1(y - 2 - y_1) + R_2(y_2 - y_1)^2$.

Example: We continue the example of section 1 adding the following quantities:

C_1	0
y_1	1
C_2	0.3

This means that one unit comes for free, other sources cost 30%. Then $y_2 = 4$. The objective function is plotted below. There is a maximum at $w^* = 5.5$. Therefore $z* = 111.82$ and $x^* = 17.32$. The optimal strategy is to invest 17.32 out of the available budget of 20 and use 2.68 for insurance. Since there is a chance for getting free money, the part of the budget that goes to insurance is smaller than in the comparable single stage case.



The expected return as a function of w

3 Why governments are not risk neutral for nondifferentiable returns

Let us recall Arrow's argument by citing the following Proposition

Proposition. *If R is a differentiable return (utility) function, satisfying $R(u) \le K(1 + |u|)$ and X is a random variable with zero expectation, then for all a*

$$\lim_{n \to \infty} n\mathbb{E}[R(a + X/n) - R(a)] = 0.$$

The proof uses the fact that $n[R(a + X/n) - R(a) - R'(a)X] \to 0$ a.s. and the dominated convergence theorem. This theorem is often taken as the basis for the argument that governments should be risk-neutral, since dividing the risk X among n individuals lets the sum of all individual risk premiums go to zero as n increases.

Small countries may not be able to spread risks because of there size (smaller n). But more remarkably, the argument does also fail in situations in which the return function has a kink. This is exactly the situation for cat-risks, as was pointed out.

The result for return with kink reads: If R is concave, but nondifferentiable at a, we get instead:

Proposition. *Let R be a return (utility) function, satisfying $R(u) \le K(1 + |u|)$. Let further $R^+(a) = \lim_{s \downarrow 0} \frac{1}{s}[R(a + s) - R(a)]$ and $R^-(a) =$*

$\lim_{s\uparrow 0} \frac{1}{s}[R(a+s) - R(a)]$. *Then*

$$\lim_{n\to\infty} n\mathbb{E}[R(a+X/n) - R(a)] = R^{+}\mathbb{E}[X^{+}] - R^{-}\mathbb{E}[X^{-}].$$

Proof. We have

$$\begin{aligned} & n\mathbb{E}[R(a+X/n) - R(a)] \\ = \;& n\mathbb{E}[(R(a+X/n) - R(a))\mathbb{1}_{\{X>0\}} + (R(a+X/n) - R(a))\mathbb{1}_{\{X<0\}}] \\ = \;& n\mathbb{E}[R^{+}X/n\mathbb{1}_{\{X<0\}}] + n\mathbb{E}[R^{-}X/n\mathbb{1}_{\{X<0\}}] + o(1) \\ = \;& R^{+}\mathbb{E}[X^{+}] - R^{-}\mathbb{E}[X^{-}] + o(1) \end{aligned}$$

The cumulative risk premium does not disappear, but tends to a non-negative limit.

4 Conclusions

We have presented a model for the decision process of cat-risk protection for governments. The model contains the ex-ante as well as the ex-post decision process. It is argued that knowing the possible amount for ex-post donations and the costs for diverting funds and tax revenue to the cat damage relief, an optimal mix of ex-ante protection and ex-post protection can be found. There is no way of simply spreading the risk among the population. Firstly, be cause this risk spreading comes at high costs, but secondly also because of the asymmetricity of the marginal return function.

References

[1] Arriens W.T.L.; Benson Ch. (1999). Post Disaster Rehabilitation: The Experience of the Asian Development Bank. Paper before IDNR-ESCAP Regional Meeting for Asia: Risk Reduction and Society in the 21st Century, Bangkok, 23 – 26 February 1999.

[2] Arrow K.J. (1971). Essays in the thory of risk-bearing. Markham, Chicago.

[3] Dong W., Shah H., Wong F. (1996). A Rational Approach to Pricing of Catastrophe Insurance. Journal of Risk and Uncertainty (Kluwer) 12, 201 – 218.

[4] Freeman Paul K. (1999). Natural Catstrophe Risk Transfer and Developing Countries. To appear in: Journal of Environment and Development.

[5] Hodgson, A., (1997). The pricing of catastrophic risk: Market and insurance perspectives. In Britton, N.R. and Oliver, J. (eds.) Financial Risk Management for Natural Catastrophes. Aon Group Australia Limited. Brisbane: Griffith University.

[6] Insurance Services Office Inc. (1999). Financing Catastrophe Risk: Capital Market Solutions,

[7] Major J.A. (1999). Uncertainty in Catastrophe Models, Part I: What it is and where it comes from; Part II: How bad is it? Financing Risk and Reinsurance, International Risk Management Institute.

[8] Pratt J. (1964). Risk aversion in the small and in the large. Econometrica 32, 122 – 136.

[9] Stone J. (1973). A Theory of Capacity and the Insurance of Catastrophe Risks (Part I and II). Journal of Risk and Insurance 40, 231 – 244 and 339 – 355.

[10] Swiss Re (1997). Natural catastrophes and major losses in 1996: High losses from man-made disasters, but no extremely costly losses from natural catastrophes. Sigma, 3. Zurich.

[11] Swiss Re (1997). Too little reinsurance of natural disasters in many markets. Sigma, 7. Zurich.

