Three applications in evolutionary dynamics

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This research proposal is motivated by ongoing work on evolutionary dynamics jointly developed with Sergio Rinaldi, my PhD Tutor at the Politecnico di Milano, and Régis Ferrière, member of the IIASA Adaptive Dynamics Network. My interest in the subject is recent and has been reinforced by attending the Winterschool on Mathematical Modelling in Ecology and Evolutionary Biology, at Paris.

Background

Ecological interactions among genetically invariant populations easily lead to cyclic or chaotic regimes at the fast ecological timescale By contrast, modelling studies often support the idea that genetical processes tend to stationary regimes. genetical changes are frequently considered separately, occur simultaneously in systems of coevolving populations. In complex cases, ecological and genetical characteristics continue to vary, even when external conditions remain constant (*Red Queen Dynamics*

The main problem in evolutionary dynamics is to derive the equations that describe the time evolution of the populations' genetical traits. These dynamics are the result of the random appearance of a mutant followed by the competition between the mutant and the resident individuals. Since the mutant is rare initially, it must be able to spread at low density, i.e. to invade. But the fate of an initially successful mutant depends on its interactions with the resident population. Either it replaces the former resident, or both the mutant and the resident stay in the system at finite densities, giving rise to a branch in the evolutionary dynamics (*polymorphism*).

Most of the studies developed in the last years focus on cases in which

1. the resident population has a single attractor
2. the attractor is an equilibrium
3. an invading mutant always replaces the former resident

Thus, if successful mutations are small and occur rarely enough, there are monomorphic evolutionary dynamics within species, and the state equations describing such dynamics, are called *monomorphic model*. This monomorphic model is often derived by explicitly taking into account the mutant characteristics, like in but sometime even without taking care of the mutant at all, like in

Research questions and work plan

The research plan is in three parts, each one dealing with a problem in which one of the three above conditions is not satisfied.

Evolutionary dynamics of populations with multiple attractors

Assumption (1) does not always hold, since many polulations have multiple attractors. For this reason we first intend to explore the consequences that such a property may have on evolutionary dynamics. To this aim, we will consider the simplest case in which the population can have two alternative stable equilibria and determine if this condition can trigger cyclic evolutionary dynamics. Technically, this will be done by using the so called *separation principle* a condition for the existence of slow-fast limit cycles.

Evolutionary dynamics of predator-prey systems

The second (major) research topic concerns the evolutionary dynamics of predator-prey systems described, in ecological time, by a couple of ordinary differential equations. Evolutionary dynamics of predator-prey systems have already been studied in terms of simple Lotka-Volterra models Here, more realistically, we assume that the predator has a saturating functional response. Thus, the model is the standard Rosenzweig-Mac Arthur model (logistic prey and Holling type II predator), where parameter values depend upon two phenotypes, one for each species.

The derivation of the monomorphic model is not easy, since for fixed phenotype values, the attractor of the predator-prey model can be a limit cycle, which is not known analytically. In turn, this implies that the monomorphic model cannot be specified analytically. In order to avoid this difficulty, we plan to consider the particular but meaningful case in which preys are much faster than predators in growing and reproducing. This implies that the limit cycle becomes a slow-fast limit cycle which is completely known and, one would hope, allows the derivation of the monomorphic model analytically.

The analysis of the monomorphic model and, in particular, the determination of all its attractors, will be solved numerically by using advanced software for bifurcation analysis The main task of this analysis is to check if under suitable environmental conditions one of the following two properties holds:

(i) coevolution can be a never-ending process

(ii) coevolution can find its end at a point where the two populations are at the edge of their most complex dynamic behavior (a cycle in the present case)

Proving that case (ii) is possible, would be because, on one hand, recent analyses of field data support the idea dynamic behavior, while, on the other hand, ecosistems often hold at the edge of the most complex dynamic behavior

Evolutionary dynamics in information technology

A third (minor) research topic concerns the area of *information technology*, where remarkable new ideas have emerged in the last fifty years. Given a fixed state of science and technology, producers and consumers interact and quickly tend to an equilibrium. But, once in a while, one of the many new ideas go through and produces a technological innovation, which, in turn, entrains a new supply-demand equilibrium. Thus, the appearance of a new idea is like the appearance of a mutant with a new technological trait, and the success of the idea is characterized by the substitution of the old technological trait to a rest, but under to diversified the birth of PC points in the history of information technology.

Some preliminary but rewarding discussions on this problem are certainly possible at IIASA, where various projects are focussing on industrial innovation. Régis Ferrière and Sergio Rinaldi will initiate contacts with researchers in the groups that are potentially interested. If clear signs of interest come from one or more research devoted to the topic.

Envisaged publications

The proposed research is planned to be part of my PhD thesis or, alternatively, be one of my two minors in my PhD program at the Politecnico di Milano. The study on evolutionary dynamics of predator-prey systems will result in a jointly authored paper, hopefully by the end of the YSSP. The studies on populations with multiple attractors and information technology may also lead to independent publications.

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