[12] Swiss Re (1998). Natural catastrophes and major losses in 1997: Exceptionally few high losses. Sigma, 3. Zurich.

[13] Swiss Re(1998). The global reinsurance market in the midst of consolidation. Sigma, 9. Zurich.

[14] Swiss Re (1999). Natural catastrophes and man-made disasters 1998: Storms, hail and ice cause billion-dollar losses. Sigma, 1. Zurich.

[15] Wall, G.R. (1997). Current developments in catastrophe modeling. In Britton, N.R. and Oliver, J. (eds.) Financial Risk Management for Natural Catastrophes. Aon Group Australia Limited. Brisbane: Griffith University.

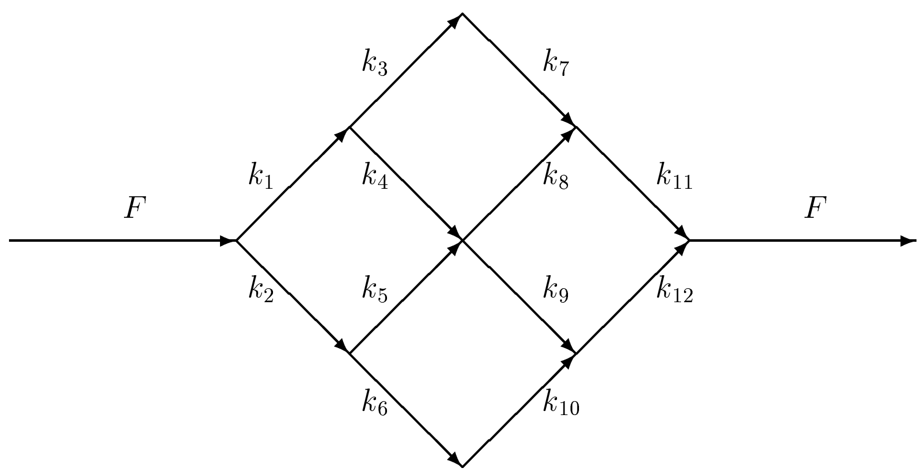
[16] World Bank (1994). World Development Report 1994: Infrastructure for Development. Washington, D.C.: Oxford University Press.

[17] Zeckhauser, R. (1996). The economics of catastrophes. Journal of Risk and Uncertainty, 12, 113-40.

5 Appendix

We illustrate the phenomenon of "kinky" return functions by a simple example from a network optimization problem.

Suppose that a network like the following is given.



A network example: The flow F represents the return. In investments can be made to increase the arc capacities k_i.

The investor may invest in the capacities of the arcs. The capacity k_i of arc i is a function $k_i = k_i(c_i)$ of the amount c_i invested in this arc. The "return" of the investment is the maximal flow through this network.

Suppose that the cost function is like $k_i = \alpha_i c_i + \beta_i c_i^2$, i.e. the marginal costs for arc capacities increase. For each budget B, we may find the optimal investment in every arc, which maximizes the total flow $F = F(B)$ through the network.

Let $c_i^*(B)$ be the optimal investment and $k_i^*(B) = \alpha_i c_i^*(B) + \beta_i c_i^{*2}(B)$ be the optimal capacity for a given budget B. This is a concave return function with decreasing marginal return.

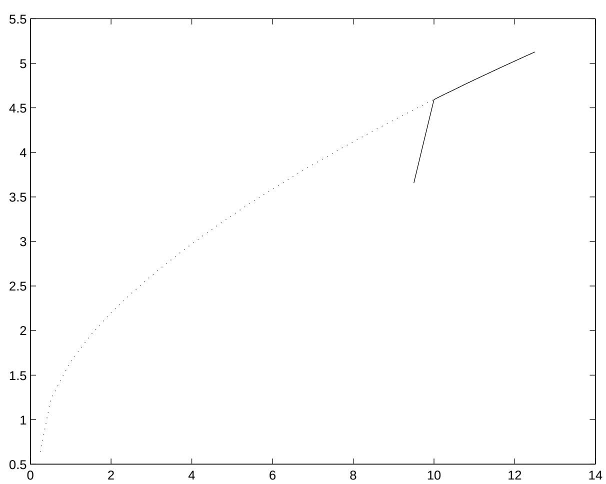
Suppose now that an arc j is chosen at random and is capacity, which was $k_i = \alpha_i c^* i + \beta_i c_i^{*2}$ is decreased to $k_i' = \alpha_i (c^* i - D) + \beta_i (c_i^* - D)^2$. All other capacities remain unchanged. Due to this random shock, the expected maximal flow decreases to $F'(D)$ (say). The expectation is taken w.r.t. the random choice of the arc, which is partially destroyed. It is evident that

$$F'(D) < F(B - D),$$

since the random shock does not take away some marginal capacity at each arc, but strikes the system much harder.

Moreover, the marginal loss in F' is higher than the marginal increase in F. Thus considering a basic budget of B^* and a return function, which equals $F(B)$ for $B > B^*$ and $F'(B^* - B)$ for $B < B^*$, this function has a kink at B^*.

This simple model shows, that assuming that return functions are kinked is very natural. Viewing the infrastructure of a country as a network, the analogy becomes evident.



The form of the return function in the network investment example. The basic investment is $B^* = 10$.